IN3070
IN4070

Autumn 2023

Exercises for the Course

# Logic

Week 5

---

## Exercise 5.1

**(Substitution and Unification)**

a) Let $\sigma = \{u\backslash y, y\backslash f(a), x\backslash g(u)\}$, $\theta = \{x\backslash f(g(y)), y\backslash u, z\backslash f(y)\}$ and $E = p(x, f(y), g(u), z)$. Compute $\sigma\theta$ using the Proposition on slide 11 of lecture 6. Show that $(\sigma\theta)E = \sigma(\theta(E))$.

b) Unify the following pairs of atomic formulae, i.e. try to find substitutions such that

$$\sigma_1(\, p(a, x, f(g(y)))\,) = \sigma_1(\, p(z, g(f(a)), f(z))\,)\,,$$
$$\sigma_2(\, p(f(y), z, y)\,) = \sigma_2(\, p(z, h(z, u), f(u))\,)\,,$$
$$\sigma_3(\, p(f(x, y), y, g(x))\,) = \sigma_3(\, p(f(v, v), z, z)\,)\,.$$

## Exercise 5.2

**(DNF)**

A formula is in disjunctive normal form (DNF) iff it is a disjunction of conjunctions of literals. Show that every propositional formula can be transformed to an equivalent one in DNF.

## Exercise 5.3

**(Propositional Clause Form)**

Transform the following set of formulae to clause form:

$$\{p, p \to ((q \vee r) \wedge \neg(q \wedge r)), p \to ((s \vee t) \wedge \neg(s \wedge t)), s \to q, \neg r \to t, t \to s\}$$

## Exercise 5.4

**(First-order Clause Form)**

Transform each of the following formulas to clausal form:

$$\forall x(p(x) \to \exists y\, q(y))$$
$$\forall x \forall y(\exists z\, p(z) \wedge \exists u(q(x, u) \to \exists v\, q(y, v)))$$
$$\exists x(\neg \exists y\, p(y) \to \exists z(q(z) \to r(x)))$$

## Exercise 5.5

**(Equivalence vs. Equisatisfiability)**

Skolemisation does not necessarily produce a logically equivalent formula. In this exercise, you prove that the logical consequence between the formula $\forall x \exists y\, A(x, y)$ and its Skolemisation holds in one direction, but not the other.

a) $\forall x\, A(x, f(x)) \models \forall x \exists y\, A(x, y)$

b) $\forall x \exists y\, A(x, y) \not\models \forall x\, A(x, f(x))$.