IN3070
IN4070

Autumn 2023

Exercises for the Course

Logic

Week 7

Exercise 7.1 **(First-Order Resolution)**

Translate the following formulae into a (skolemized) clausal form. Show that the first two formulae are valid and the third formula is invalid by using the resolution calculus.

a) $\forall x\,(q \to p(x)) \to (q \to \forall y\,p(y))$

b) $\forall y\,(p(y) \land q(y)) \to \forall x\,(\forall x\,p(x) \land q(x))$

c) $\forall x\,\exists y\,p(x,y) \to \exists v\,\forall u\,p(u,v)$

Exercise 7.2 **(Clause Subsumption)**

Given two propositional clauses C_1 and C_2, we say that C_1 *subsumes* C_2 if $C_1 \subseteq C_2$.

E.g. $\{p, \neg r\}$ subsumes $\{p, q, \neg r\}$. And the empty clause subsumes every other clause.

The intuition is that if an interpretation satisfies C_1 (i.e. makes one of the literals true) then it certainly also satisfies C_2 which has more literals to choose from. C_1 'says more' than C_2.

Syntactically, if we use the longer clause C_2 in a proof, we would think that it should be possible to use C_1 instead and get a simpler proof… and this is indeed the case!

> Prove that the resolution calculus remains complete if we allow to remove a clause $C_2 \in S$ from S if it is subsumed by a different clause $C_1 \in S$.

Hints:

- *If a node n in a semantic tree falsifies a clause C_2, and $C_1 \subseteq C_2$, can you conclude that n falsifies C_1?*
- *Show that if n is a failure node for a clause set S, then it falsifies some* non-subsumed clause*, i.e. a clause that is not subsumed by any other clause in S.*
- *Conclude that for an unsatisfiable set S not containing the empty clause, there is a resolution step between two* non-subsumed *clauses.*
- *Explain why a clause that is subsumed by another (different) clause in S will continue to be subsumed if we add resolvents to S or remove subsumed clauses.*