



Deadline: 4 October 2023, 23:59

Exercise O1.1

(Validity and Proof Calculi)

Consider the following formulae.

$$F_1: ((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r) \quad (\text{“Transitivity”})$$

$$F_2: \forall x \exists y (p(x) \wedge (p(y) \rightarrow q(x))) \rightarrow \forall z q(z) \quad (\text{“A modus ponens”})$$

$$F_3: ((p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow q) \quad (\text{“Permuted transitivity”})$$

- Prove the validity of formulae F_1 and F_2 in the *sequent calculus* (make sure you respect the Eigenvariable condition when proving F_2).
- Show that formula F_3 is invalid by specifying a counter model using the *sequent calculus*.

Exercise O1.2

(Adding a Logical Operator)

The logical operator \uparrow is defined as follows: $A \uparrow B \equiv \neg(A \wedge B)$.

Extend the LK calculus by rules for the \uparrow operator, such that the operator is supported “natively”, i.e. the premises/assumptions of the new rules should only contain A , B , and $A \uparrow B$ (and no additional logical operators, such as \neg or \wedge should be used or introduced).

- Specify the two new rules \uparrow -left and \uparrow -right that have to be added to the rules of the *sequent calculus LK* as defined in Lecture 2 (slides 35 and 36).
- There is a central part of the soundness proof for propositional LK, where a property is shown separately for each of the rules of the calculus.

What is this property? Show that it holds for your two rules.