

# Mandatory Assignment 3, IN3130, 2021

**Deadline: TUESDAY November 8, at 23:59**

## **Comments concerning this mandatory assignment:**

- This assignment consists of two exercises, and each student must turn in their own, independent solution to both.
- Exercise 1 and 2 requires a textual answer, including some discussion.
- In the event of minor errors or ambiguities in the assignment text, you are generally expected to state your reasonable assumptions in the PDF or commented in the source code as to the intended meaning in your answer.
- We might give further comments or corrections to this assignment. If so, they will appear as messages on the course page. Check regularly.
- For 3-day extension, submit your request in the nettskjema at the course page.

## ***Exercise 1 (Undecidability)***

We say that a Turing machine  $M$  is a *composition* of Turing machines  $M_1$  and  $M_2$ , and write:

$$M = M_1 \& M_2$$

if, for every input  $x$ ,  $M$  produces the same result or output as the one that results when  $M_1$  is first run on input  $x$ , and then  $M_2$  is run with the output of  $M_1$  as input. Or put more simply, when  $M$  always produces the same output as  $M_1$  “followed by”  $M_2$ .

### **Question a)**

Formulate the Turing machine composition problem (the problem of deciding whether, when given three machines as input, the first is a composition of the remaining two) as a formal language.

### **Question b)**

Prove that the Turing machine composition problem is undecidable. Your proof should be complete – *i.e.*, include the proof that the reduction algorithm exists, by describing that algorithm in sufficient detail.

## ***Exercise 2 (NP-completeness)***

We know that 3-SATISFIABILITY is NP-complete. But there is of course nothing special about the number 3.

### **Question a)**

Define 10-SATISFIABILITY and prove that it is NP-complete.

### **Question b)**

What is the complexity of 10-SATISFIABILITY when the number of variables is limited to 100?

Prove your answer.

### ***Handing-in format***

- Hand-in the PDF-document called oblig3-<username>.pdf at:  
<https://devilry.ifi.uio.no>

[end of mandatory assignment 3]