Group Session 05.05.2021

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Mandatory assignment 4 – Deadline

- Remember the deadline for Mandatory assignment 4
- **Monday, 10. May, 2021, 11:59 PM**

Todays plan

- Mandatory Assignment 3 – Solution

(I will remove the solution from this powerpoint after some time)

- Control Theory – Exam problems

- The plan for future group sessions

Mandatory Solution - Start

!REMOVED!

Mandatory Solution - Finish

Control Theory – Exercise 1

Control Theory – Exercise 4, 2017

Exercise 4 (20 %)

We have the system $J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = \tau$. When we use the Laplace transform on this system we get $Js^2\theta(s) + Bs\theta(s) + K\theta(s) = \tau$.

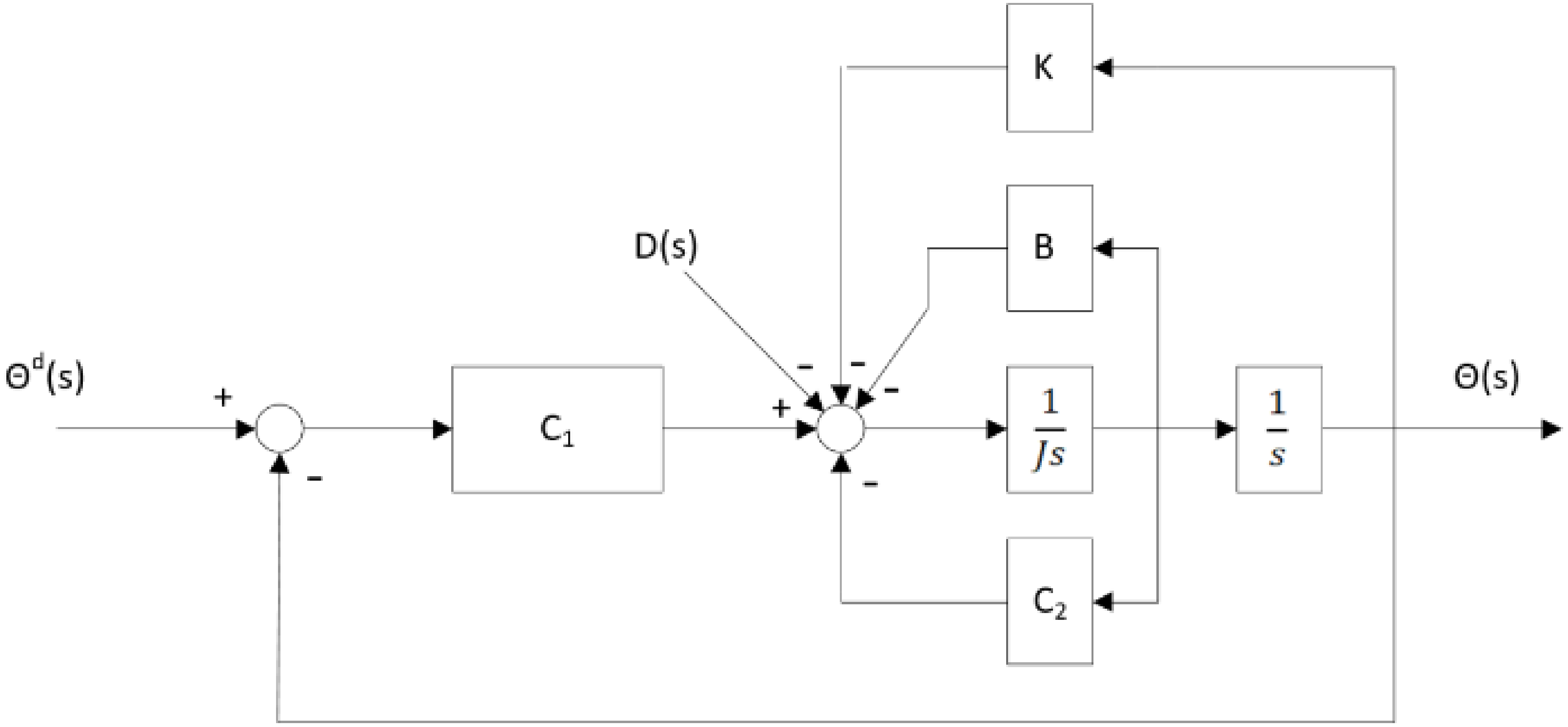

Figure 4: Control system

Control Theory – Exercise 4, 2017

a) (2.5 %) Figure 4 shows the system with controller in Laplace domain. What is the name of the controller used here?

b) (10 %) Working further with the controller in figure 4, how can we remove the steady state error, and still have a fast responsive system that reacts to the rate of change of the process value? What is the name of this new controller? Find the closed loop transfer function between the input value ($\Theta^d(s)$ - desired angle) and output value ($\Theta(s)$ - actual/measured angle) for the system with this new improved controller. Use the final value theorem to calculate the steady state error for the closed loop control system with this new improved controller, when both the desired angle $\Theta^d(s)$ and the disturbance $D(s)$ are "step inputs". Comment on the result.

c) (2.5 %) In general, how would you examine the stability of a control system like the one in task a? What is required to get a stable system?

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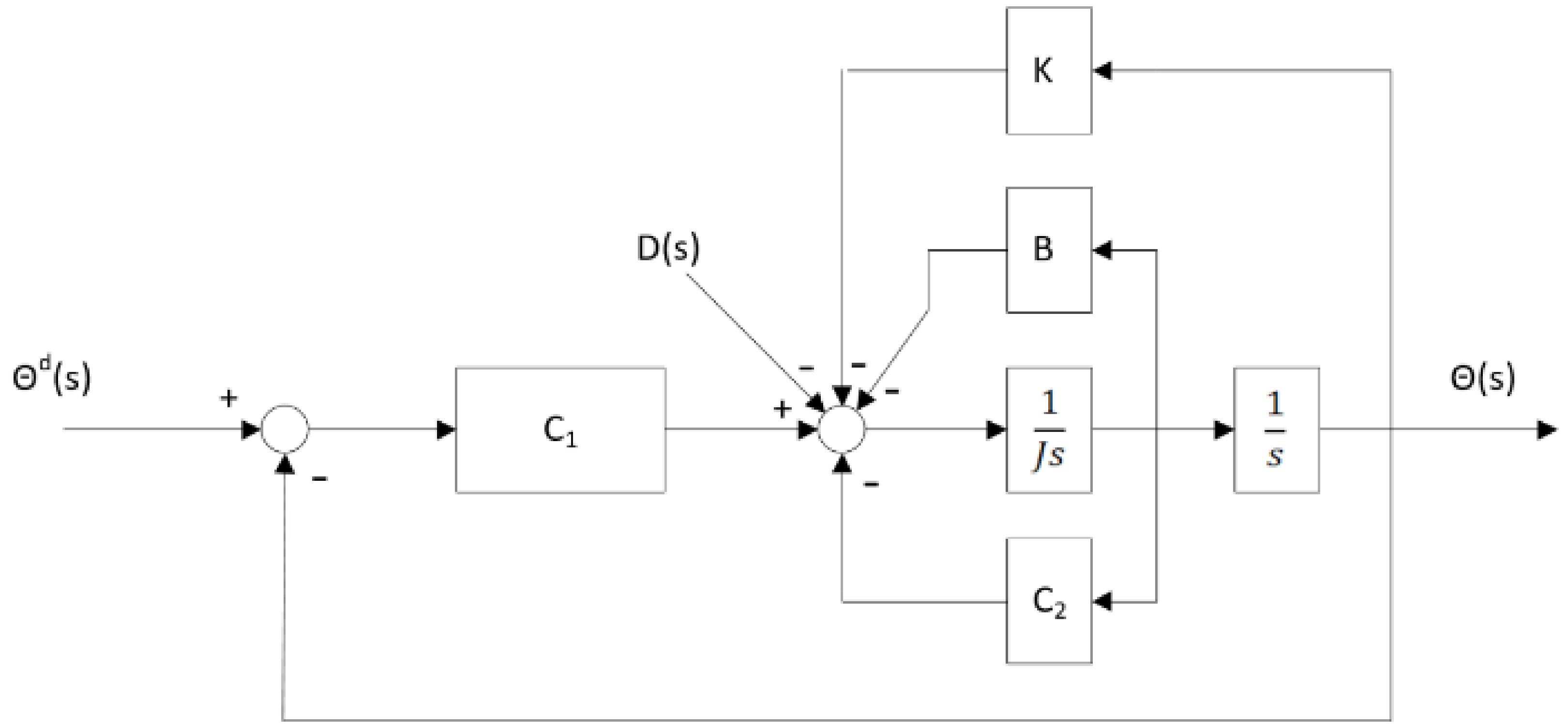

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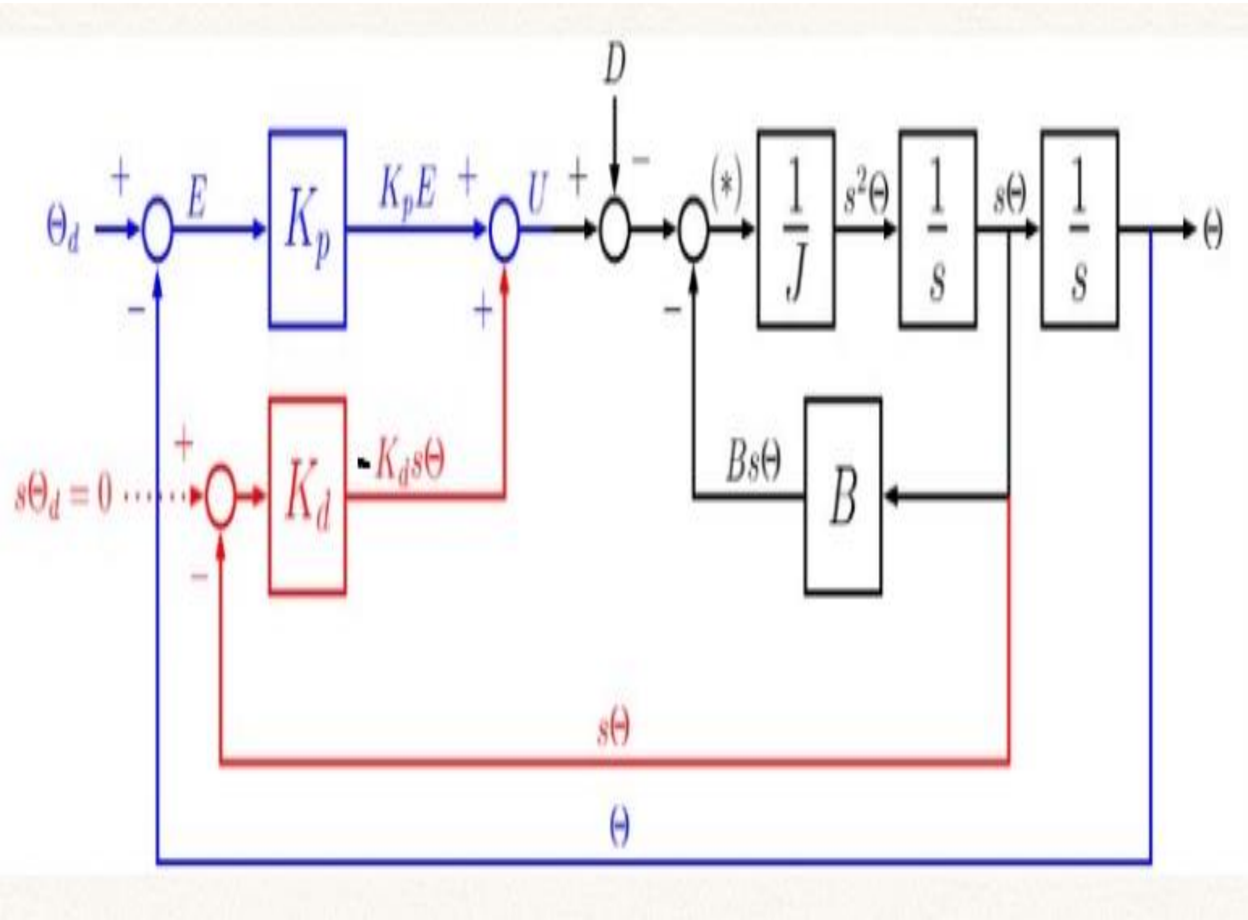

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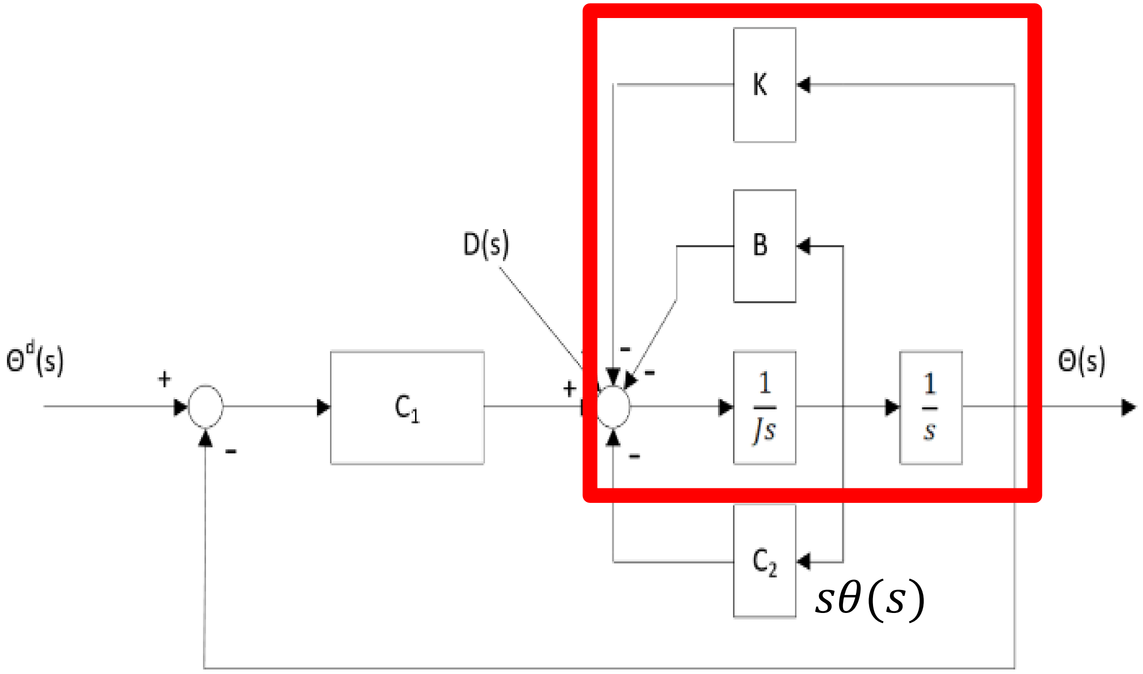

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Control Theory – Exercise 4, 2017

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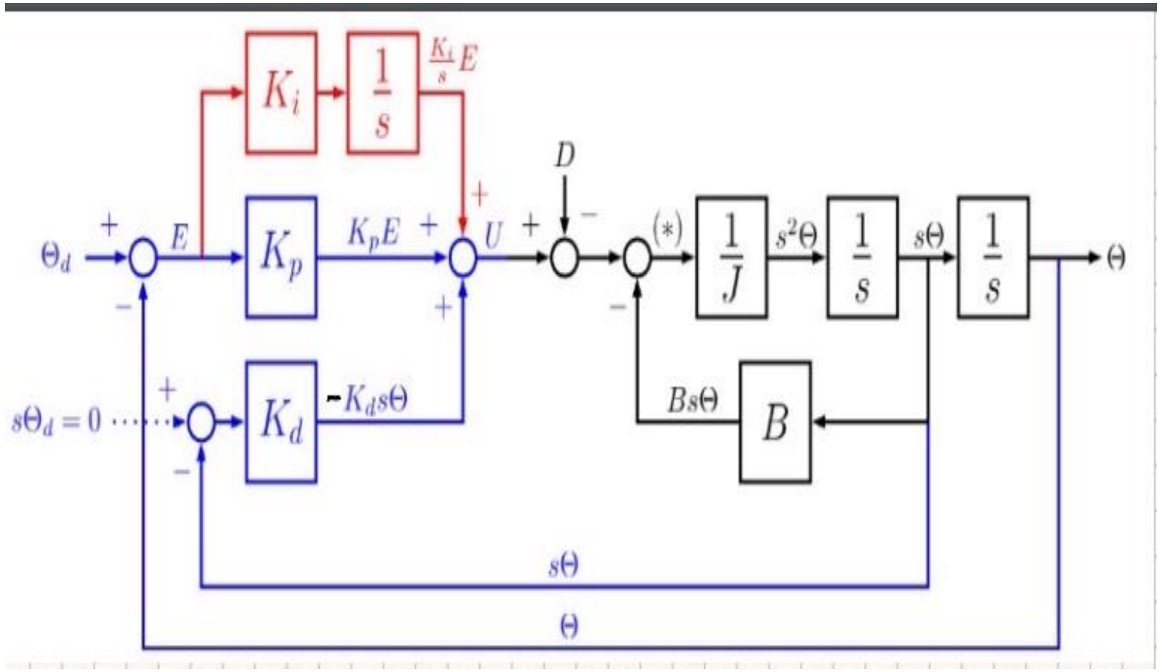


$$e(s) = \theta^d(s) - \theta(s)$$

$$\dot{e}(s) = s\theta^d(s) - s\theta(s)$$

$$u_{controller} = K_p e(s) + \frac{K_i}{s} e(s) + K_d \dot{e}(s)$$

$$u_{controller} = K_p \left(\theta^d(s) - \theta(s)\right) + \frac{K_i}{s}\left(\theta^d(s) - \theta(s)\right) + K_d \left(s\theta^d(s) - s\theta(s)\right)$$

$$u_{controller} = K_p\theta^d(s) - K_p\theta(s) + \frac{K_i}{s}\theta^d(s) - \frac{K_i}{s}\theta(s) + \underbrace{K_d s\theta^d(s)}_{0} - K_d s\theta(s)$$

$$u_{controller} = K_p\theta^d(s) - K_p\theta(s) + \frac{K_i}{s}\theta^d(s) - \frac{K_i}{s}\theta(s) - K_d s\theta(s)$$

$$u_{actuator} = Js^2\theta(s) + Bs\theta(s) + K\theta(s) - D(s)$$

Set $\boldsymbol{u_{controller}}$ = $\boldsymbol{u_{actuator}}$ and solve for $\boldsymbol{\theta(s)}$

Transfer function = $H(s) = \frac{Y(s)}{X(s)} = \frac{\theta(s)}{\theta^d(s)}$

$$K_p\theta^d(s) - K_p\theta(s) + \frac{K_i}{s}\theta^d(s) - \frac{K_i}{s}\theta(s) - K_d s\theta(s) = Js^2\theta(s) + Bs\theta(s) + K\theta(s)$$

$$K_p\theta^d(s) + \frac{K_i}{s}\theta^d(s) = Js^2\theta(s) + Bs\theta(s) + K\theta(s) + K_p\theta(s) + \frac{K_i}{s}\theta(s) + K_d s\theta(s)$$

$$\theta^d(s)(K_p + \frac{K_i}{s}) = \theta(s)(Js^2 + Bs + K + K_p + \frac{K_i}{s} + K_d s)$$

$$\frac{\theta^d(s)\left(K_p + \frac{K_i}{s}\right) + D(s)}{(Js^2 + Bs + K + K_p + \frac{K_i}{s} + K_d s)} = \theta(s)$$

$$\theta(s) = \frac{\theta^d(s)\left(K_p + \frac{K_i}{s}\right) + D(s)}{(Js^2 + Bs + K + K_p + \frac{K_i}{s} + K_d s)}$$

Transfer function

$$\frac{\theta(s)}{\theta^d(s)} = \frac{\left(K_p + \frac{K_i}{s}\right) + D(s)}{(Js^2 + Bs + K + K_p + \frac{K_i}{s} + K_d s)}$$

$$\Phi = (Js^2 + Bs + K + K_p + \frac{K_i}{s} + K_d s)$$

Step input reference

$$\theta^d(s) = \frac{\phi^d}{s}$$

Constant disturbance

$$D(s) = \frac{D}{s}$$

Set transfer function in this theorem

Final state-theorem

$$\lim_{s \to 0} s\theta(s)$$

$$\theta(s) = \frac{\theta^d(s)\left(K_p + \frac{K_i}{s}\right) - D(s)}{\phi}$$

$$\lim_{s \to 0} s \frac{\theta^d(s)\left(K_p + \frac{K_i}{s}\right) - D(s)}{\phi}$$

Replace θ^d with step input reference and D(s) with constant disturbance

$$\lim_{s \to 0} s \frac{\frac{\phi^d}{s}\left(K_p + \frac{K_i}{s}\right) - \frac{D}{s}}{\phi}$$

$$\lim_{s \to 0} \frac{\phi^d\left(K_p + \frac{K_i}{s}\right) - D}{\phi}$$

$$\lim_{s\to 0} \frac{\phi^d \left(K_p + \frac{K_i}{s}\right) - D}{\left(Js^2 + Bs + K + K_p + \frac{K_i}{s} + K_d s\right)} \quad \begin{matrix} | * s \\ | * s \end{matrix}$$

Prevent dividing by zero

$$\lim_{s\to 0} \frac{\phi^d \left(K_p + \frac{K_i}{s}\right) s - Ds}{\left(Js^3 + Bs^2 + Ks + K_p s + K_i + K_d s^2\right)}$$

Use limit and observe when, s, approaches 0, i.e, s = 0

$$\lim_{s\to 0} \frac{\phi^d \left(K_p + \frac{K_i}{s}\right) s - Ds}{\left(Js^3 + Bs^2 + Ks + K_p s + K_i + K_d s^2\right)}$$

0 0

0 0 0 0 0

$$= \frac{\phi^d (K_i)}{K_i} = \phi^d$$

We reach our targeted value

Control Theory – Exercise 4, 2017

c) (2.5 %) In general, how would you examine the stability of a control system like the one in task a? What is required to get a stable system?

$$\theta(s) = \frac{\theta^d(s)(K_p + \frac{K_i}{s}) - D(s)}{(Js^2 + Bs + K + K_p + \frac{K_i}{s} + K_d s)}$$

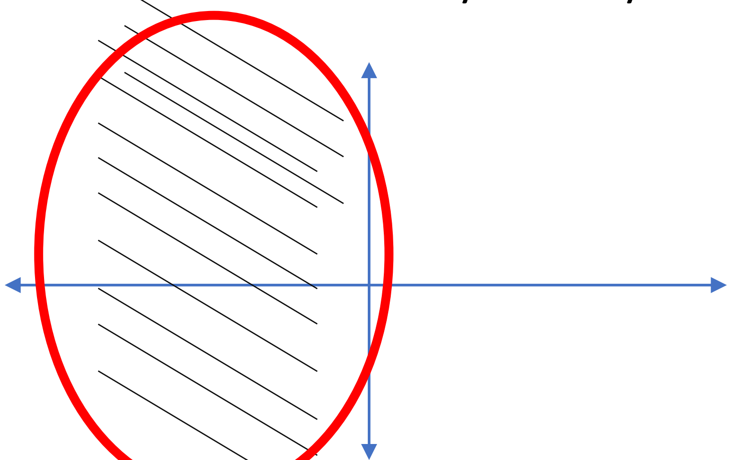
Zeros (numerator)

Poles (denominator)

Solve the denominator by itself by setting s=0

Solve the nominator by itself by setting s=0

Poles and zero solutions should be negative and within the red circle



Control Theory – Exercise 2

Control Theory – Exercise 4, 2018

Exercise 4 (25%)

We have the system $J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = \tau$. When we use the Laplace transform on the system we get $Js^2\theta(s) + Bs\theta(s) + K\theta(s) = \tau$.

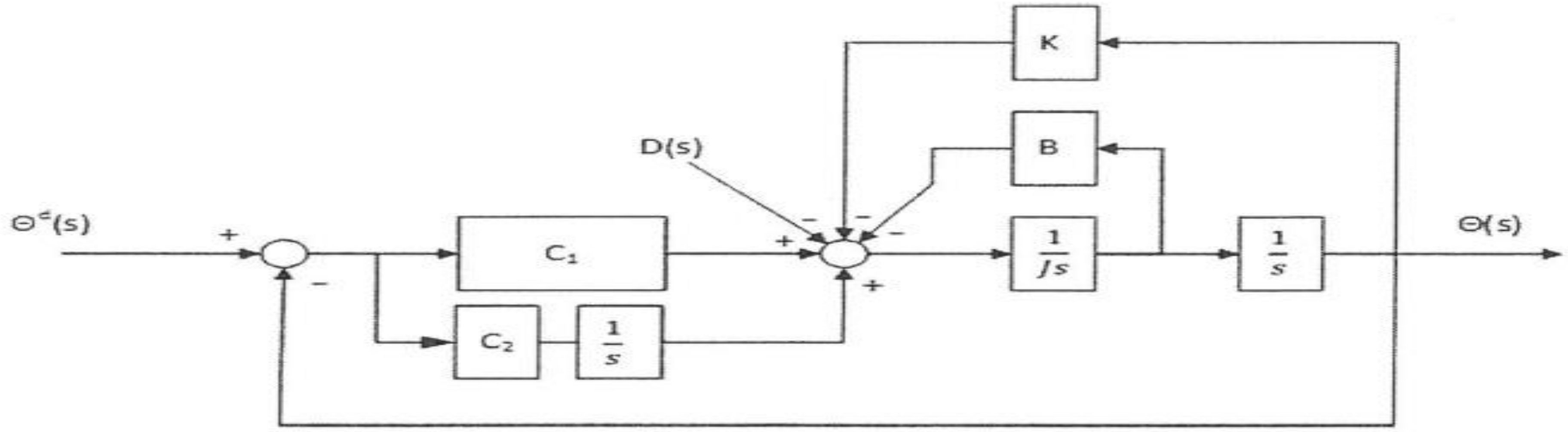


Figure 3: Control system

a) (2.5%) Figure 3 shows a set-point tracking control system in the s domain. What is the name of the controller used here? What properties does it provide to the system?

b) (2.5%) The system with the controller in Figure 3 have oscillations, how can we remove the oscillations while keeping the other system properties? What is the name of this new controller?

c) (5%) Find the closed loop transfer function between the input value ($\Theta^d(s)$ - desired angle) and output value ($\Theta(s)$ - actual/measured angle) for the system with this new improved controller.

d) (5%) Use the final value theorem to calculate the steady state error for the closed loop control system with this new improved controller, when both the desired angle $\Theta^d(s)$ and the disturbance D(s) are "step inputs". Comment on the result.

Control Theory – Exercise 4, 2018

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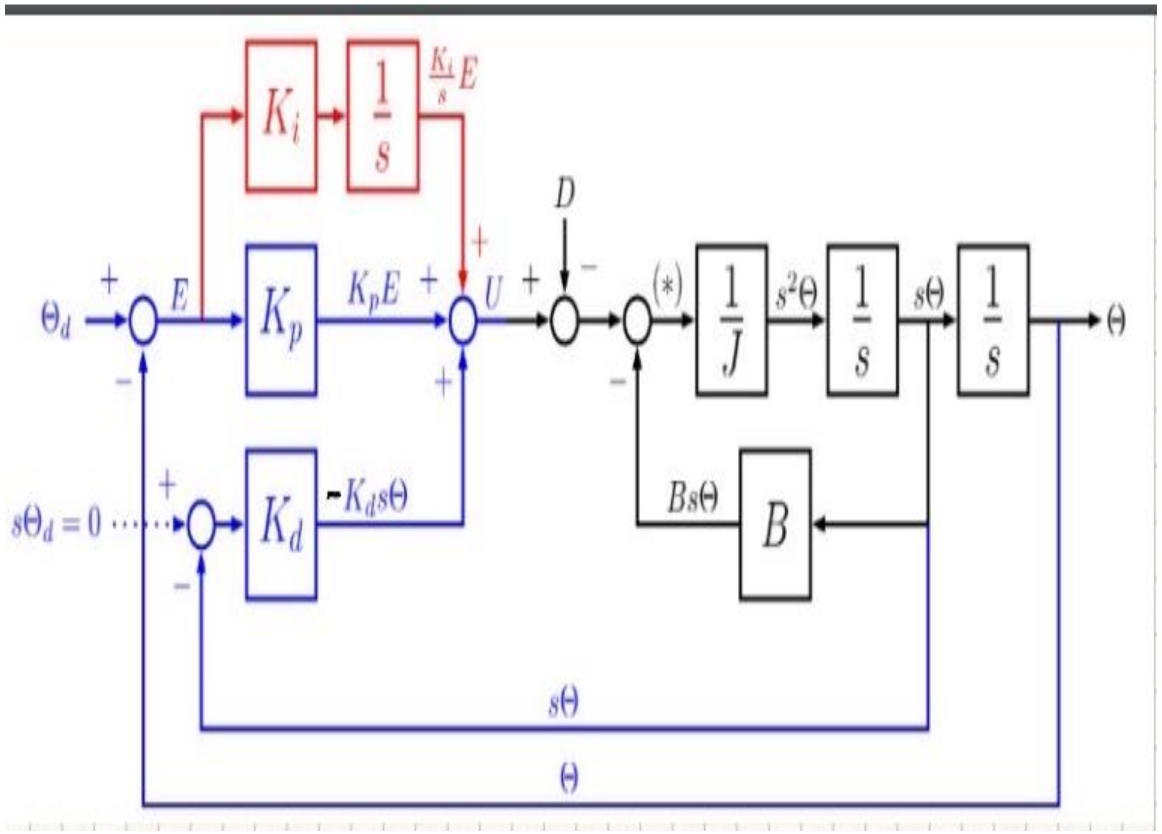

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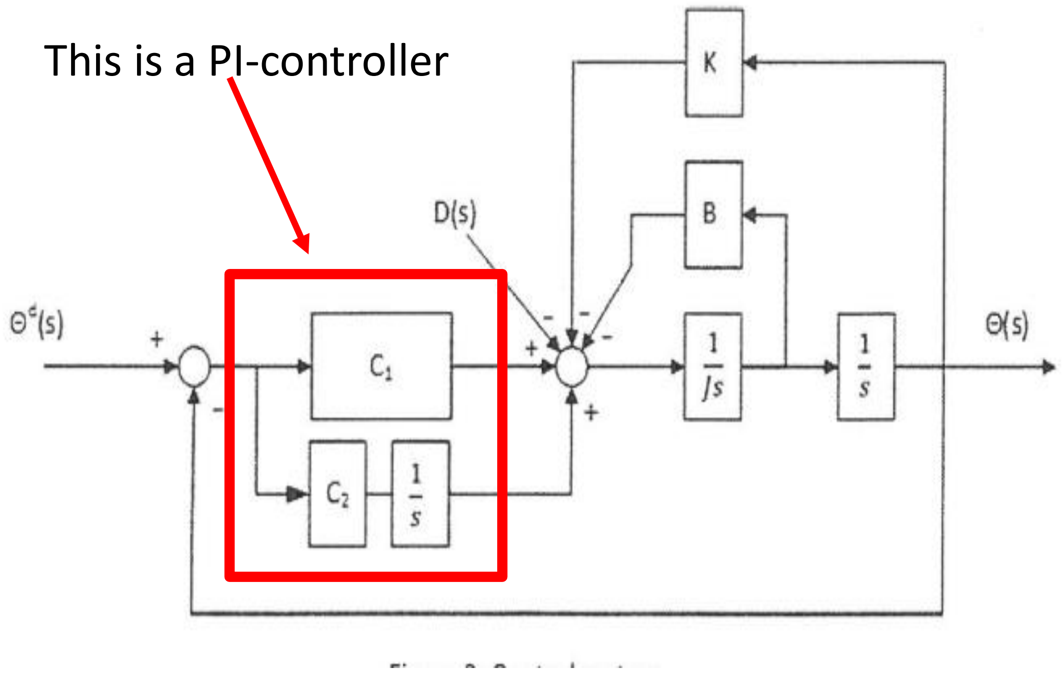

Control Theory – Exercise 4, 2018

b) (2.5%) The system with the controller in Figure 3 have oscillations, how can we remove the oscillations while keeping the other system properties? What is the name of this new controller?

- Add damping to reduce the oscillations with a derivative term known as K_d

Control Theory – Exercise 4, 2018

c) (5%) Find the closed loop transfer function between the input value ($\Theta^d(s)$ - desired angle) and output value ($\Theta(s)$ - actual/measured angle) for the system with this new improved controller.

See ‘**previous exercise’** and follow the same steps and approach

Control Theory – Exercise 4, 2018

d) (5%) Use the final value theorem to calculate the steady state error for the closed loop control system with this new improved controller, when both the desired angle $\Theta^d(s)$ and the disturbance D(s) are "step inputs". Comment on the result.

See ‘**previous exercise’** and follow the same steps and approach

Hint and tips for assignment 4

$$K_p = 3K_d$$

$$K_D = \frac{K_p}{3}$$

Usually good

- Find the fastest response for settling or convergence, i.e $t_s \sim 0$

- The error should be as small as possible for $t_s \sim 0$ given some values for K_p and K_D

Group sessions from now on

Week 18, **5. May** and **7. May**

- Mandatory assignment 4 - Walkthrough solution

- Control Theory - Exam problems

Week 19, ***12. May*** and ***14. May***

- Homogeneous Transform - Summary and Exam problems
- Forward-Kinematics - Summary and Exam problems

(Inverse-kinematics) - Included if time

Week 20, ***19. May*** and ***21. May***

- Inverse-Kinematics - Summary and Exam problems
- Dynamics - Summary and Exam problems

(Control Theory) - Included if time

(ROS) - Included if time

Week 21, ***26.May*** or ***28.May***

- Last official group session - Feel free to request any topics you want further explanation
- Additional information or questions about the exam might be explained further here

Week 23, ***11. June***

- Examination in IN3140