

# Group Session 07.05.2021

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# Mandatory assignment 4 – Deadline

- Remember the deadline for Mandatory assignment 4
- **Monday, 10. May, 2021, 11:59 PM**

# Today's plan

- Mandatory Assignment 3 – Solution

(I will remove the solution from this powerpoint after some time)

- Control Theory – Exam problems
- The plan for future group sessions

**Mandatory Solution - Start**

**!REMOVED!**

**Mandatory Solution - Finish**

# Control Theory – Exercise 1

# Control Theory – Exercise 4, 2017

## Exercise 4 (20 %)

We have the system  $J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = \tau$ . When we use the Laplace transform on this system we get  $J s^2 \theta(s) + B s \theta(s) + K \theta(s) = \tau$ .

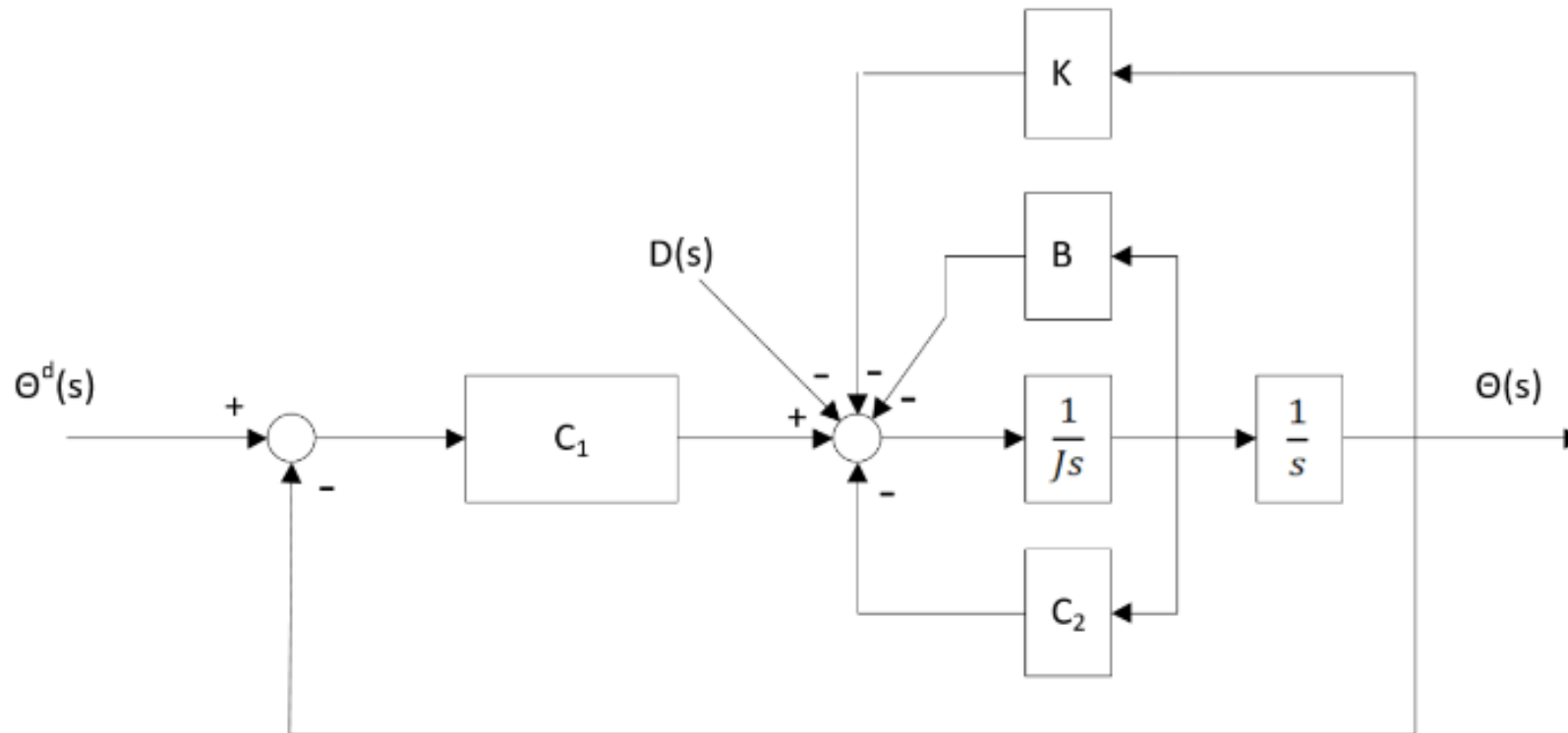


Figure 4: Control system



# Control Theory – Exercise 4, 2017

- a) (2.5 %) Figure 4 shows the system with controller in Laplace domain. What is the name of the controller used here?
- b) (10 %) Working further with the controller in figure 4, how can we remove the steady state error, and still have a fast responsive system that reacts to the rate of change of the process value? What is the name of this new controller? Find the closed loop transfer function between the input value ( $\Theta^d(s)$  - desired angle) and output value ( $\Theta(s)$  - actual/measured angle) for the system with this new improved controller. Use the final value theorem to calculate the steady state error for the closed loop control system with this new improved controller, when both the desired angle  $\Theta^d(s)$  and the disturbance  $D(s)$  are "step inputs". Comment on the result.
- c) (2.5 %) In general, how would you examine the stability of a control system like the one in task a? What is required to get a stable system?

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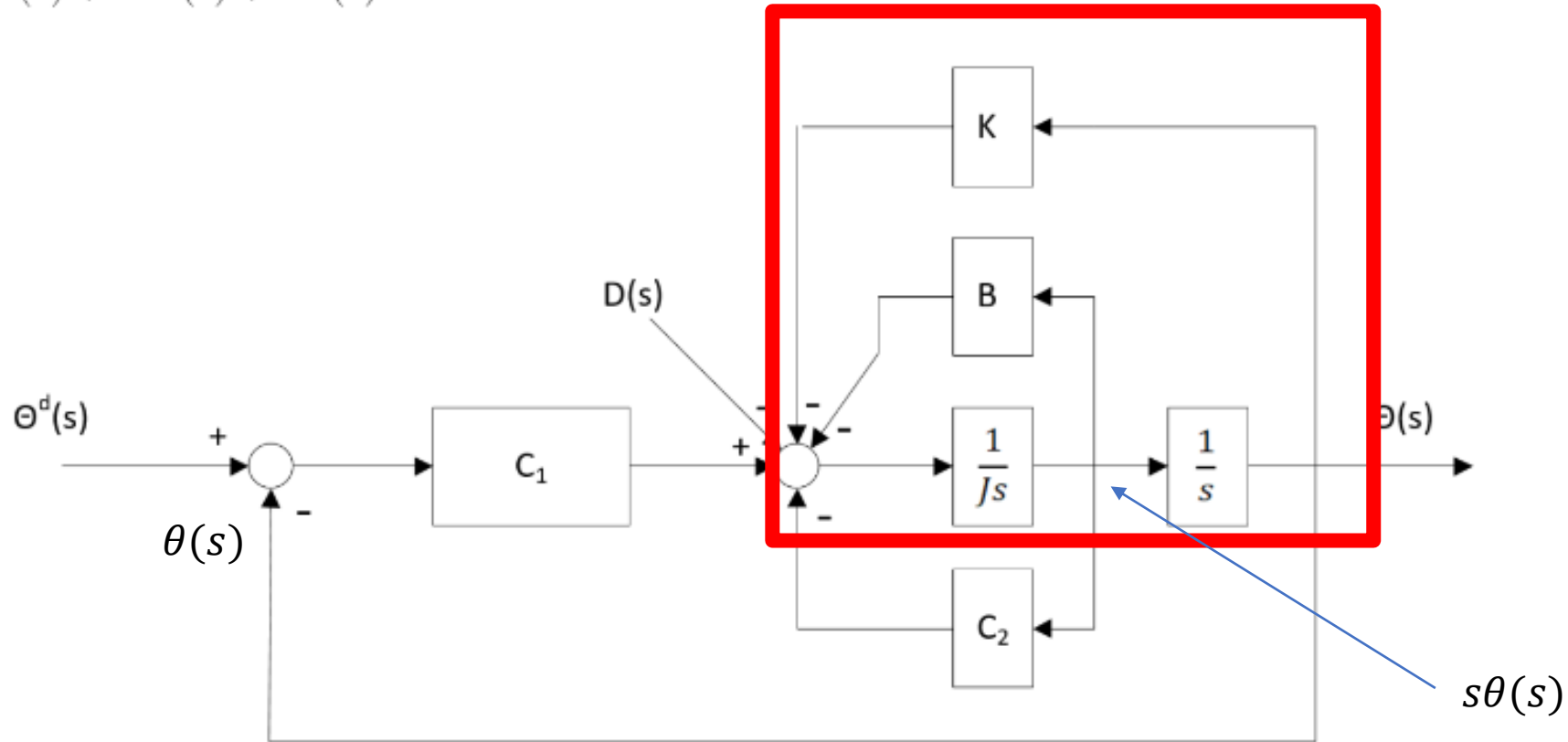
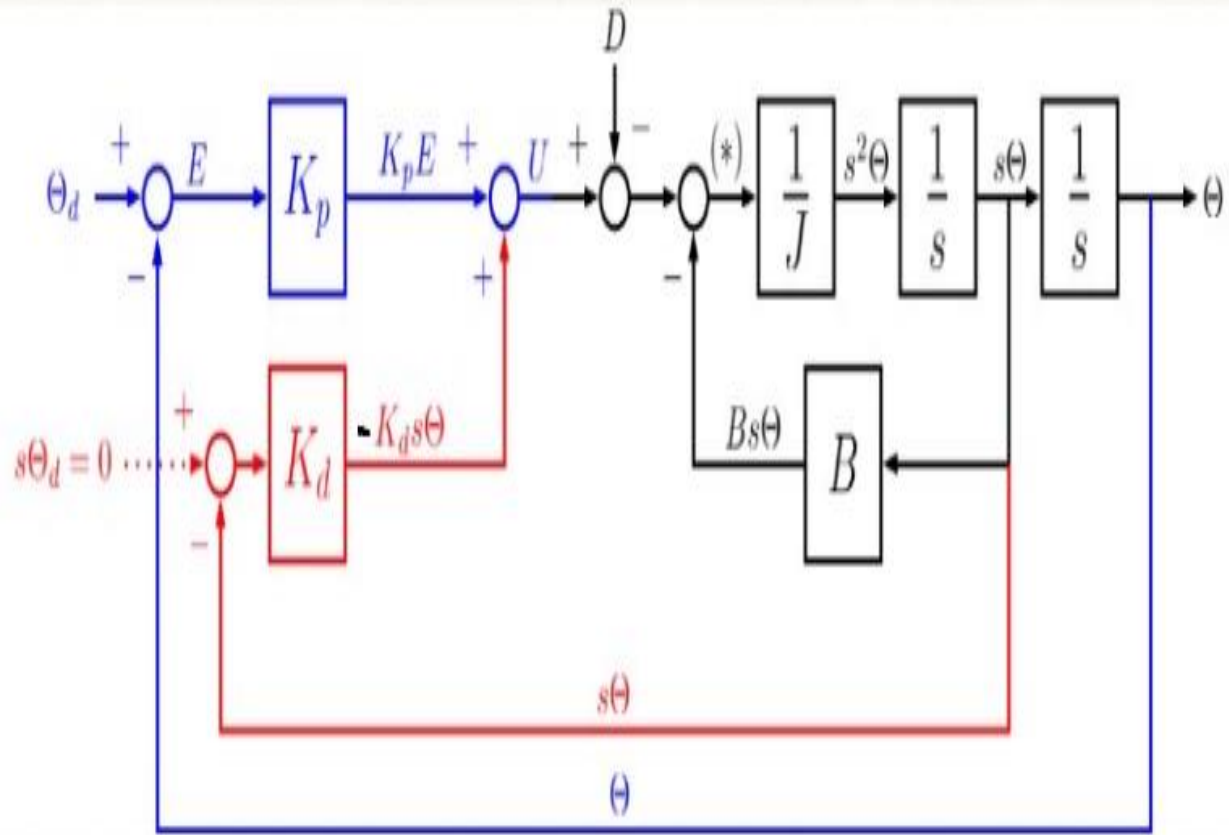


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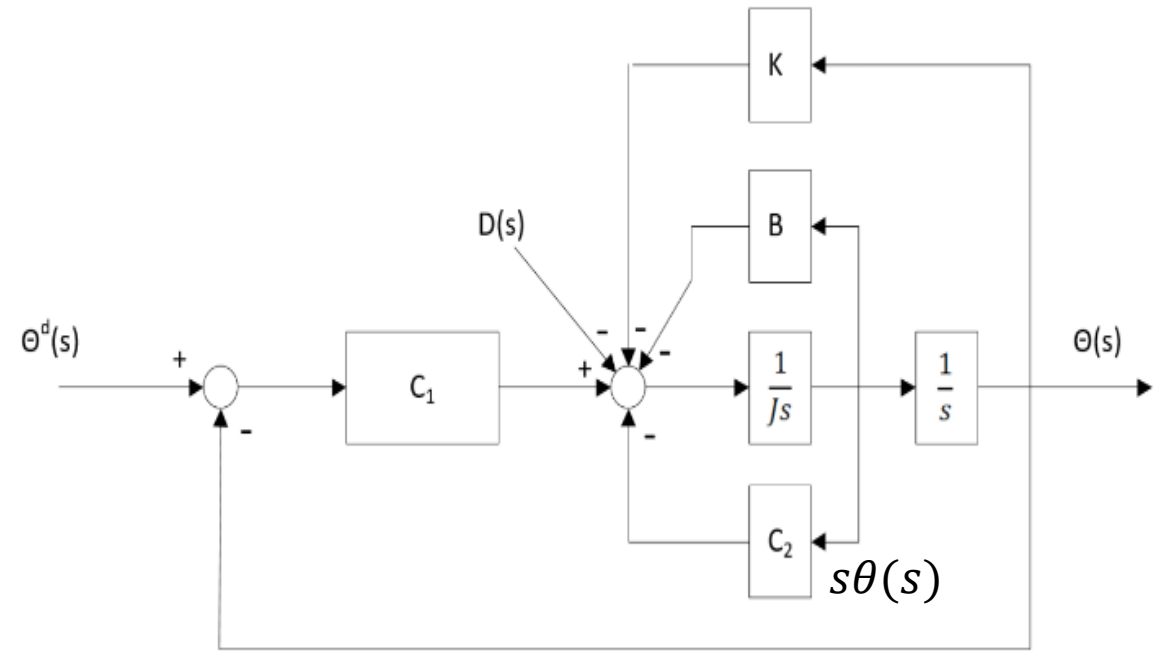
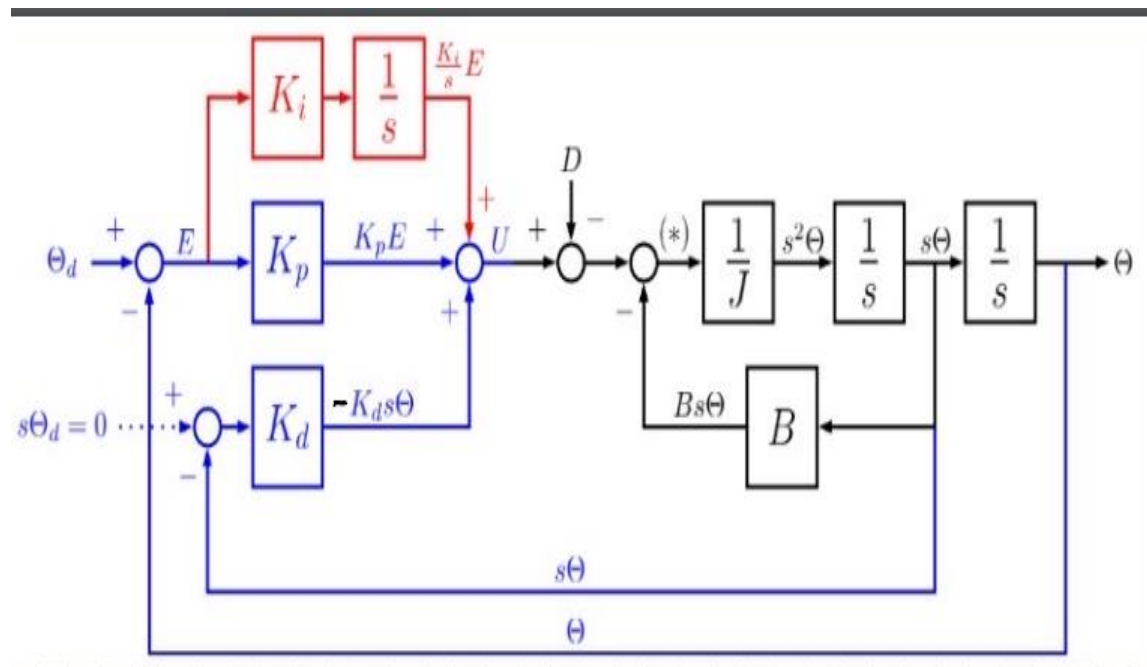


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# Control Theory – Exercise 4, 2017

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$$u_{actuator} = Js^2\theta(s) + Bs\theta(s) + K\theta(s) - D(s)$$

$$e(s) = \theta^d(s) - \theta(s)$$

$$e\dot{(s)} = s\theta^d(s) - s\theta(s)$$

$$u_{controller} = K_p e(s) + K_D \dot{e}(s) + \frac{K_i}{s} e(s)$$

$$u_{controller} = K_p(\theta^d - \theta) + K_D(\dot{\theta}^d - \dot{\theta}) + \frac{K_i}{s}(\theta^d - \theta)$$

$$u_{controller} = K_p\theta^d(s) - K_p\theta(s) + K_D s\theta^d(s) - K_D s\theta(s) + \frac{K_i}{s}\theta^d(s) - \frac{K_i}{s}\theta(s)$$

0

We want the velocity to reach zero as it reaches its targeted value, hence this term becomes zero

$$u_{controller} = K_p\theta^d(s) - K_p\theta(s) - K_D s\theta(s) + \frac{K_i}{s}\theta^d(s) - \frac{K_i}{s}\theta(s)$$

**Find the transferfunction setting  $u_{actuator} = u_{controller}$  and by solving for  $\theta$**

$$K_p\theta^d(s) - K_p\theta(s) - K_D s\theta(s) + \frac{K_i}{s}\theta^d(s) - \frac{K_i}{s}\theta(s) = Js^2\theta(s) + Bs\theta(s) + K\theta(s) - D(s)$$

$$K_p \theta^d(s) - K_p \theta(s) - K_D s \theta(s) + \frac{K_i}{s} \theta^d(s) - \frac{K_i}{s} \theta(s) = J s^2 \theta(s) + B s \theta(s) + K \theta(s) - D(s)$$

$$K_p \theta^d(s) + \frac{K_i}{s} \theta^d(s) + D(s) = J s^2 \theta(s) + B s \theta(s) + K \theta(s) + \frac{K_i}{s} \theta(s) + K_D s \theta(s) + K_p \theta(s)$$

$$\theta^d(s) \left( K_p + \frac{K_i}{s} \right) + D(s) = \theta(s) \left( J s^2 + B s + K + \frac{K_i}{s} + K_D s + K_p \right)$$

$$\frac{\theta^d(s) \left( K_p + \frac{K_i}{s} \right) + D(s)}{\left( J s^2 + B s + K + \frac{K_i}{s} + K_D s + K_p \right)} = \theta(s)$$

Step input reference

$$\theta^d = \frac{\phi^d}{s}$$

Constant disturbance

$$D(s) = \frac{D}{s}$$

Using **Final Value theorem**

$$\lim_{s \rightarrow 0} s\theta(s)$$

Set your transferfunction in here

$$\frac{\theta^d(s)(K_p + \frac{K_i}{s}) + D(s)}{(Js^2 + Bs + K + \frac{K_i}{s} + K_Ds + K_p)} = \theta(s)$$

$$\lim_{s \rightarrow 0} s \frac{\theta^d(s)(K_p + \frac{K_i}{s}) + D(s)}{(Js^2 + Bs + K + \frac{K_i}{s} + K_Ds + K_p)}$$

Replace  $\theta^d(s)$  with **step input reference** and **the disturbance**

$$\lim_{s \rightarrow 0} s \frac{\frac{\phi^d}{s} (K_p + \frac{K_i}{s}) + \frac{D}{s}}{(Js^2 + Bs + K + \frac{K_i}{s} + K_Ds + K_p)}$$

$$\lim_{s \rightarrow 0} s \frac{\frac{\phi^d}{s} (K_p + \frac{K_i}{s}) + \frac{D}{s}}{(Js^2 + Bs + K + \frac{K_i}{s} + K_D s + K_p)}$$

$$\lim_{s \rightarrow 0} \frac{\phi^d (K_p + \frac{K_i}{s}) + D}{(Js^2 + Bs + K + \frac{K_i}{s} + K_D s + K_p)}$$

| \* s      Prevent dividing by  
| \* s      zero

$$\lim_{s \rightarrow 0} \frac{s\phi^d (K_p + \frac{K_i}{s}) + Ds}{(Js^3 + Bs^2 + Ks + K_i + K_D s^2 + K_p s)}$$

**Now we can use the lim**

$$\lim_{s \rightarrow 0} \frac{s\phi^d (K_p + \frac{K_i}{s}) + Ds}{(Js^3 + Bs^2 + Ks + K_i + K_D s^2 + K_p s)}$$

The diagram shows the limit expression with arrows pointing from '0' to each term in the numerator and denominator. In the numerator, two arrows point to  $s\phi^d (K_p + \frac{K_i}{s})$  and  $Ds$ . In the denominator, five arrows point to  $Js^3$ ,  $Bs^2$ ,  $Ks$ ,  $K_i$ , and  $K_p s$ .



$$= \frac{\phi^d(K_i)}{(K_i)}$$

$$= \phi^d$$

The result will be the targeted  $\theta^d$

# Control Theory – Exercise 4, 2017

- c) (2.5 %) In general, how would you examine the stability of a control system like the one in task a?  
What is required to get a stable system?

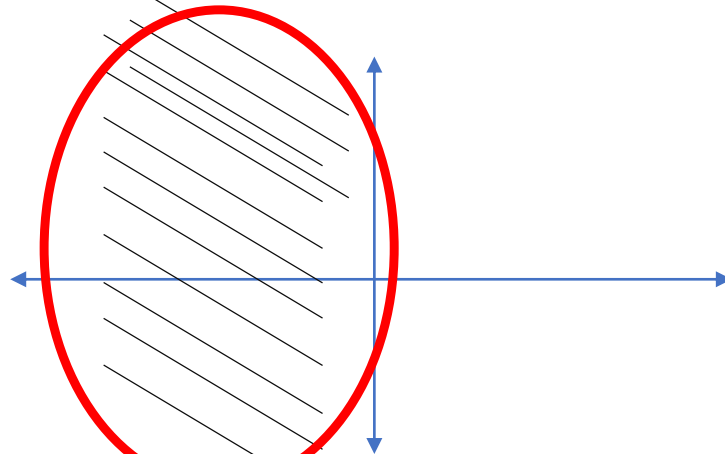
$$\theta(s) = \frac{\theta^d(s)(K_p + \frac{K_i}{s}) - D(s)}{(Js^2 + Bs + K + K_p + \frac{K_i}{s} + K_d s)}$$

Zeros  
Poles

Poles and zero solutions should be negative and within the red circle

Solve the denominator by solving for s

Solve the nominator by solving for s



# Control Theory – Exercise 2

# Control Theory – Exercise 4, 2018

## Exercise 4 (25%)

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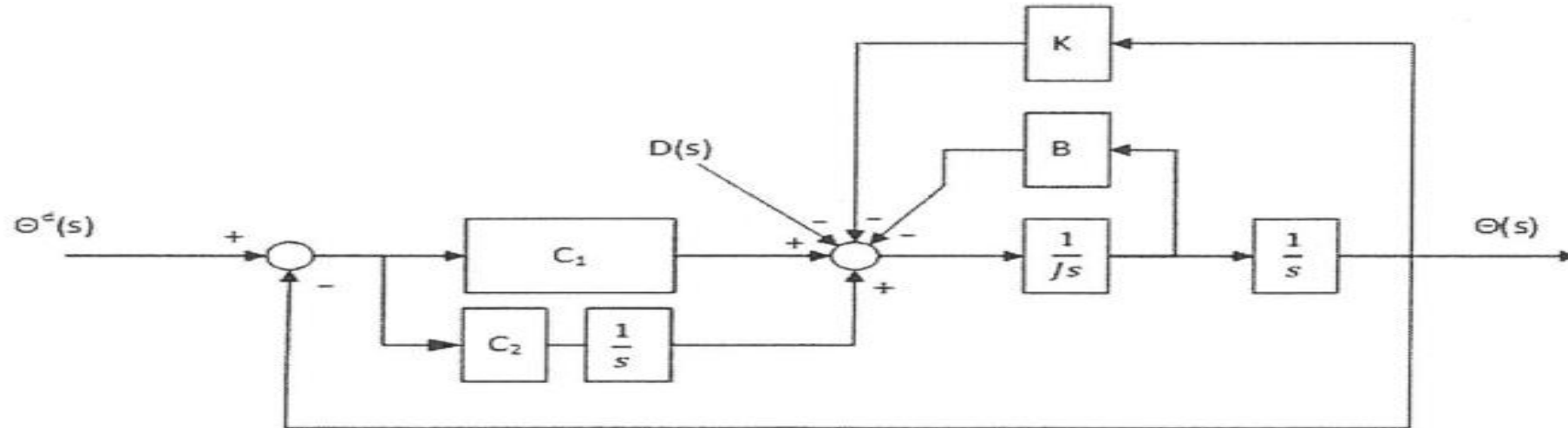
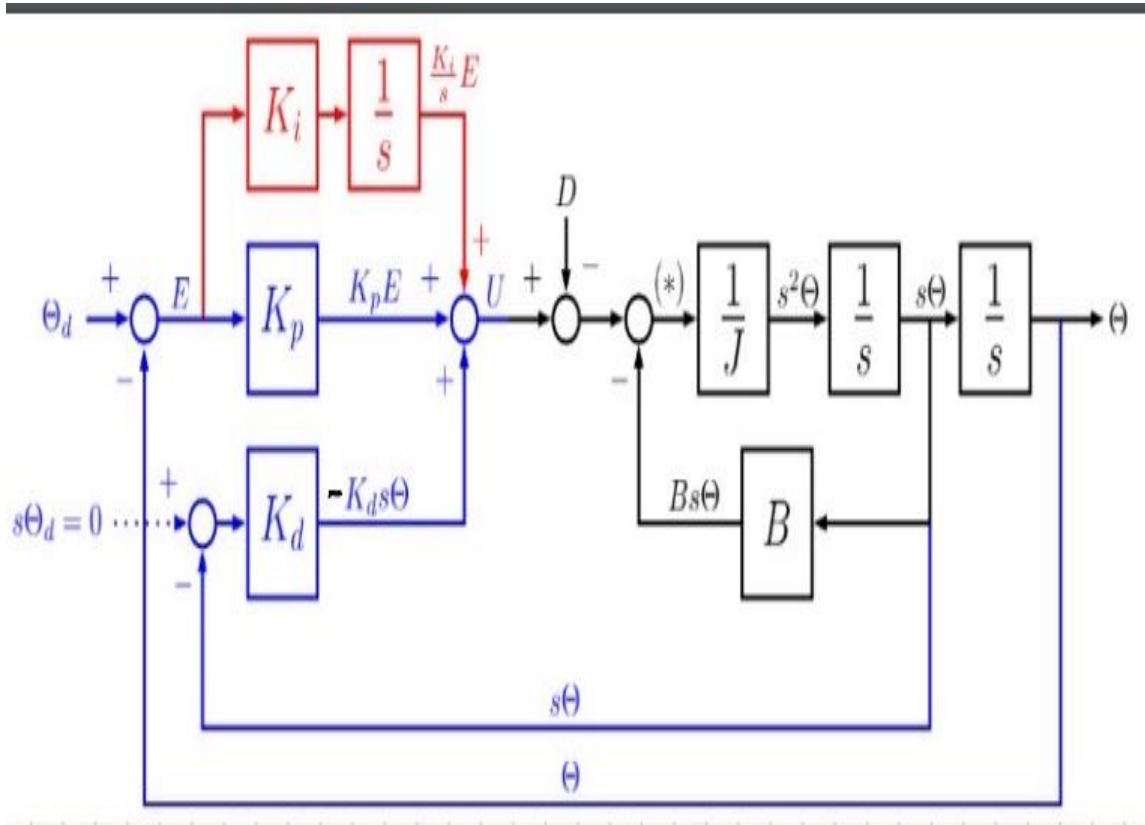


Figure 3: Control system

- (2.5%) Figure 3 shows a set-point tracking control system in the  $s$  domain. What is the name of the controller used here? What properties does it provide to the system?
- (2.5%) The system with the controller in Figure 3 have oscillations, how can we remove the oscillations while keeping the other system properties? What is the name of this new controller?
- (5%) Find the closed loop transfer function between the input value  $\Theta^d(s)$  - desired angle) and output value  $\Theta(s)$  - actual/measured angle) for the system with this new improved controller.
- (5%) Use the final value theorem to calculate the steady state error for the closed loop control system with this new improved controller, when both the desired angle  $\Theta^d(s)$  and the disturbance  $D(s)$  are "step inputs". Comment on the result.

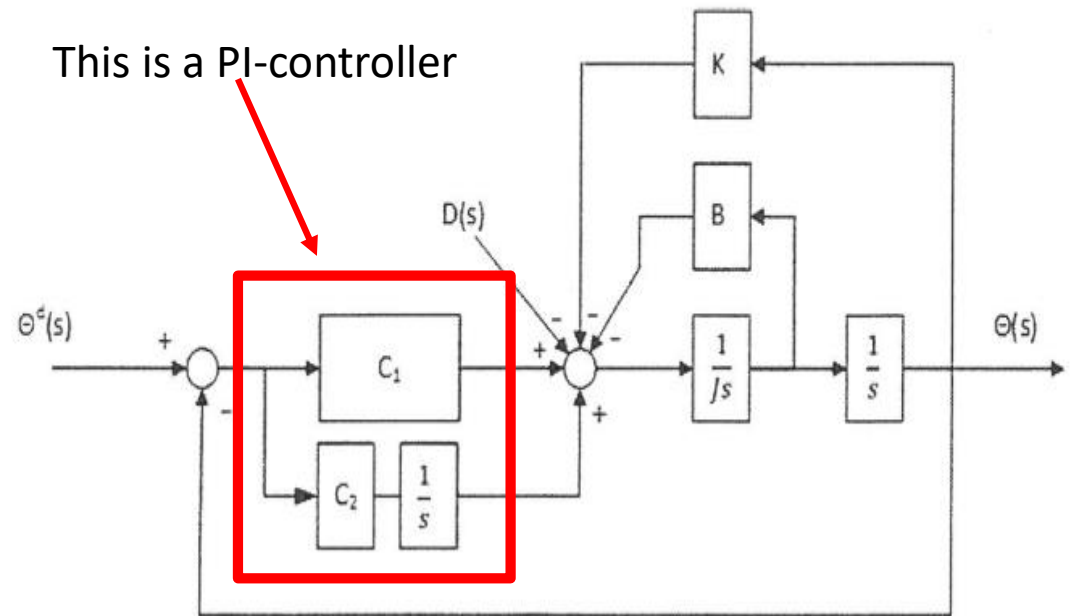
# Control Theory – Exercise 4, 2018

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## Exercise 4 (25%)

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# Control Theory – Exercise 4, 2018

b) (2.5%) The system with the controller in Figure 3 have oscillations, how can we remove the oscillations while keeping the other system properties? What is the name of this new controller?

- Add damping to reduce the oscillations with a derivative term known as  $K_d$

# Control Theory – Exercise 4, 2018

- c) (5%) Find the closed loop transfer function between the input value ( $\Theta^d(s)$  - desired angle) and output value ( $\Theta(s)$  - actual/measured angle) for the system with this new improved controller.

See '**previous exercise**' and follow the same steps and approach

# Control Theory – Exercise 4, 2018

- d) (5%) Use the final value theorem to calculate the steady state error for the closed loop control system with this new improved controller, when both the desired angle  $\Theta^d(s)$  and the disturbance  $D(s)$  are "step inputs". Comment on the result.

See '**previous exercise**' and follow the same steps and approach



# Hint and tips for assignment 4

$$\begin{array}{l} K_p = 3K_d \\ K_D = \frac{K_p}{3} \end{array} \left. \vphantom{\begin{array}{l} K_p = 3K_d \\ K_D = \frac{K_p}{3} \end{array}} \right\} \text{Usually good}$$

- Find the fastest response for settling or convergence, i.e  $t_s \sim 0$
- The error should be as small as possible for  $t_s \sim 0$  given some values for  $K_p$  and  $K_D$

Adjust set\_point

Adjust  $K_p$

Adjust  $K_d$

- Observe the time-response, time\_settling
- Observe the error and explain

Final value theorem

Kapittel 6.3 formel (6.21 og 6.22)

# Group sessions from now on

Week 18, **5. May** and **7. May**

- Mandatory assignment 4
- Control Theory
- Walkthrough solution
- Exam problems

Week 19, **12. May** and **14. May**

- Homogeneous Transform
- Forward-Kinematics
- \*(Inverse-kinematics)\*
- Summary and Exam problems
- Summary and Exam problems
- Included if time

Week 20, **19. May** and **21. May**

- Inverse-Kinematics
- Dynamics
- \*(Control Theory)\*
- \*(ROS)\*
- Summary and Exam problems
- Summary and Exam problems
- Included if time
- Included if time

Week 21, **26. May** or **28. May**

- Last official group session
- Additional information or questions about the exam might be explained further here
- Feel free to request any topics you want further explanation

Week 23, **11. June**

- Examination in IN3140