Forward kinematics: Find the end effector given the joint variables.

All we need is the transformations H_1^0, H_2^1... and multiply them!

DH-convention: Streamlines the process -> "recipe for forward kinematics"
- Provides a universal language to describe a manipulator.
-- As long as we follow the rules set by DH.

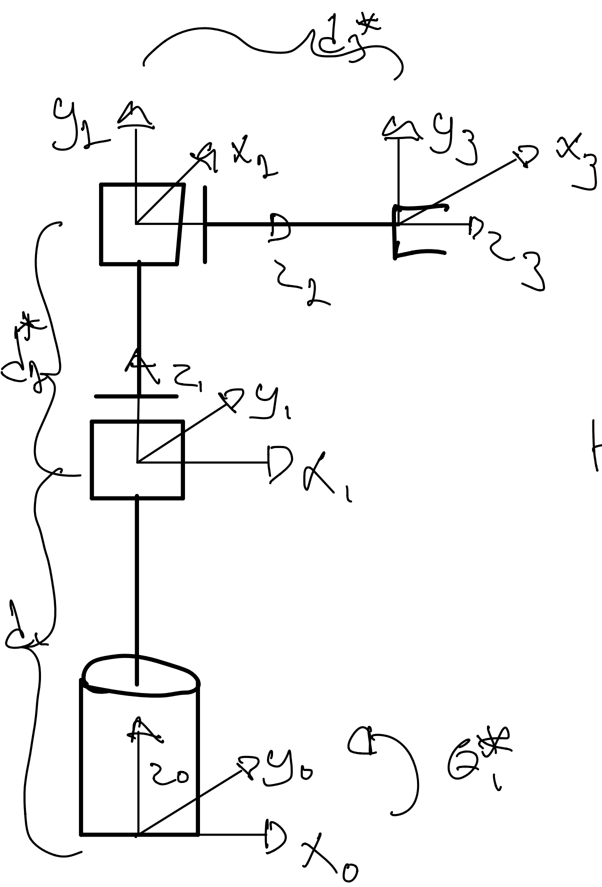
Coordinate systems:

Rule 1: $x_i \perp z_{i-1}$

Rule 2: x_i has to intersect z_{i-1}
↳ move O_i if not

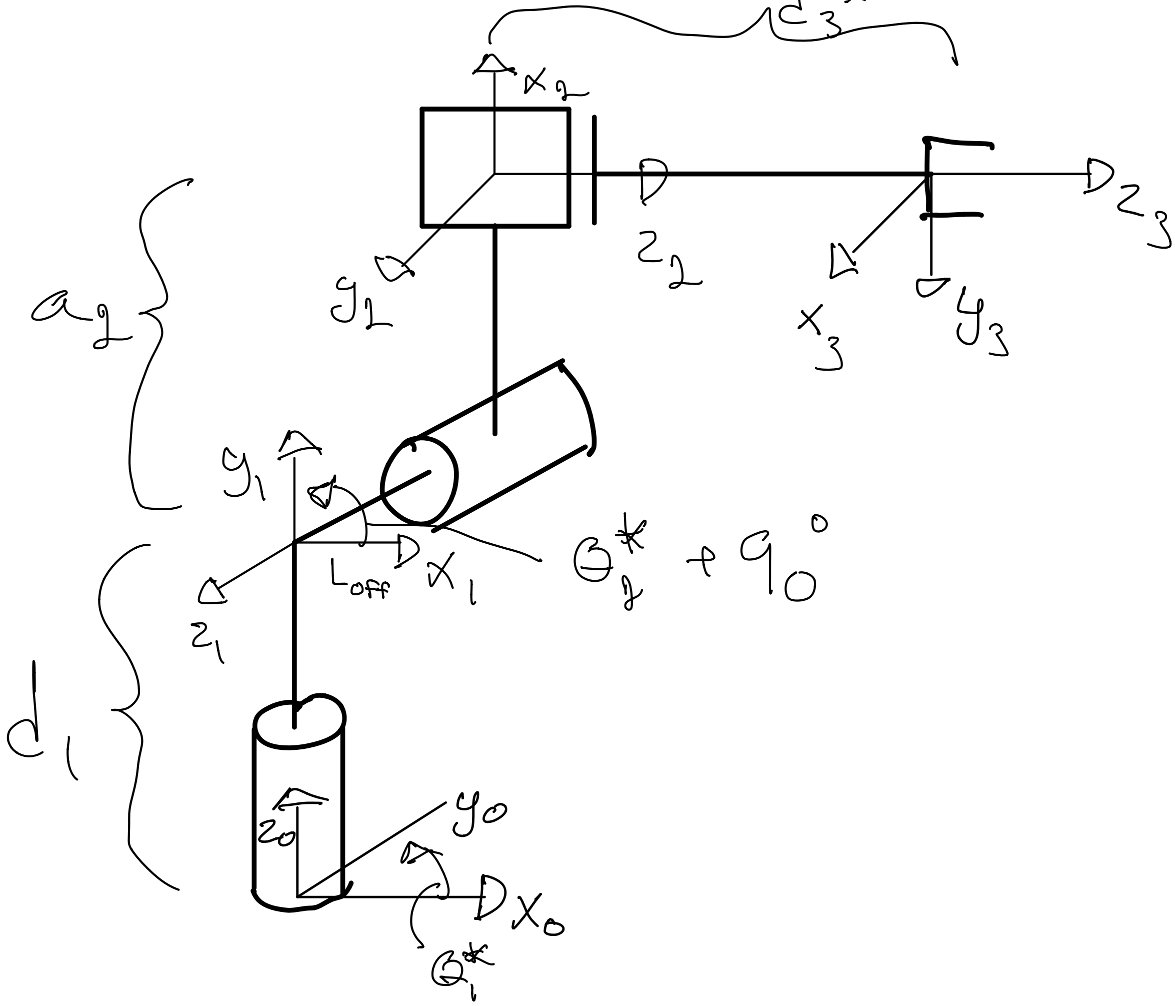
z in the direction of action.
y from right hand rule.

Example: Coordinate frames for the cylindrical manipulator.



$$H_3^0 = \underbrace{H_1^0 H_2^1}\, H_3^2$$

Assignment: Assign coordinate frames to the manipulator from the 2018 exam.



DH parameters: (page 110 step 7)

Θ_i: Joint angle, the angle from x_{i-1} to x_i measured about z_{i-1}

d_i: Link offset, distance from O_{i-1} about z_{i-1} to the intersection of x_i and z_{i-1}

a_i: Link length, distance from the intersection of x_i and z_{i-1} to O_i

α_i: Link twist, the angle from z_{i-1} to z_i measured about x_i

Example: Parameter table for the cylindrical manipulator

Link	Θ_i	d_i	a_i	α_i
1	Θ_1^*	d_1	0	0
2	$90°$	d_2^*	0	$90°$

3	0	d_3^*	0	0

Assignment: Fill in the parameter table for the 2018 manipulator.

Link	θ_i	d_i	a_i	α_i
1	θ_1^*	d_1	0	90°
2	$\theta_2^* + 90^\circ$	L_{off}	a_2	90°
3	90°	d_3^*	0	0

--- BREAK ---

The special matrix (page 77)

$$
\begin{aligned}
A_i &= Rot_{z,\theta_i} \text{Trans}_{z,d_i} \text{Trans}_{x,a_i} Rot_{x,\alpha_i} \qquad (3.10) \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Example: Forward kinematics for the cylindrical manipulator (two first joints only)

Assignment: Calculate the forward kinematics for the 2018 manipulator (two first joints only)

$$A_1 \; (H_1^0) = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 \; (H_2^1) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = H_1^0 H_2^1$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ \dots \end{bmatrix} \begin{bmatrix} -s_1 & 0 & c_1 & 0 \\ c_1 & \dots \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 1 & [illegible] & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & s_1 & 0 \\ 0 & 1 & 0 & d_1 + d_2^{*} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$