Forward kinematics: Find the end effector given the joint variables.

All we need is the transformations H_1^0, H_2^1... and multiply them!

DH-convention: Streamlines the process -> "recipe for forward kinematics"
- Provides a universal language to describe a manipulator.
-- As long as we follow the rules set by DH.

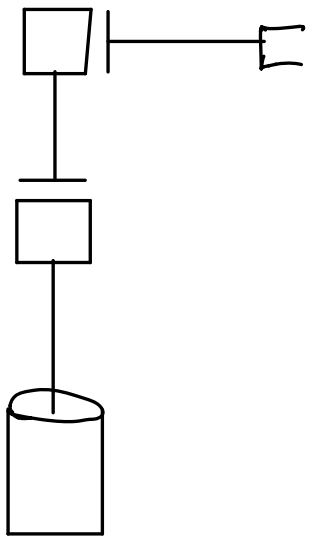
Coordinate systems:

Rule 1: $X_i \perp Z_{i-1}$

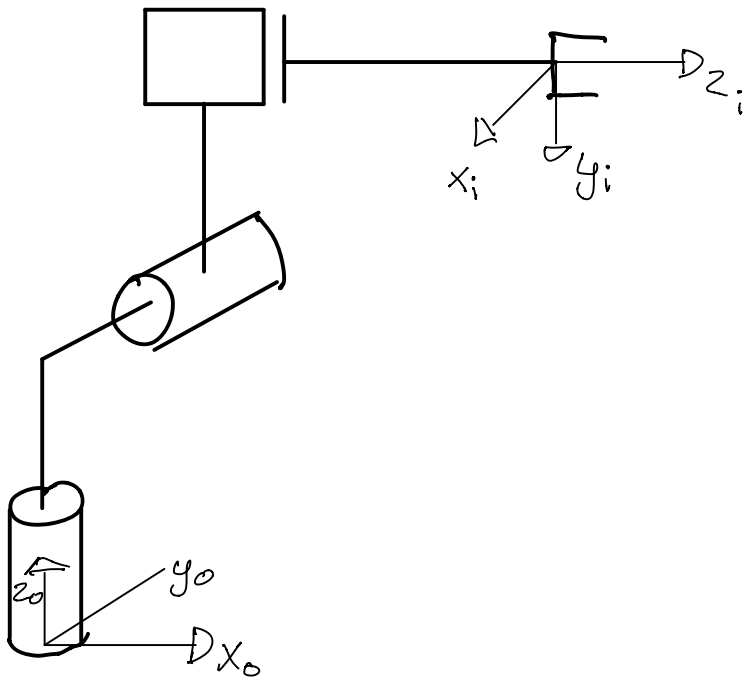
Rule 2: X_i has to intersect Z_{i-1}
to move O_i if not

z in the direction of action.
y from right hand rule.

Example: Coordinate frames for the cylindrical manipulator.



Assignment: Assign coordinate frames to the manipulator from the 2018 exam.



DH parameters: (page 110 step 7)
Θ_i: Joint angle, the angle from x_{i-1} to x_i measured about z_{i-1}

d_i: Link offset, distance from O_{i-1} about z_{i-1} to the intersection of x_i and z_{i-1}

a_i: Link length, distance from the intersection of x_i and z_{i-1} to O_i

α_i: Link twist, the angle from z_{i-1} to z_i measured about x_i

Example: Parameter table for the cylindrical manipulator

Link	Θ_i	d_i	a_i	α_i
1				
2				

3				

Assignment: Fill in the parameter table for the 2018 manipulator.

Link	θ_i	d_i	a_i	α_i
1				
2				
3				

--- BREAK ---

The special matrix (page 77)

$$
\begin{aligned}
A_i &= Rot_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,a_i}Rot_{x,\alpha_i} \qquad (3.10)\\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Example: Forward kinematics for the cylindrical manipulator (two first joints only)

Assignment: Calculate the forward kinematics for the 2018 manipulator (two first joints only)