Group session 4

Inverse Kinematics, Two weeks

3.12 and Kristians Example from previous group session

Email: hpnguyen@ifi.uio.no

General info

- Mandatory assignment deadline next week!!!
- Mattermost channel should be **coming up** soon. Hopefully in this week or the next week.
- Questions??

Forward-kinematics

The forward-kinematics is concerned with the relationship between the individual joints of the robot manipulator and the position and orientation of the tool or end effector.

Given a set of joint angles for each joint, we get the cartesian coordinates of the end-effector.

DH-convention – commonly used for selecting frames of reference in robotic applications.

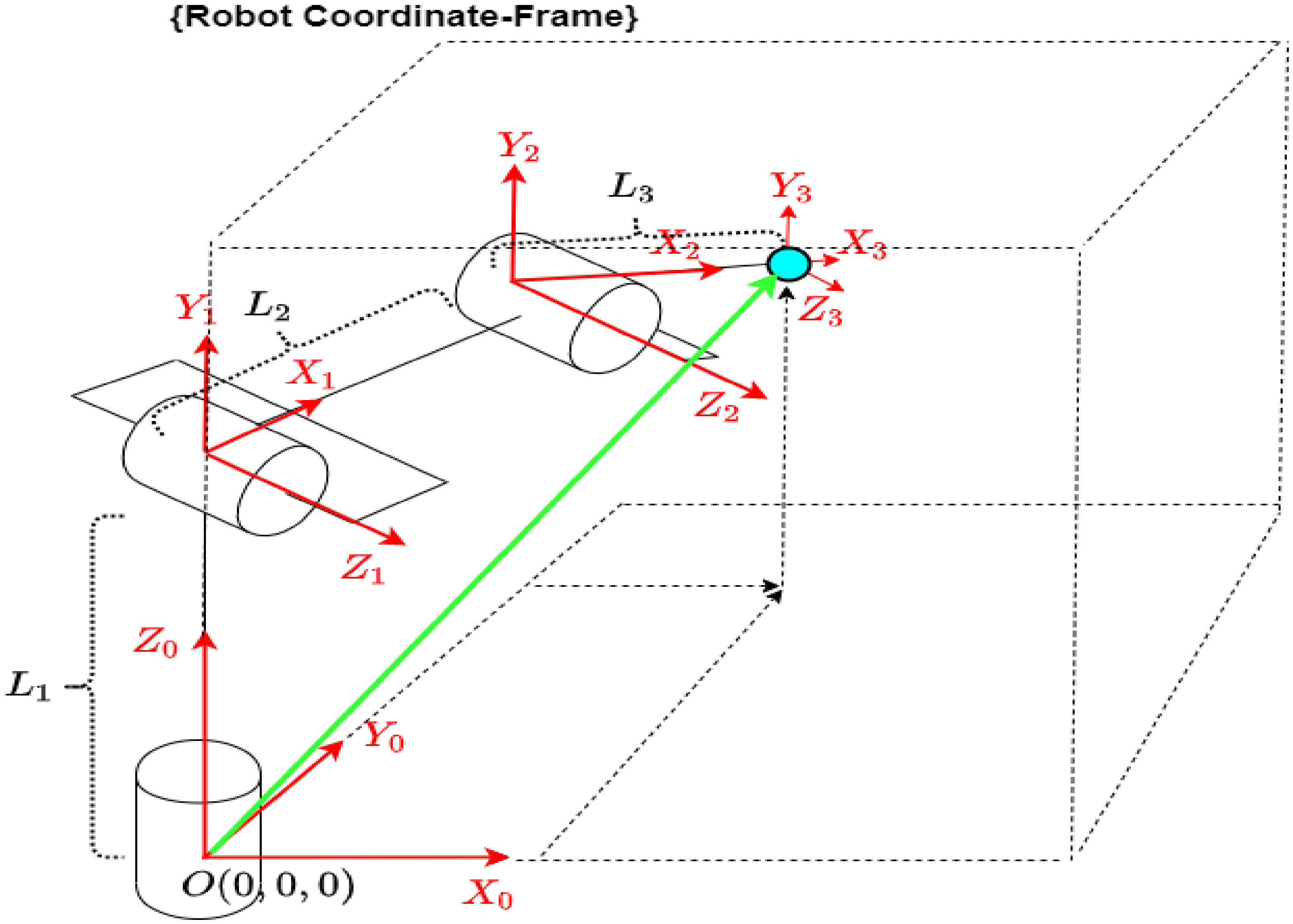
4 – parameters –

- α_i - *link twist*
- θ_i- Joint angle
- a_i- link length
- d_i- link offset

Homogeneous Transformation matrix

$$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



{Robot Coordinate-Frame}
Y2
L3
Y3
X2
X3
Z3
Y1
L2
X1
Z2
Z1
Z0
L1
Y0
O(0, 0, 0)
X0

Forward-kinematics cont.

$$H_j^i = H_{i+1}^i \dots H_{j+1}^{j-1} = \begin{bmatrix} R_{i+1}^i & d_{i+1}^i \\ 0 & 1 \end{bmatrix} \dots \begin{bmatrix} R_j^{j-1} & d_j^{j-1} \\ 0 & 1 \end{bmatrix}$$

$$T_j^i = T_{i+1}^i \dots T_{j+1}^{j-1} = \begin{bmatrix} R_{i+1}^i & d_{i+1}^i \\ 0 & 1 \end{bmatrix} \dots \begin{bmatrix} R_j^{j-1} & d_j^{j-1} \\ 0 & 1 \end{bmatrix}$$

$$T_j^i = A_i \dots A_j = \begin{bmatrix} R_{i+1}^i & d_{i+1}^i \\ 0 & 1 \end{bmatrix} \dots \begin{bmatrix} R_j^{j-1} & d_j^{j-1} \\ 0 & 1 \end{bmatrix}$$

$$T_j^i(q_1 \dots q_n) = A_i(q_1) \dots A_j(q_j) = \begin{bmatrix} R_{i+1}^i & d_{i+1}^i \\ 0 & 1 \end{bmatrix} \dots \begin{bmatrix} R_j^{j-1} & d_j^{j-1} \\ 0 & 1 \end{bmatrix}$$

Inverse-Kinematics

- This is concerned with the inverse problem of finding the joint variables in terms of end effector's position and orientation.

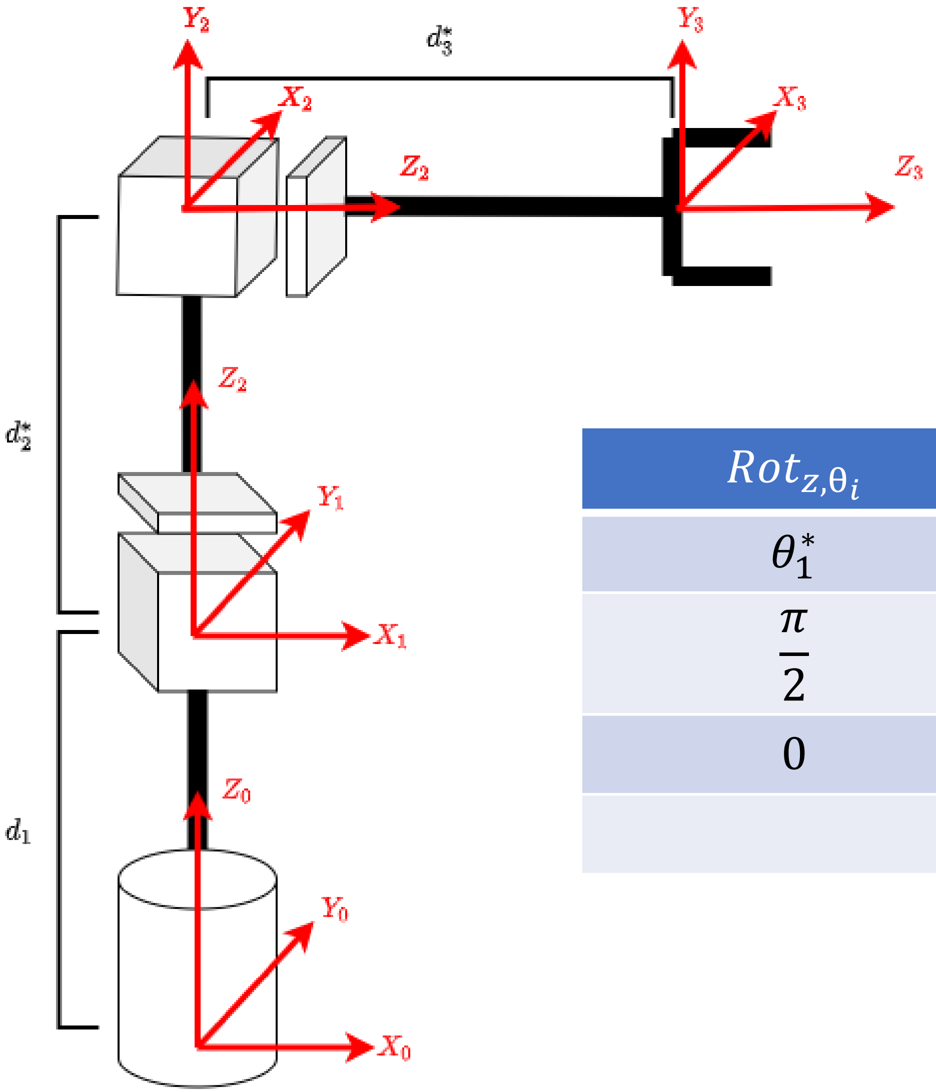
- Given a set of end-effector position, find the joint angles

- In general more difficult than forward-kinematics.

- Two approaches – Analytical and Geometric

- The first is about solving a set of equations given forward-kinematic
- The second one is solving the joint angles using trigonometry and geometry.

Kristians Example.



DH-parameters

Rot_{z,θ_i}	$Trans_{z,d_i}$	$Trans_{x,a_i}$	Rot_{x,α_i}
θ_1^*	d_1	0	0
$\frac{\pi}{2}$	d_2^*	0	$\frac{\pi}{2}$
0	d_3^*	0	0

Forward-kinematic of manipulator

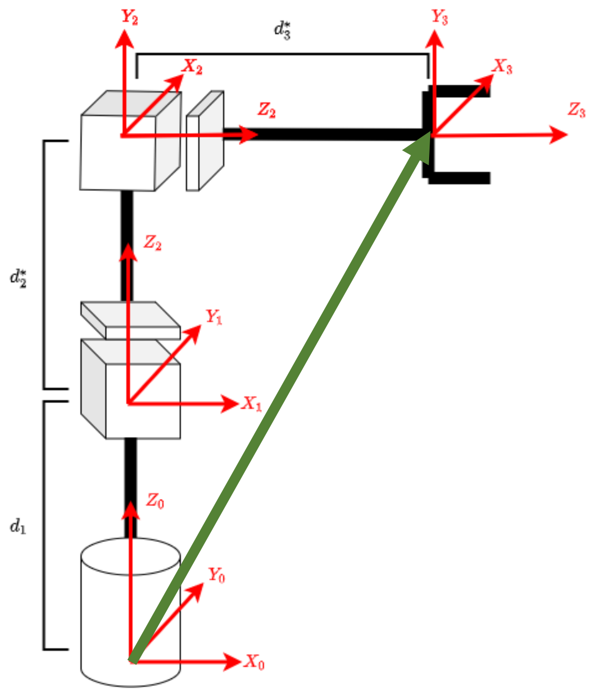
$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

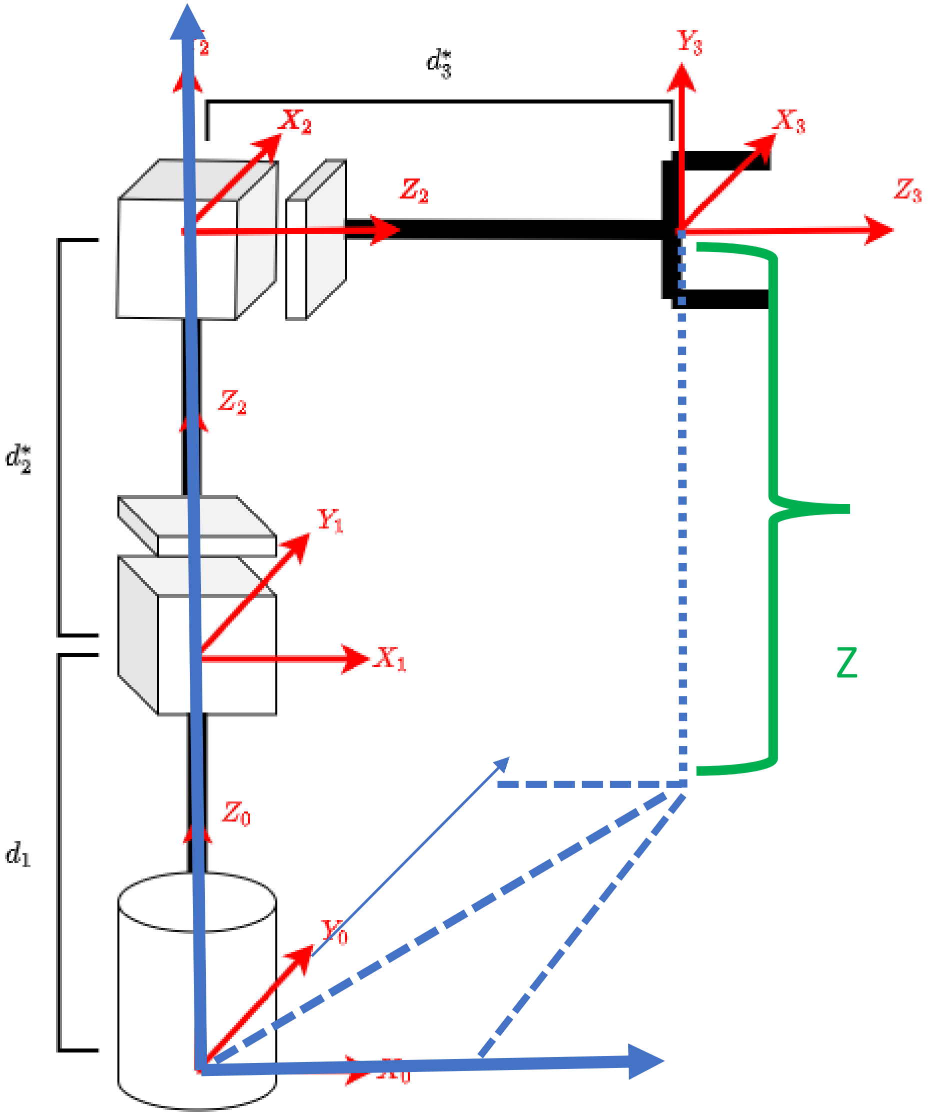
$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

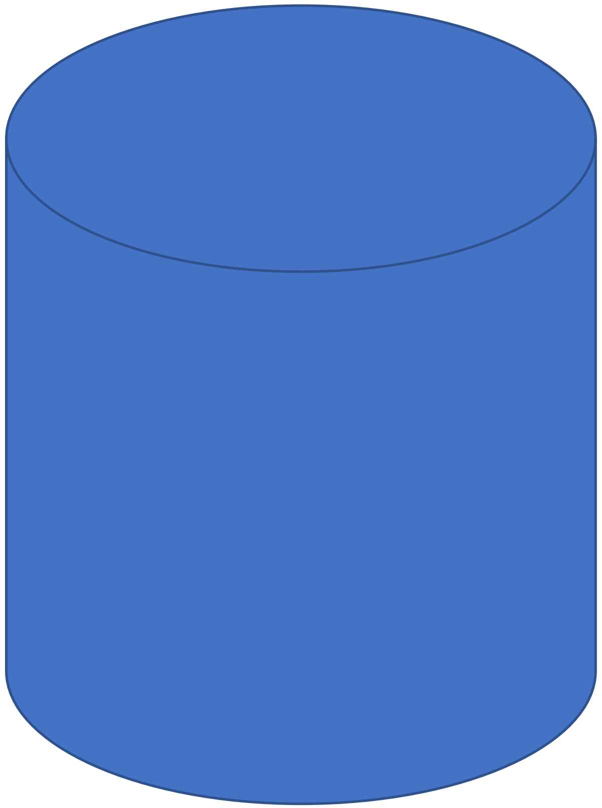
$$= \begin{bmatrix} -s & 0 & c_1 & c_1 d_3^* \\ c_1 & 0 & s_1 & s_1 d_3^* \\ 0 & 1 & 0 & d_1 + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



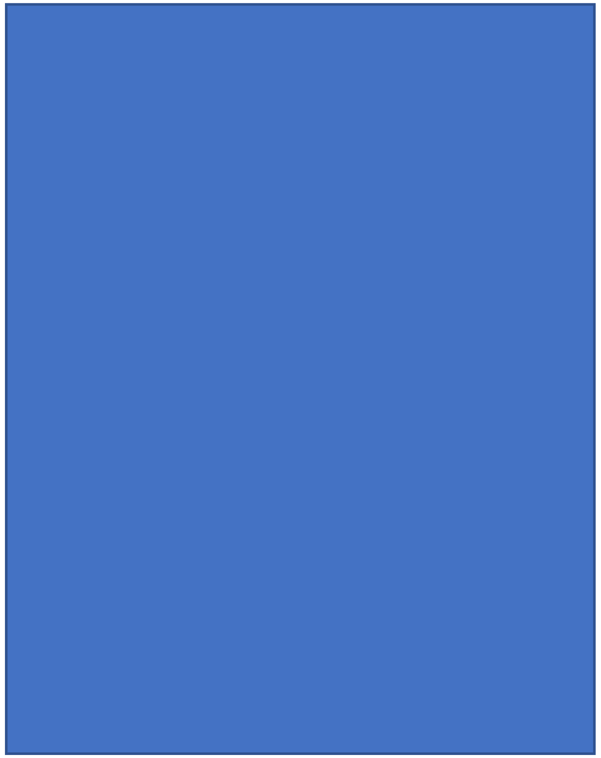
Workspace and possible configurations



3D



2D



Inverse-Kinematic – Analytic approach

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s & 0 & c_1 & c_1 d_3^* \\ c_1 & 0 & s_1 & s_1 d_3^* \\ 0 & 1 & 0 & d_1 + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Tan x = $\frac{\sin x}{\cos x}$

We know x,y,z coordinate

We must solve θ_1^*, d_2^*, d_3^*

$$\theta_1 = \arctan(\frac{y}{x})$$

$$x = c_1 d_3^* \longrightarrow d_3^* = \frac{x}{c_1}$$

$$y = s_1 d_3^* \longrightarrow d_3^* = \frac{y}{s_1}$$

$$z = d_1 + d_2^* \longrightarrow -d_1 + z = d_2^*$$

IK-solutions:

$$\theta_1^* = \arctan(\frac{y}{x})$$

$$d_2^* = z - d_1$$

$$d_3^* = \sqrt{x^2 + y^2}$$

$d_3^* = \frac{x}{c_1}$ or $\frac{y}{s_1}$ if we solve for θ_1^* first

Solution for d_3^*

$$r^2 = x^2 + y^2$$

$$r^2 = (c_1 d_3^*)^2 + (s_1 d_3^*)^2$$

$$r^2 = c_1^2 {d_3^*}^2 + s_1^2 {d_3^*}^2$$

$$r^2 = {d_3^*}^2 (c_1^2 + s_1^2)$$

$$r^2 = {d_3^*}^2 (1)$$

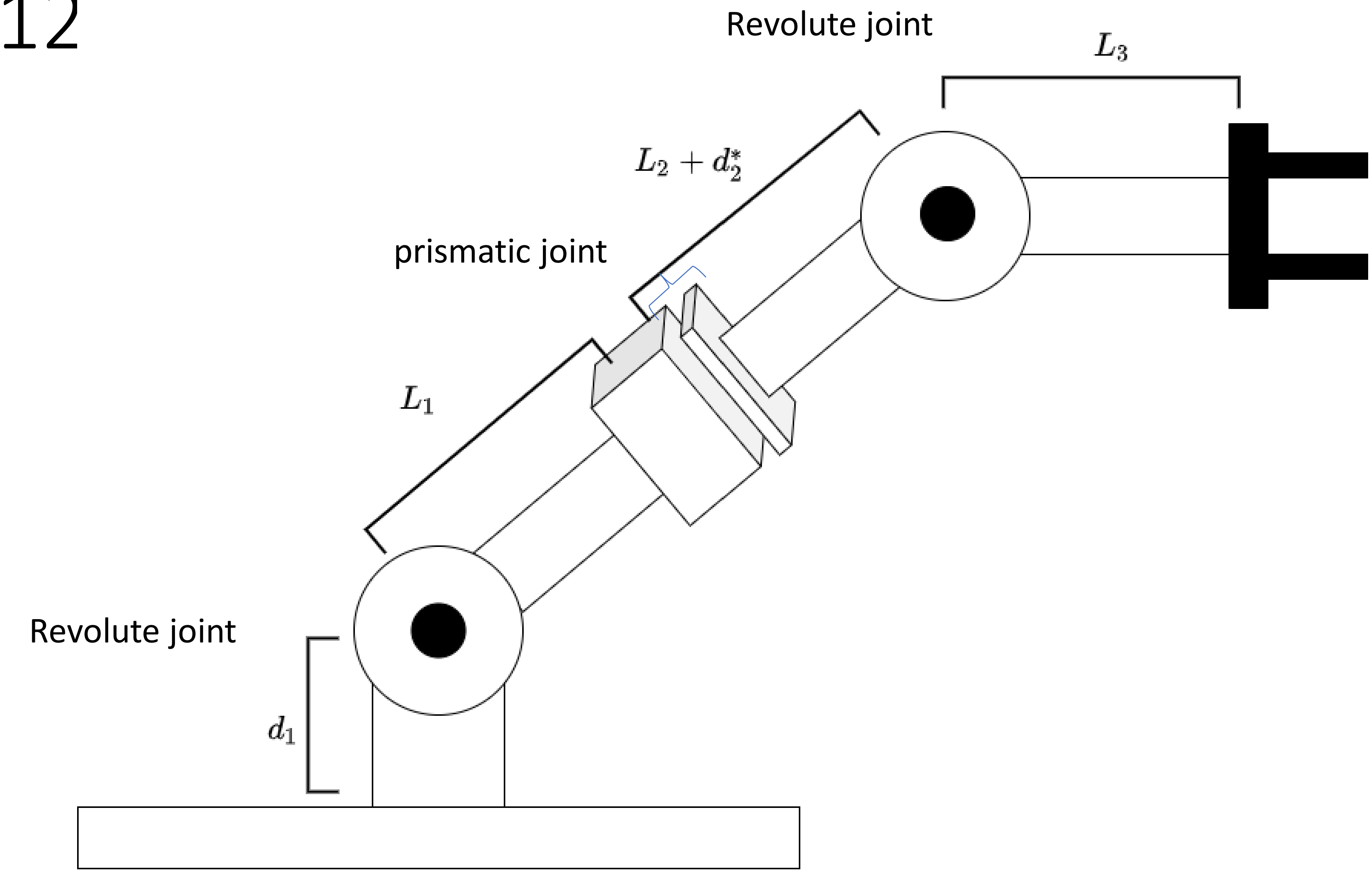
$$r^2 = {d_3^*}^2$$

$${d_3^*}^2 = r^2$$

$$\sqrt{{d_3^*}^2} = \sqrt{r^2}$$

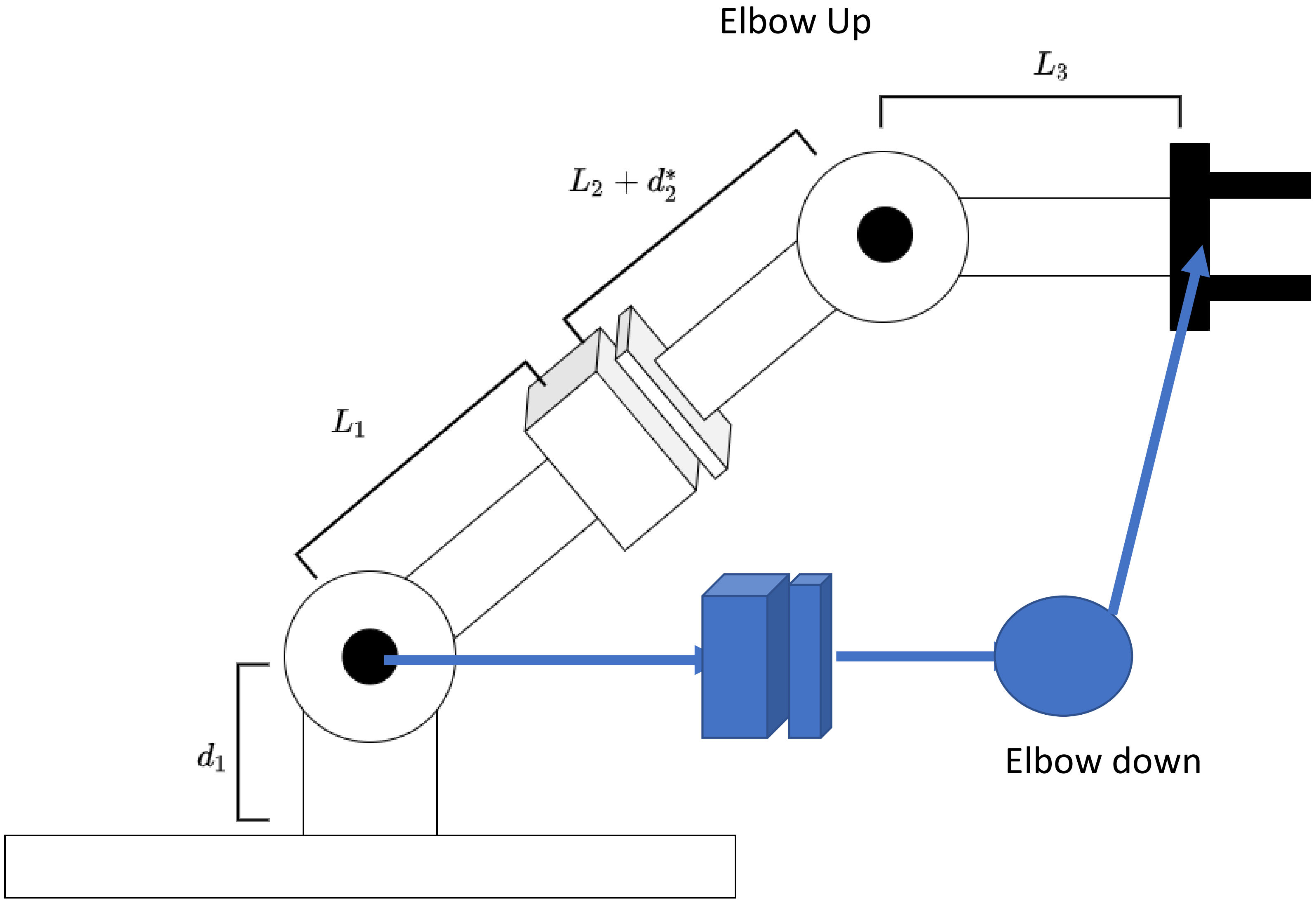
$$d_3^* = \sqrt{r^2} = \sqrt{x^2 + y^2}$$

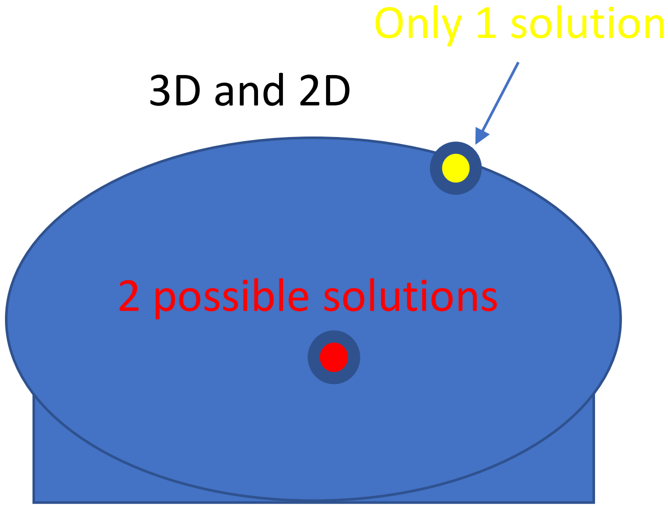
3.12

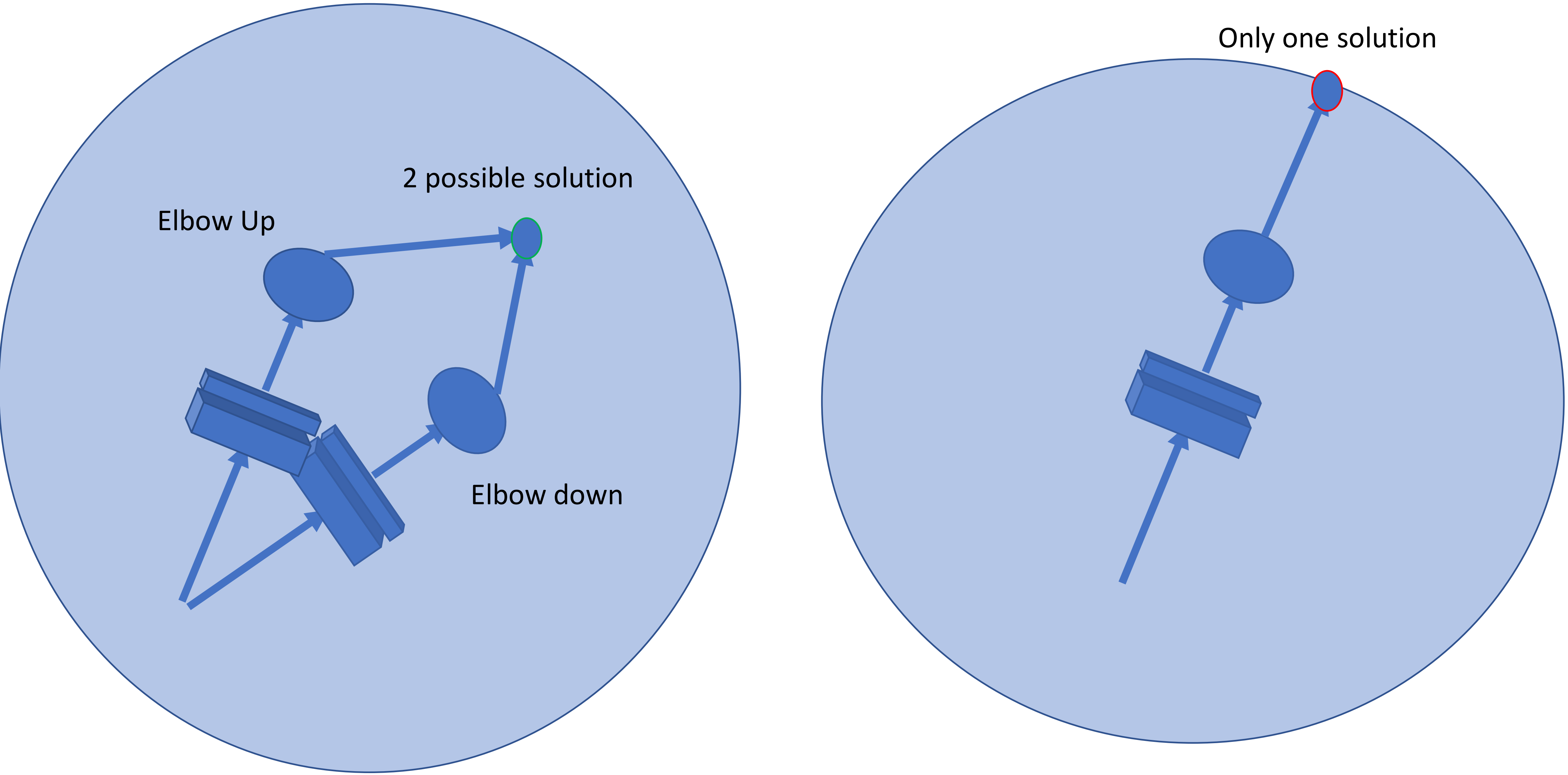


Revolute joint
L_3
$L_2 + d_2^*$
prismatic joint
L_1
Revolute joint
d_1

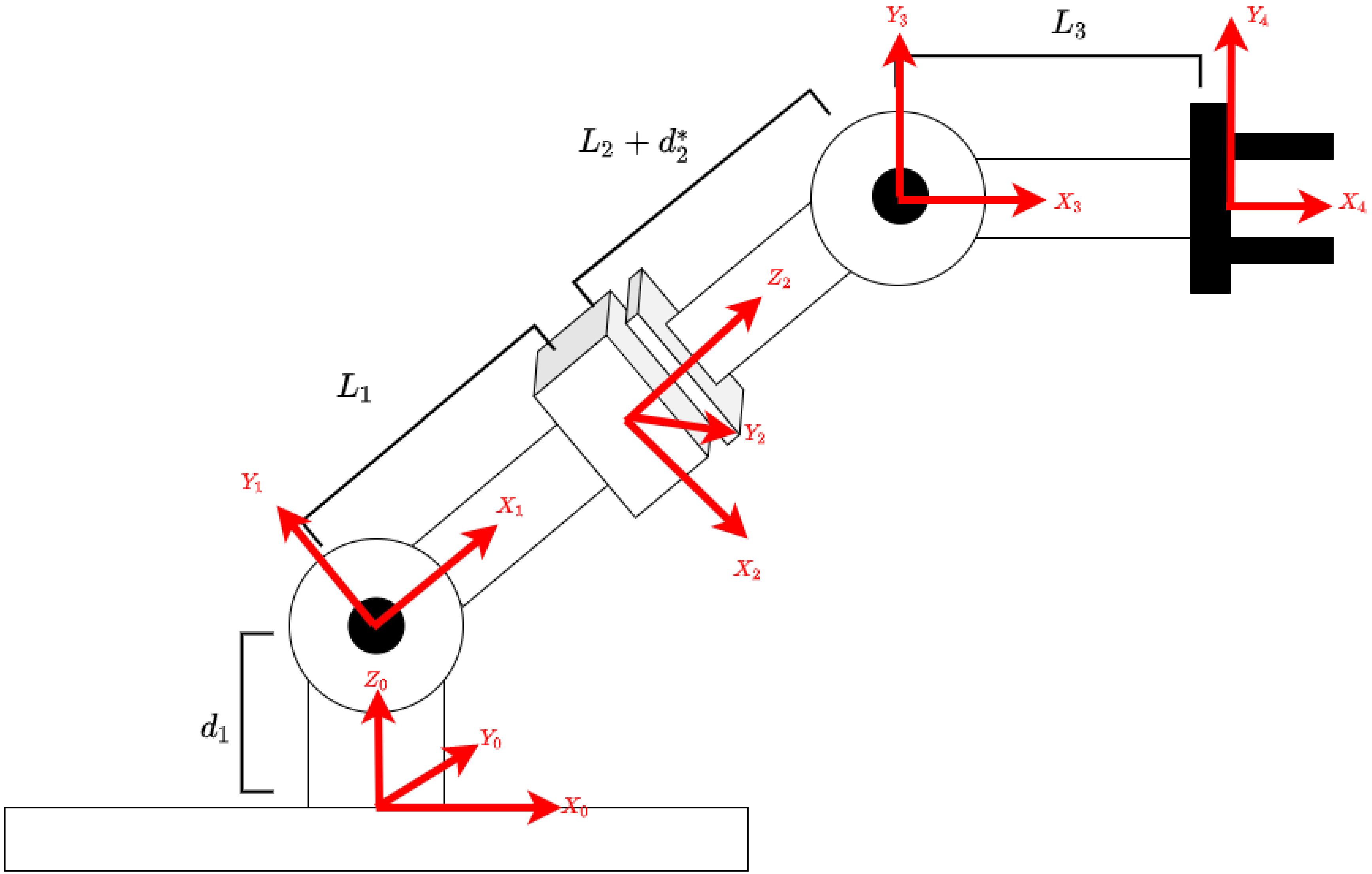
Workspace and possible configurations?

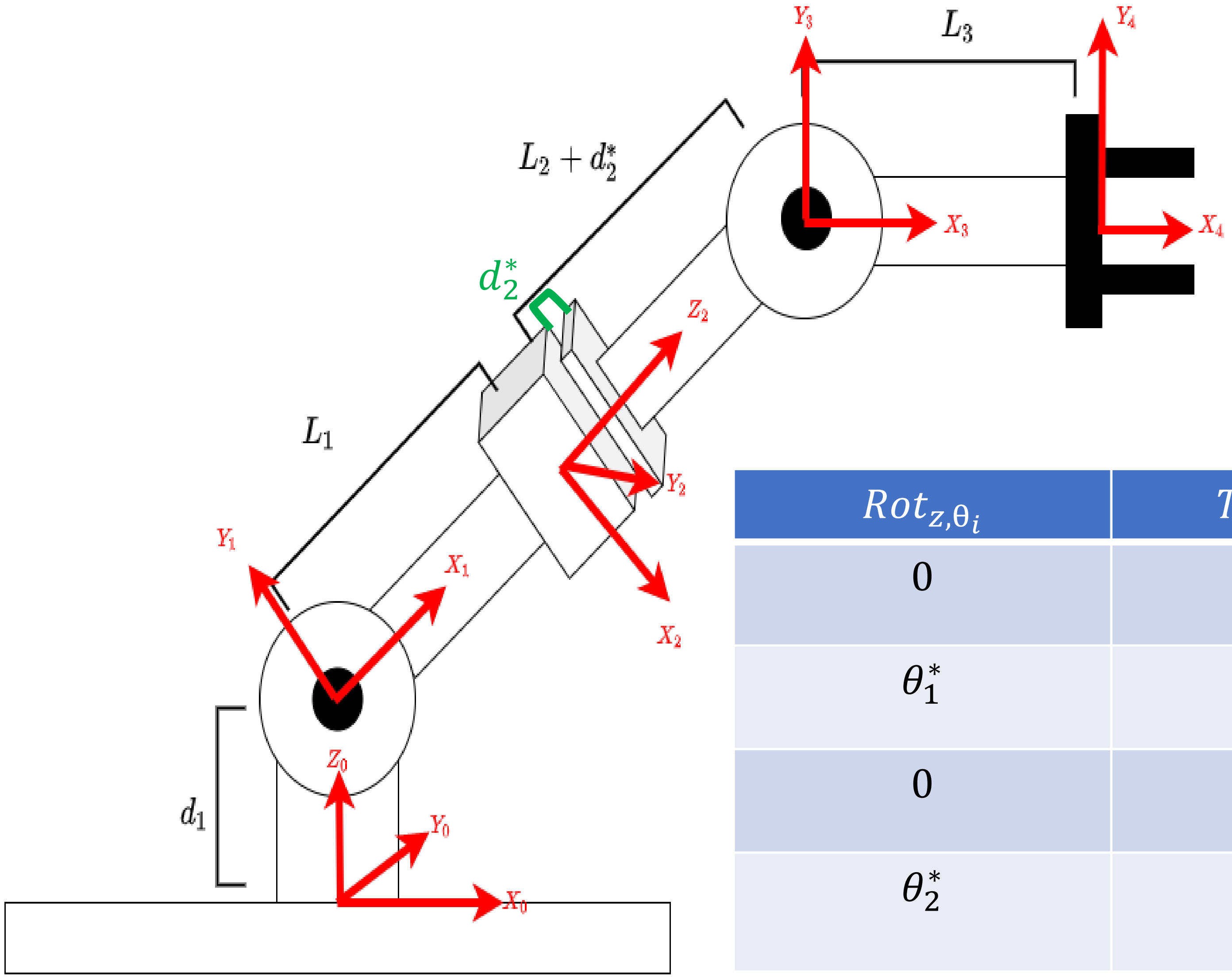





2 possible solution
Elbow Up
Elbow down
Only one solution

3.12 cont





Rot_{z,θ_i}	$Trans_{z,d_i}$	$Trans_{x,a_i}$	Rot_{x,α_i}
0	d_1	0	$\frac{\pi}{2}$
θ_1^*	0	L_2	$-\frac{\pi}{2}$
0	$L_2 + d_2^*$	0	$\frac{\pi}{2}$
θ_2^*	0	L_2	0

Forward-kinematic of manipulator

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

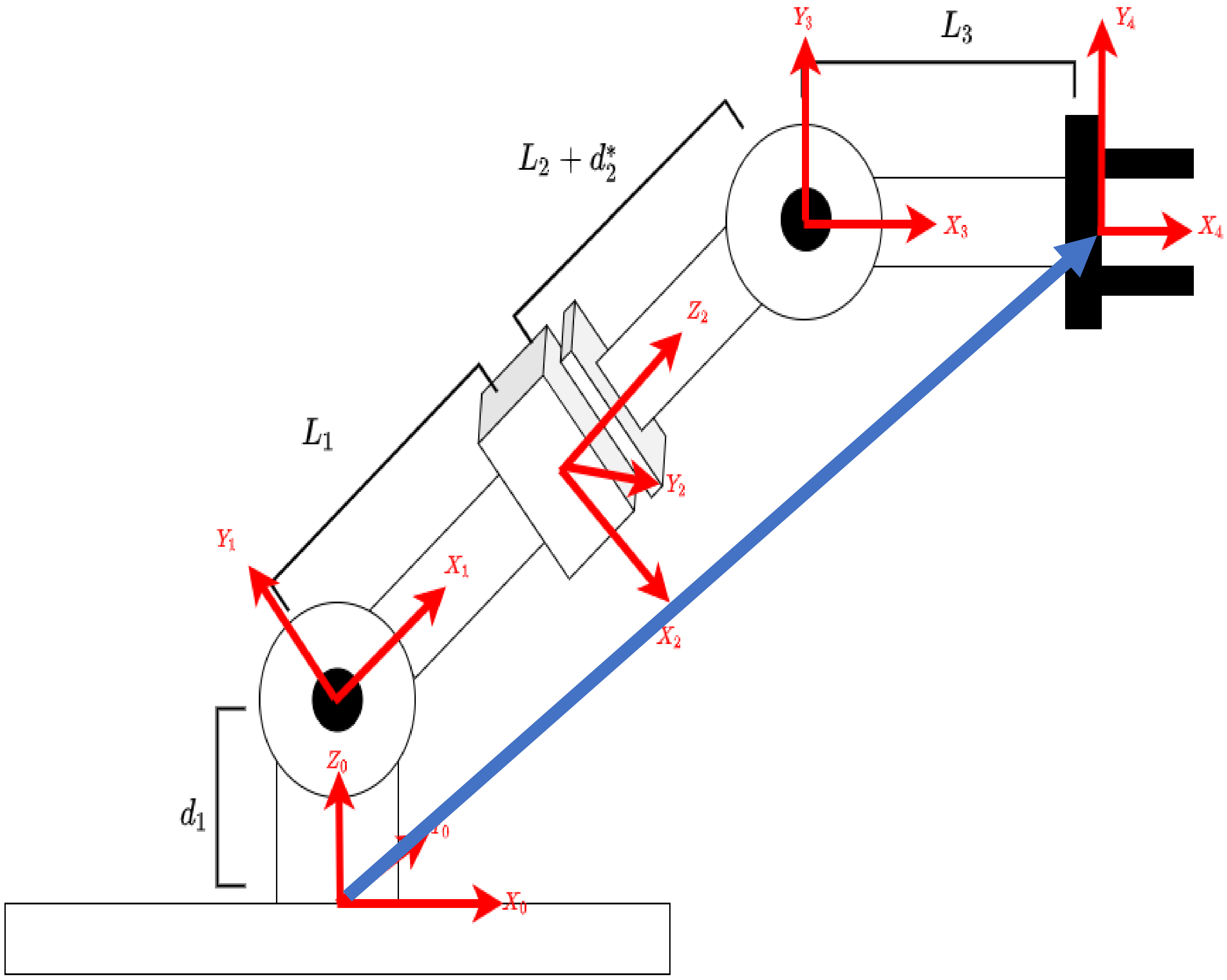
$$A_2 = \begin{bmatrix} c_1 & 0 & -s_1 & L_1 c_1 \\ s_1 & 0 & c_1 & L_1 s_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & L_2 + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_2 & -s_2 & 0 & L_3 c_2 \\ s_2 & c_2 & 0 & L_3 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = A_1 A_2 A_3 A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 & 0 & -s_1 & L_1 c_1 \\ s_1 & 0 & c_1 & L_1 s_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & L_2 + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & L_3 c_2 \\ s_2 & c_2 & 0 & L_3 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = A_1 A_2 A_3 A_4 = \begin{bmatrix} c_{12} & s_{12} & 0 & L_3 c_{12} - s_1 (L_2 + d_2^*) \\ 0 & 0 & 1 & 0 \\ s_{12} & c_{12} & 0 & L_3 s_{12} + c_1 (L_2 + d_2^*) + L_1 s_1 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



L_3
Y_3
Y_4
$L_2 + d_2^*$
X_3
X_4
Z_2
L_1
Y_2
Y_1
X_1
X_2
Z_0
d_1
X_0

Inverse-kinematic – Geometric approach

$$T_4^0 = A_1A_2A_3A_4 = \begin{bmatrix} c_{12} & s_{12} & 0 & L_3c_{12} - s_1(L_2 + d_2^*) \\ 0 & 0 & 1 & 0 \\ s_{12} & c_{12} & 0 & L_3s_{12} + c_1(L_2 + d_2^*) + L_1s_1 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = L_3c_{12} - s_1(L_2 + d_2^*)$$
$$y = 0$$
$$z = L_3s_{12} + c_1(L_2 + d_2^*) + L_1s_1 + d_1$$

Three unknowns:
$\theta_1, \theta_2, d_2^*$

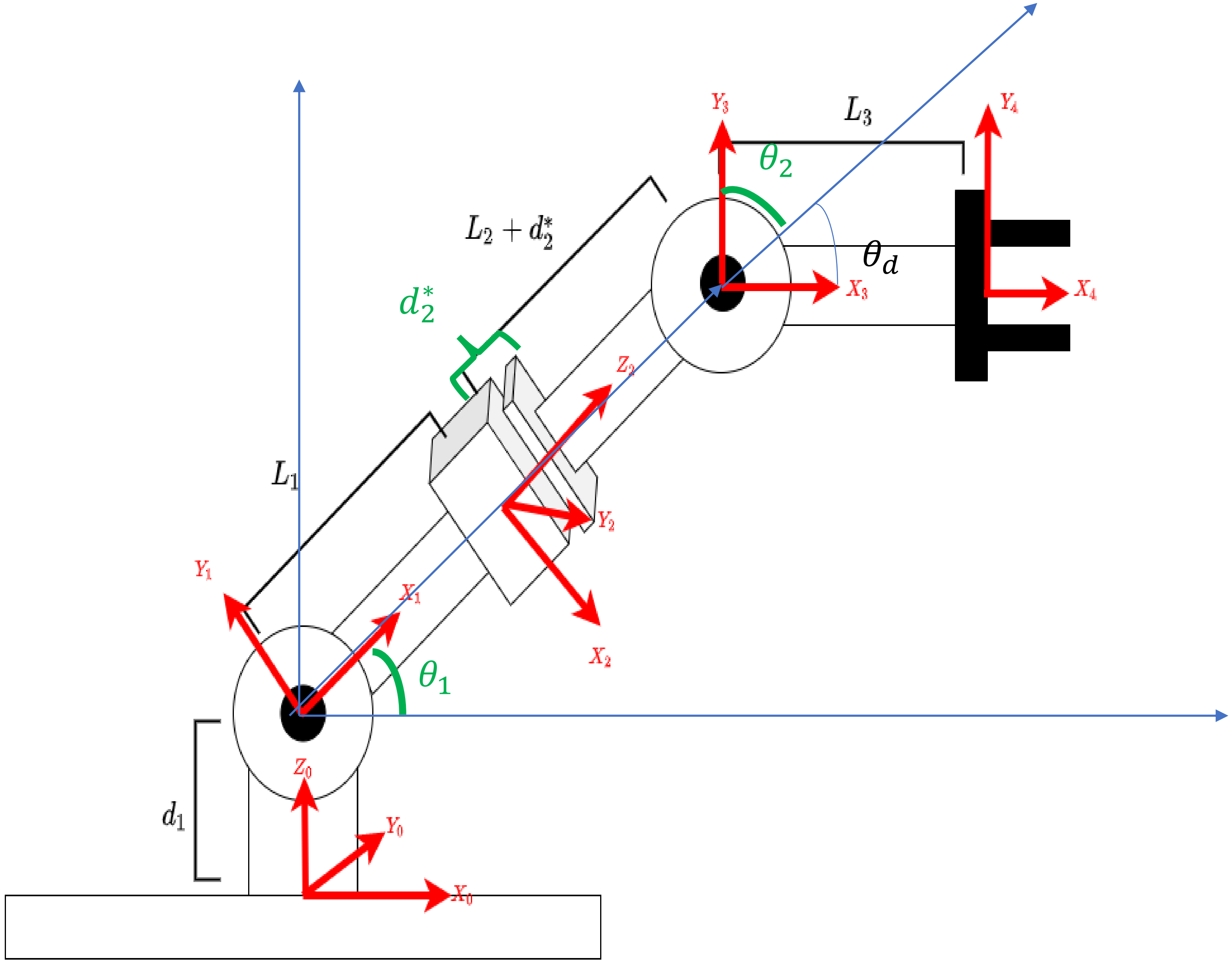
Hard to solve analytical, unless one knows the trigonometric identities or relationship between the equations.

-Try to sketch lots of drawings with possible configurations and preferably on 2D-plane.

-Try to find the unknown variables with respect to your own sketch, ie the angles matches perfectly to your drawing.

Our task is to find

θ_1, d_2^* and θ_2



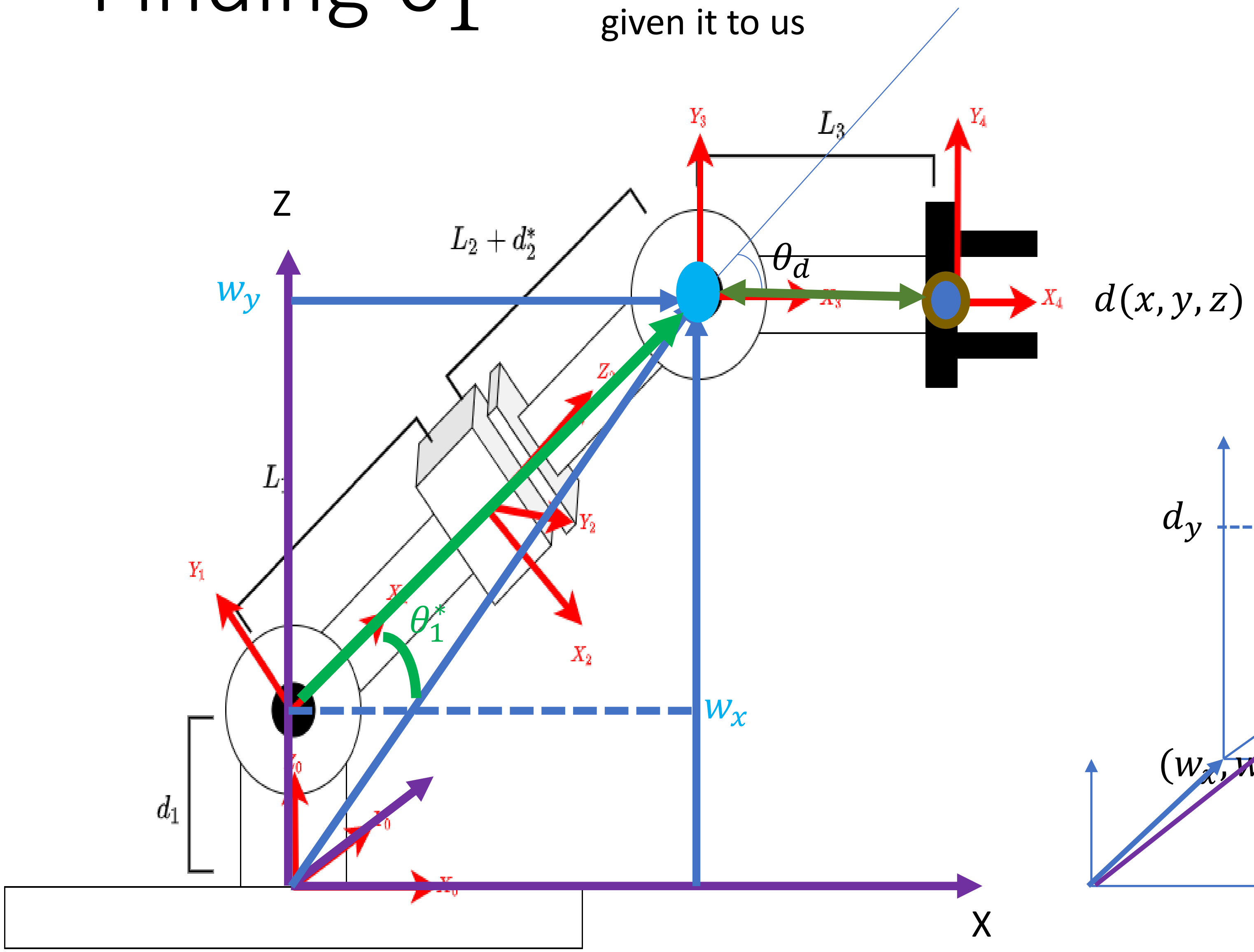
Finding θ_1

We know θ_d because the task has given it to us

$w_x = d_x - L_3 \cos(\theta_d)$

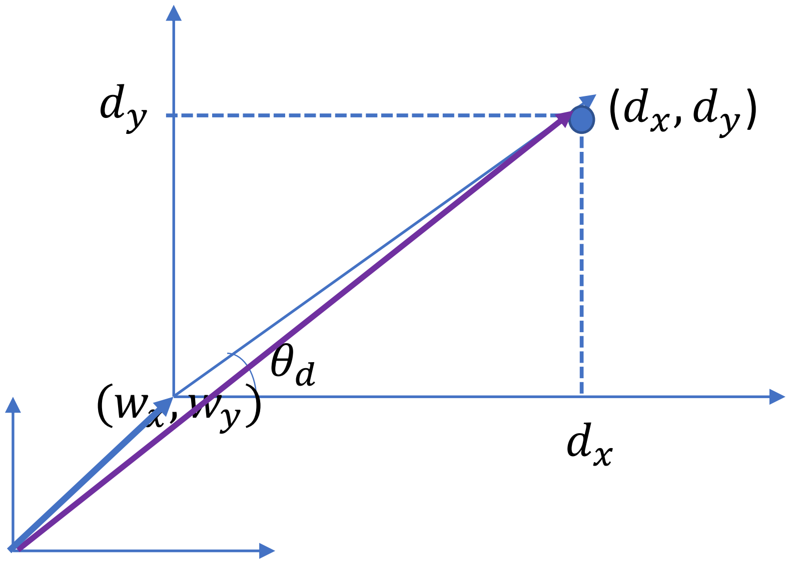
$w_y = d_y - L_3 \sin(\theta_d) - d_1$

$\theta_1^* = \tan^{-1}(w_x, w_y)$

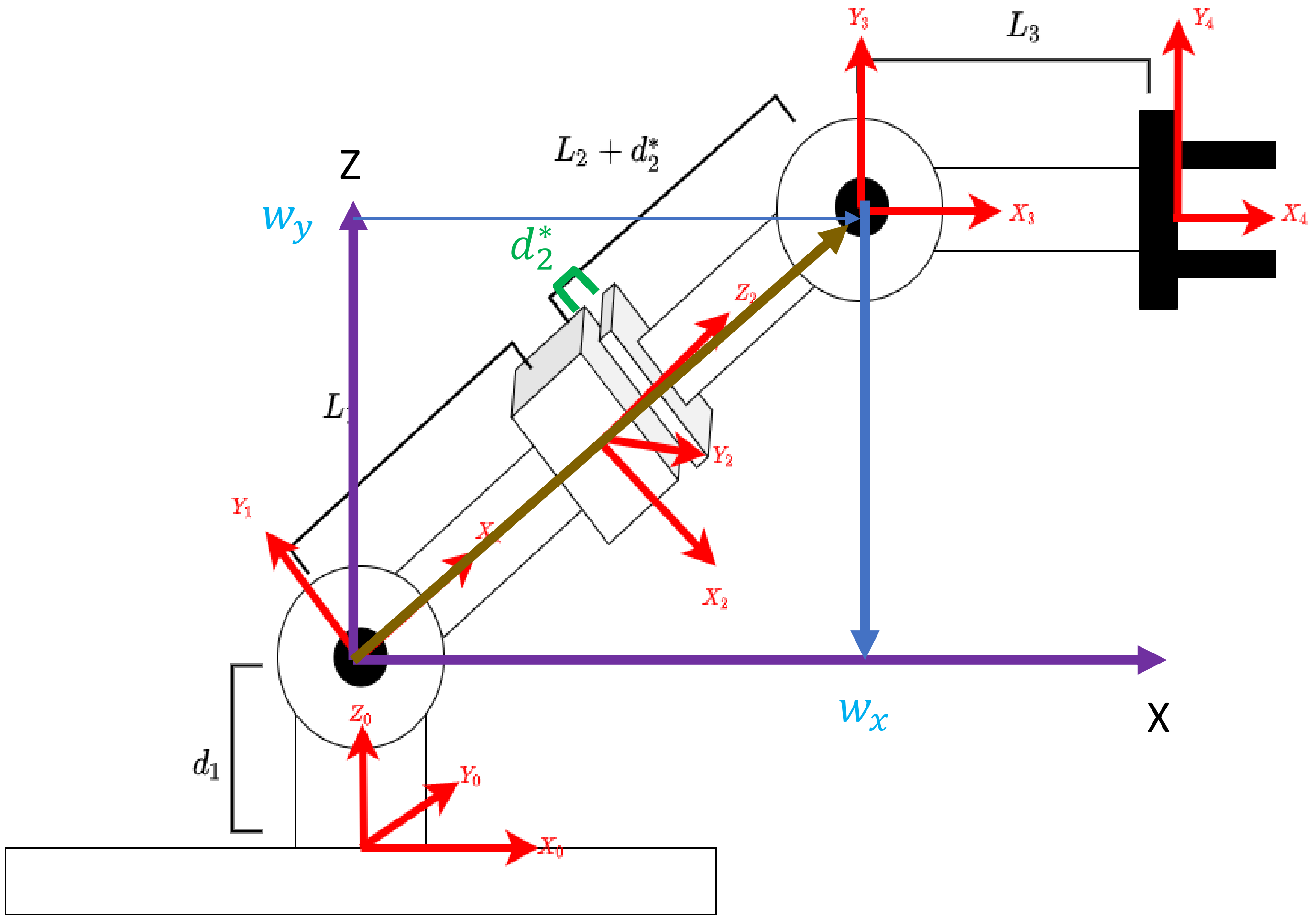


sin (x) = opposite/hypotenus

cos (x) = adjacent/hypotenus



Finding d_2^*



$r^2 = {w_x}^2 + {w_y}^2$

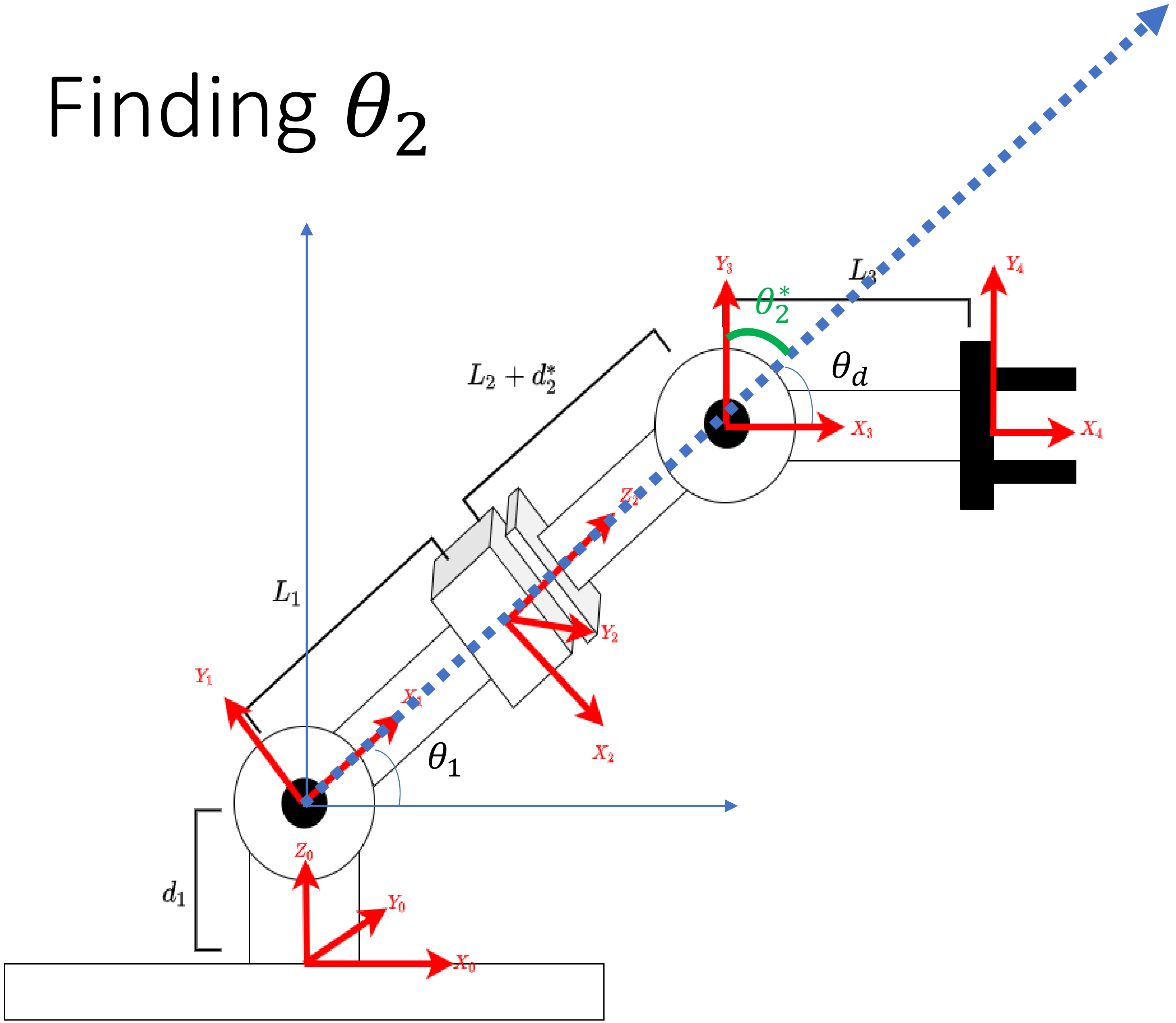
$r = \sqrt{{w_x}^2 + {w_y}^2}$

$r = \sqrt{{w_x}^2 + {w_y}^2} = L_1 + L_2 + d_2^*$

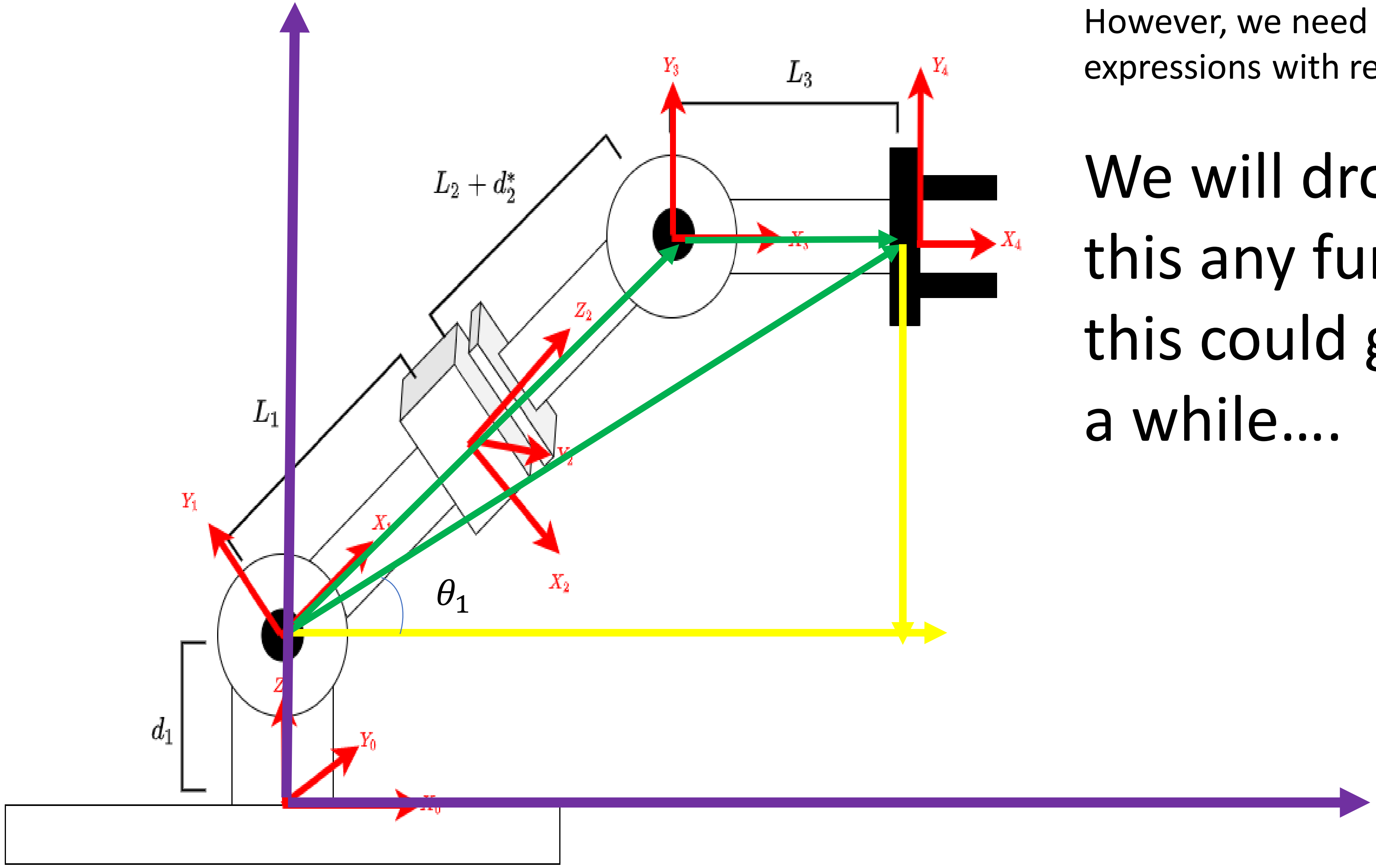
$-L_1 - L_2 + \sqrt{{w_x}^2 + {w_y}^2} = d_2^*$

Finding θ_2

$\theta_2^* = \theta_d - \theta_1$



Alternate solution, theta_1



However, we need to find the expressions with respect to theta_1.

We will drop solving this any further as this could go on for a while….

Tips: Relevant for mandatory assignment

$\cos y = x$

$Y = \arccos(x)$

$sin^2 y + cos^2 y = 1$

$\sin y = \sqrt{1 - cos^2 y} \longrightarrow \sin y = \sqrt{1 - x^2}$

$\tan y = \frac{\sin y}{\cos y} \longrightarrow y = \arctan\left(\frac{\sin y}{\cos y}\right) = atan2(\sin y, \cos y)$

$y = \arctan\left(\frac{\pm\sqrt{1 - x^2}}{x}\right) = atan2(\pm\sqrt{1 - x^2}, x)$