Group Seminar Week 2: Homogeneous Transform

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- Responsible for correcting the mandatory assignment
- Backup Group-teacher

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Session's plan

- Brief talk on mandatory assignments

- A short example on homogeneous transform

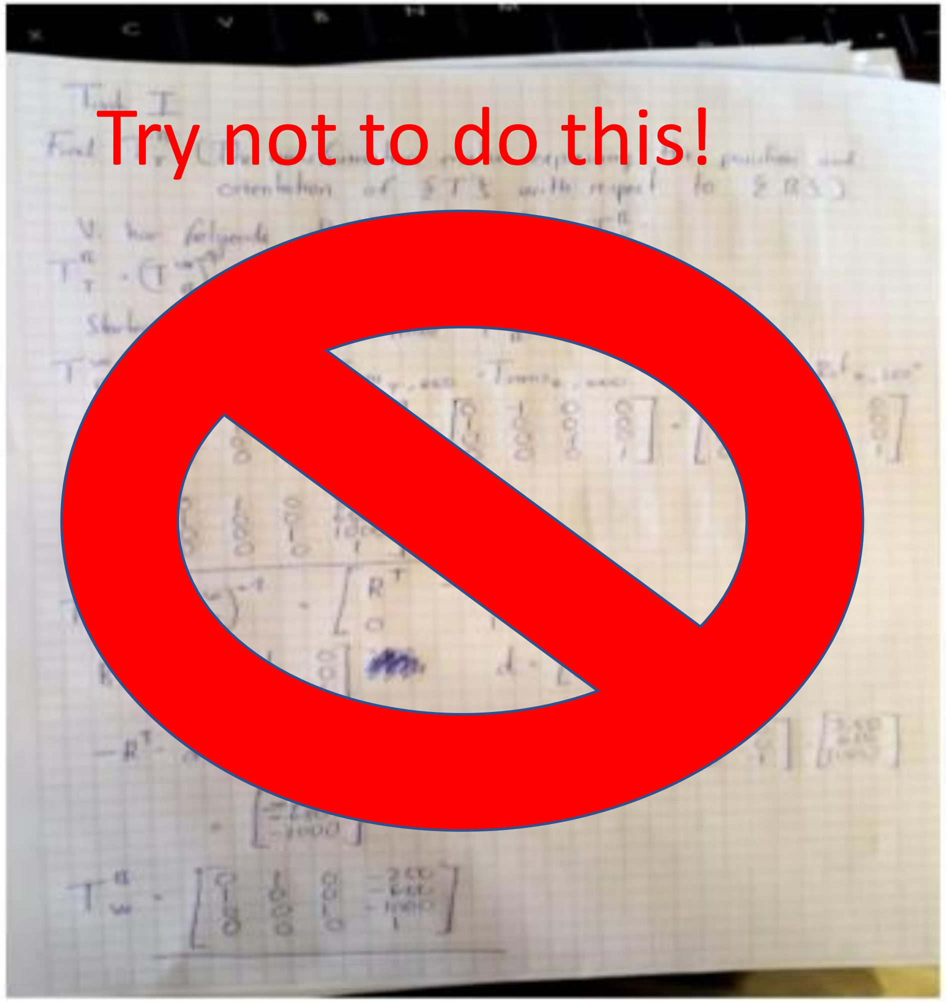
- Walkthrough on weekly exercise 2.10-2.11, 2.38.

Delivery

- Start early on the assignments, each task becomes solvable for each lecture
- All assignments must be **”PASSED”** in order to take exam
- Devilry -
- Postponing the delivery, send an e-mail to one of us

  \- Preferably early and couple of days beforehand

Delivering the assignment



- Do one of these two methods:

Word

Task I:

Task 1a) → Løst i matlab, se vedlegg

Task 1b) → Løst i matlab-filene, se vedlegg

Task 1c) - Use the functions to show how you can verify that the inverse and forward kinematics are correctly derived.

Ved å først ta å beregne DH-matrisene for hele robot-manipulatoren, så utledes forward kinematic og inverse kinematic. Men dette ble gjort i forrige oblig og jeg antar og forutsetter at det fremdeles samme manipulator og ingen endringer i linkene slik at forward –og inverse kinematic er det samme.

Ved å bruke de kartetiske punktene og vinklene fra forrige oblig, altså:

[redacted]

skal man få punktet ovenfor

Disse punktene kommer i fra forrige oblig.

Task II:

a)

Vi har følgende matriser til forward kinematics som er:

$$A_1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & L_2C_2 \\ S_2 & C_2 & 0 & L_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & L_3C_3 \\ S_3 & C_3 & 0 & L_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_1^0 = A_1,$

$$T_2^0 = T_1^0 * T_2^1 = A_1 * A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & L_2C_2 \\ S_2 & C_2 & 0 & L_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} C_3 & -S_3 & 0 & L_3C_3 \\ S_3 & C_3 & 0 & L_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_2C_1 & -S_2C_1 & -S_1 & L_2C_2C_1 \\ S_1C_2 & -S_2S_1 & C_1 & L_2C_2S_1 \\ -S_2 & -C_2 & 0 & L_1 - L_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[redacted]

Hpnguyen Page 2

LaTeX

1 Rotation and Translation

A rotation matrix is a matrix that is used to perform a rotation in euclidean space. In 3 dimensional space a basic rotation is a rotation about one of the axes of a coordinate system. As such we can perform a single rotation θ along one of three different axis which are following x, y, z in the respective coordinate frame.

Rotation matrix for rotation around x-axis with θ degree and observe that x-axis every axis are perpendicular with one another. This is equal for all of the rotation matrices.

$$Rot_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \tag{1}$$

Rotation matrix for rotation around y-axis with θ degree

$$Rot_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \tag{2}$$

Rotation matrix for rotation around z-axis with θ degree

$$Rot_{z,\theta} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Translation matrix is a very useful matrix because its linearity allows for selective translation in 3d in either one, two or all of the three of axis'. For example, if we only want translation along z-axis we can extract only the rotation matrix, a $3x3$ diagonal matrix in the translation matrix (4) and setting the t_x and t_y to 0 and only replace t_z with the designated translation.

$$Trans_x = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Combining the rotation and translation matrices we can then

$$A_i = \tag{5}$$

A_i is represented as a product of four basic transformations and it is known as Denavit Hartenberg parameters.

$$A_i^{i-1} = Rot_{z,\theta} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha} = \tag{6}$$

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \tag{7}$$

LaTeX

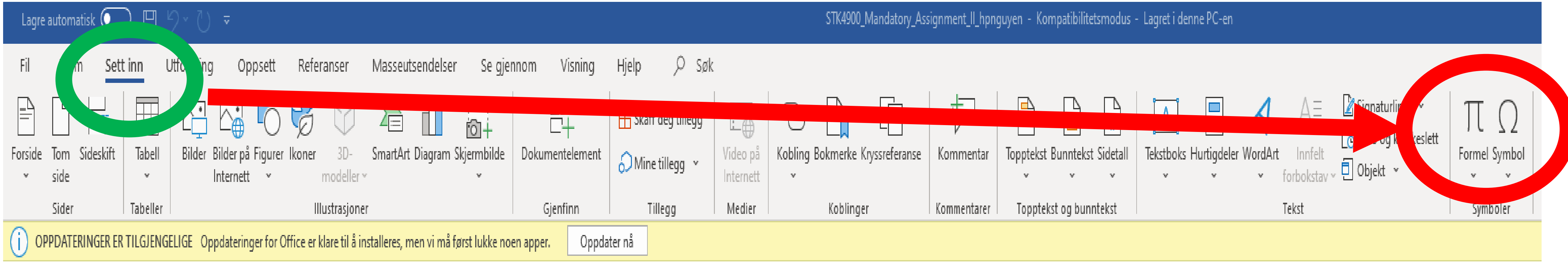
- Download on your personal laptops
- Use **Overleaf.com → I highly recommend this**

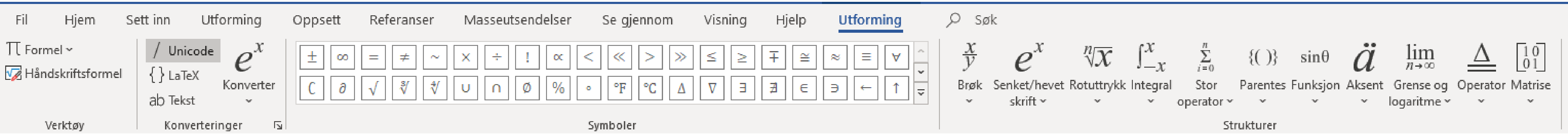
Question to you guys:

- Raise your hand if you want me to put out a simple template on LaTeX on the course page.

Word

- Intuitively simple to use, but can be tedious on long equations.





Raise your hand if you want me to put out a simple template on Word on the course-page or type in chat if you guys want me to create a template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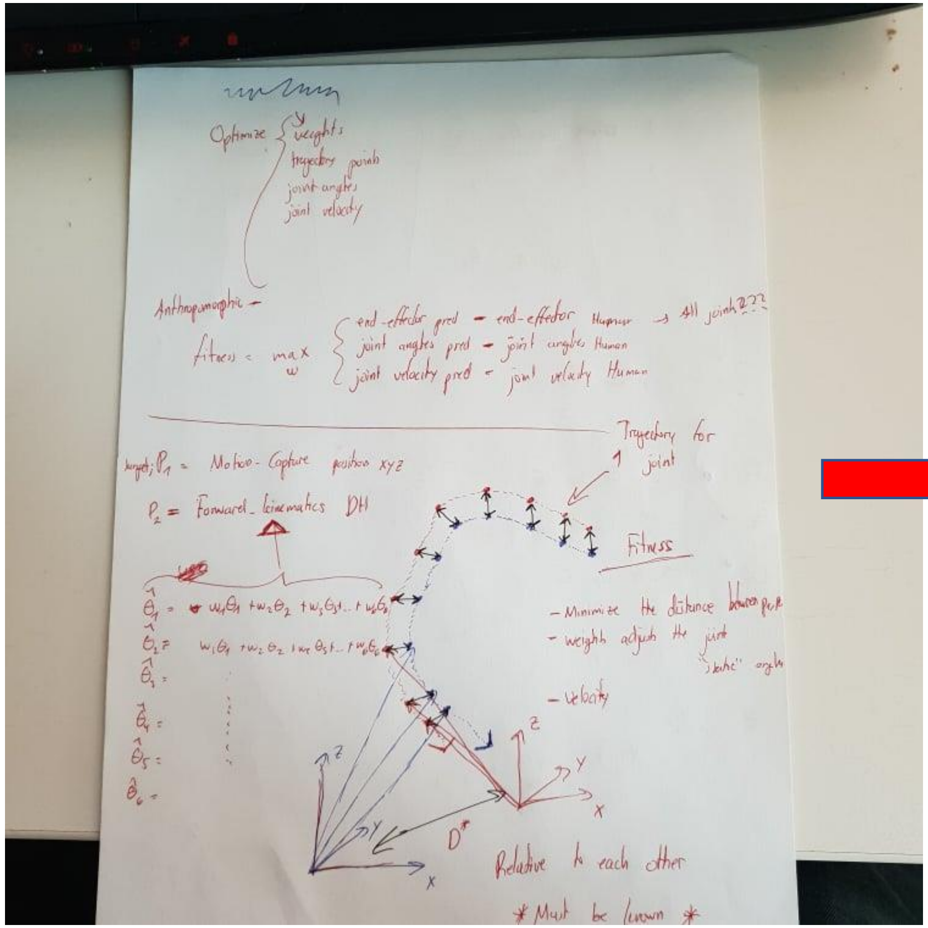
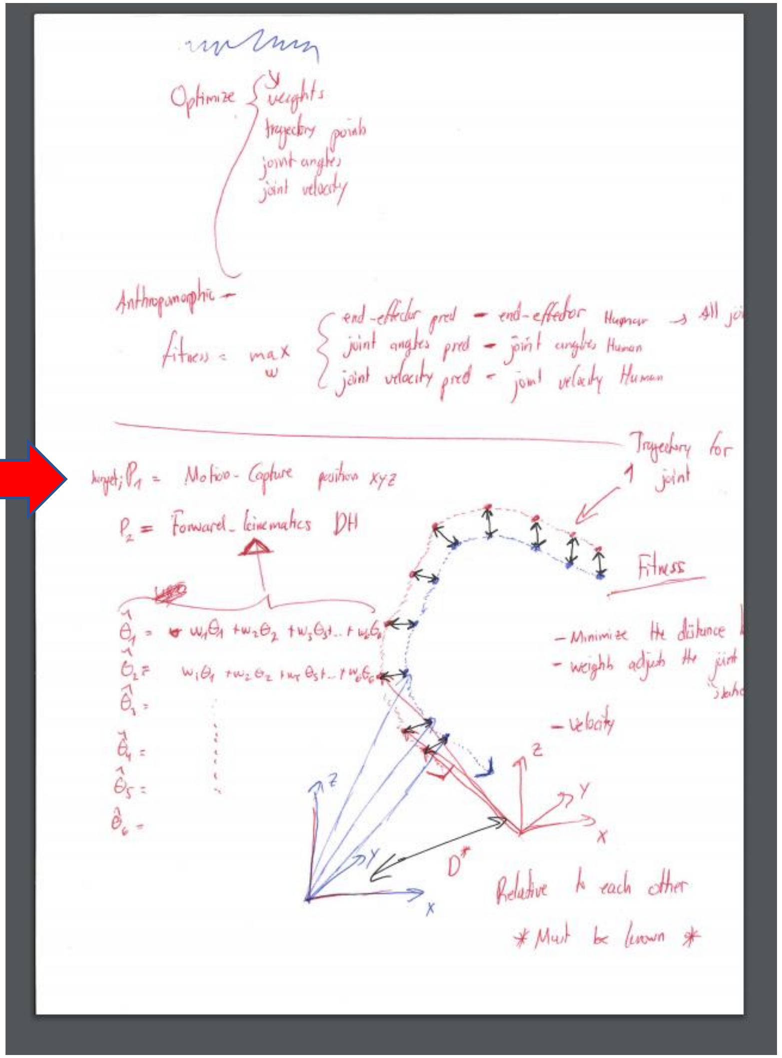
Please wait.

Before Scan

After Scan





Programming Code:

- Don’t
- Write … ing
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  Examp… and

  etc…”
- You M…

```python
import numpy as np

"""This Function solves x+y+z and returns it
   params: x = x-coordinates
           y = y-coordinates
           z = z-coordinates
"""
def function1(x,y,z):
    return x+y*z

"""This Function counts how many pancakes there are if there exists some on the moon and
   returns the amount.
   params: cool_param1 = boolean value, if there are pancakes at the moment
           cool_param2 = How many pancakes on the moon at the moment
"""
def foo(cool_param1, cool_param2):
    if cool_param1:
       return cool_param2
    return cool_param1
```

Homogeneous Transform

Represents position and orientation of rigid-body relative to another frame

H=$\begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$, R is rotation matrix (3x3),

d is translation vector (1x3)

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

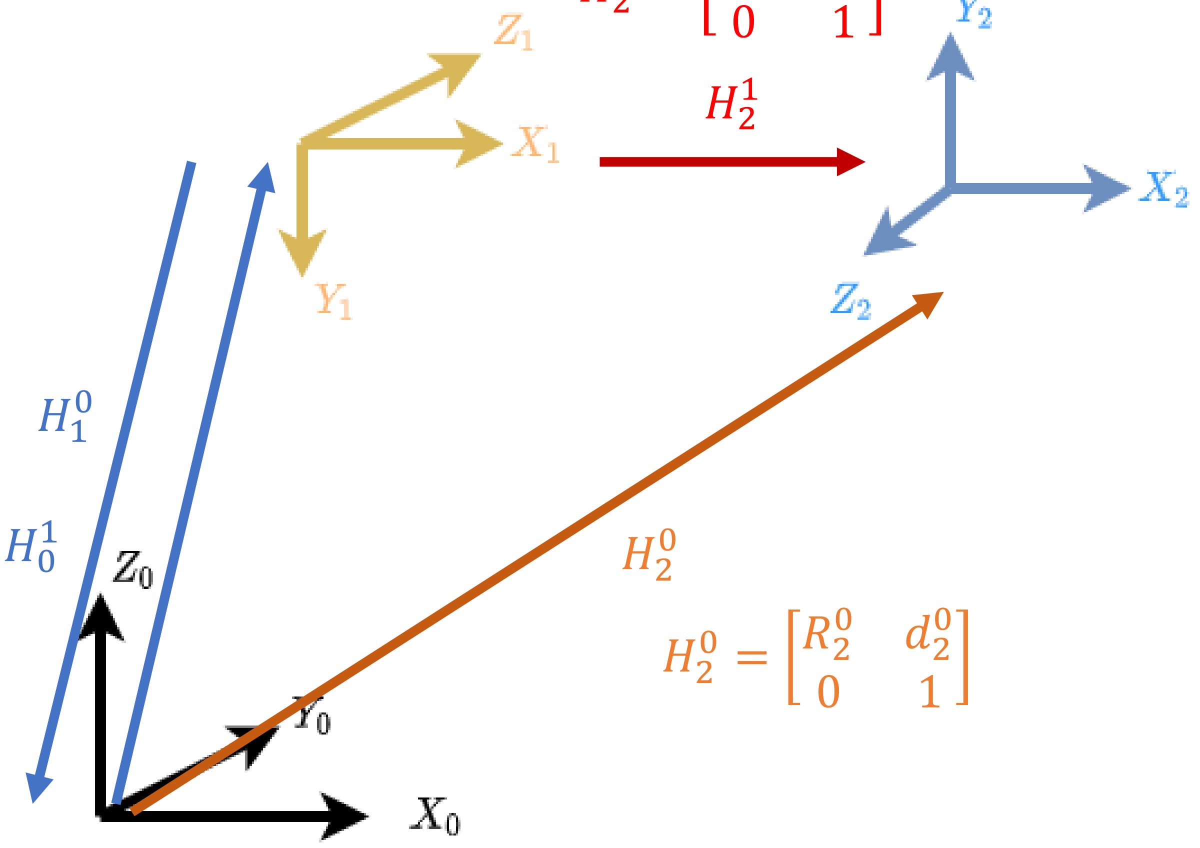
$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_1^0 = \begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix}$$

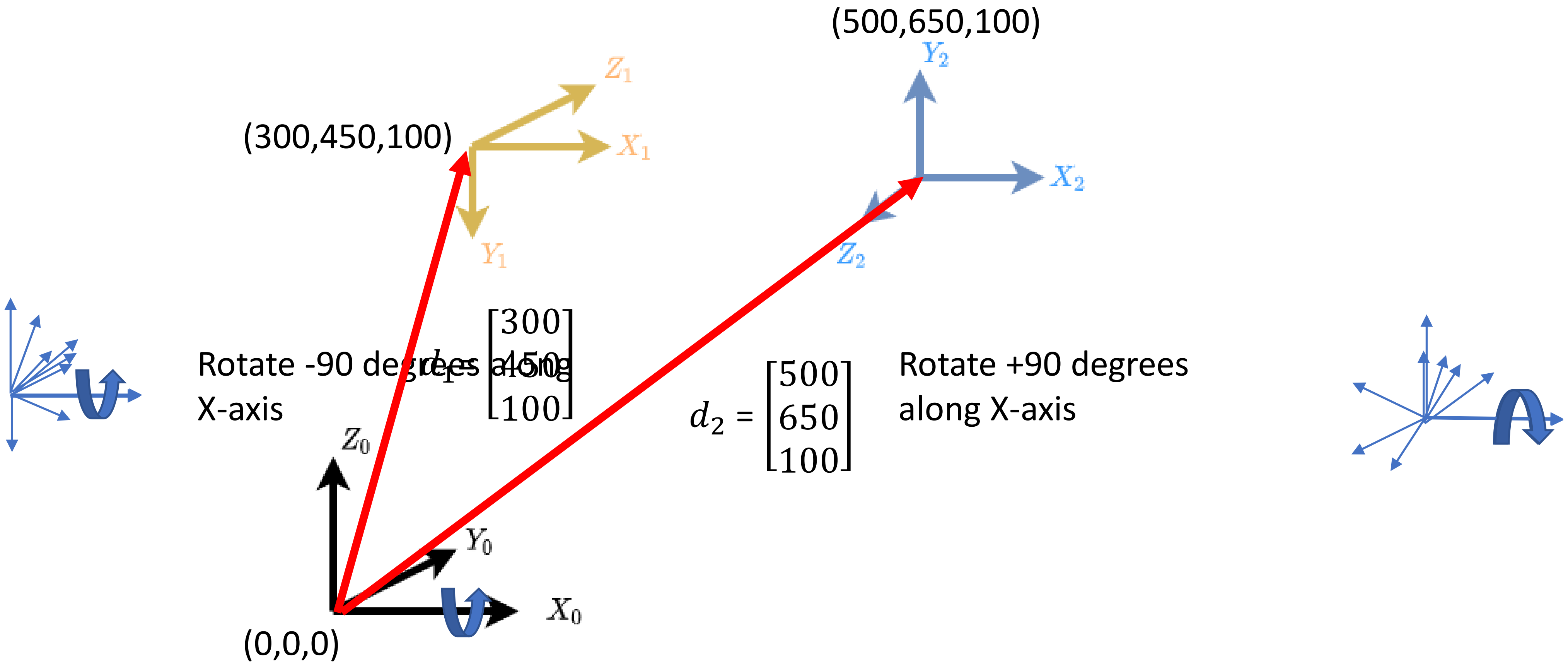
$$H_0^1 = (H_1^0)^{-1} = \begin{bmatrix} {R_1^0}^T & -{R_1^0}^T d_1^0 \\ 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} R_2^1 & d_2^1 \\ 0 & 1 \end{bmatrix}$$

$$H_2^0 = \begin{bmatrix} R_2^0 & d_2^0 \\ 0 & 1 \end{bmatrix}$$

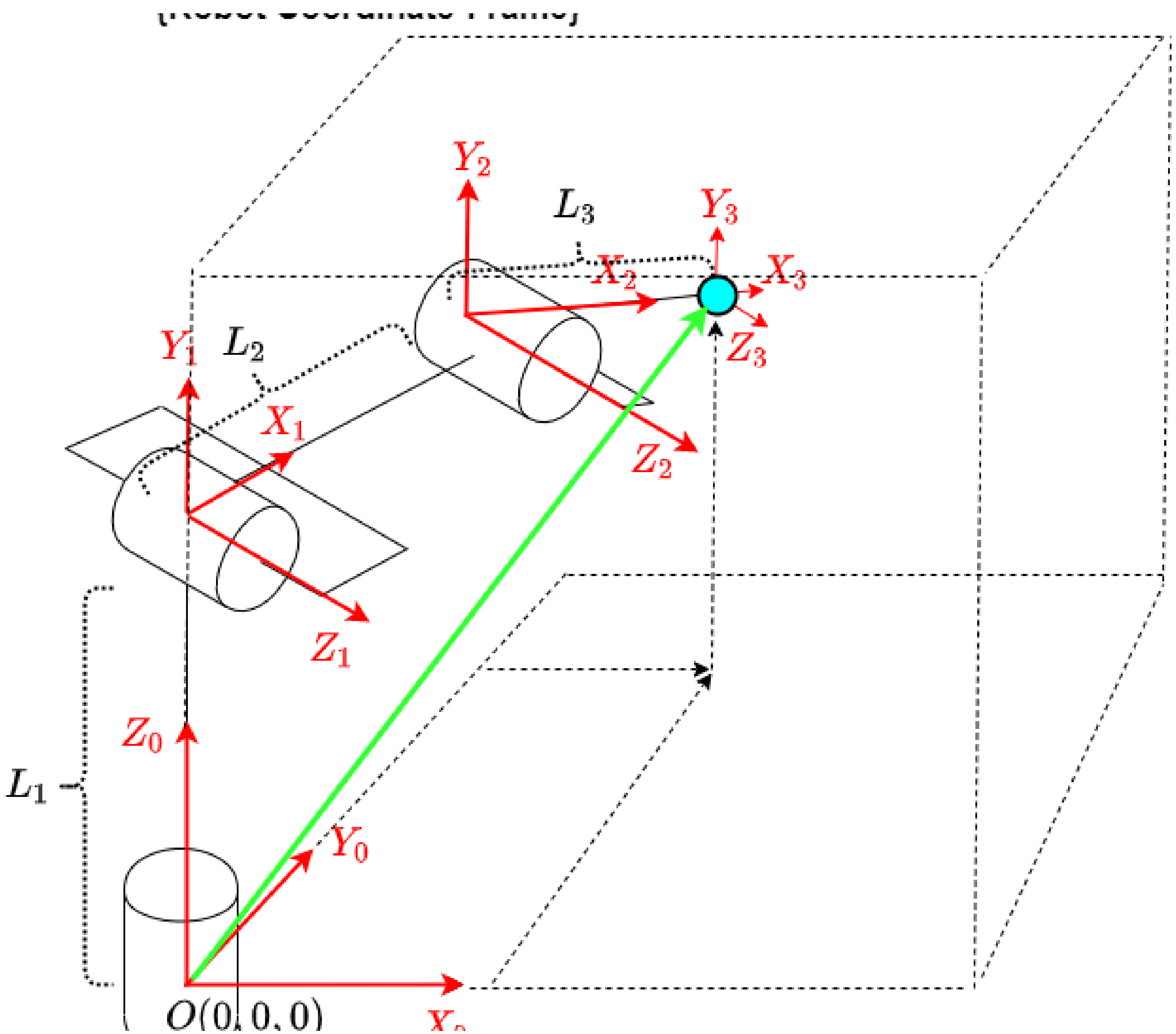


Homogeneous Transform Cont.



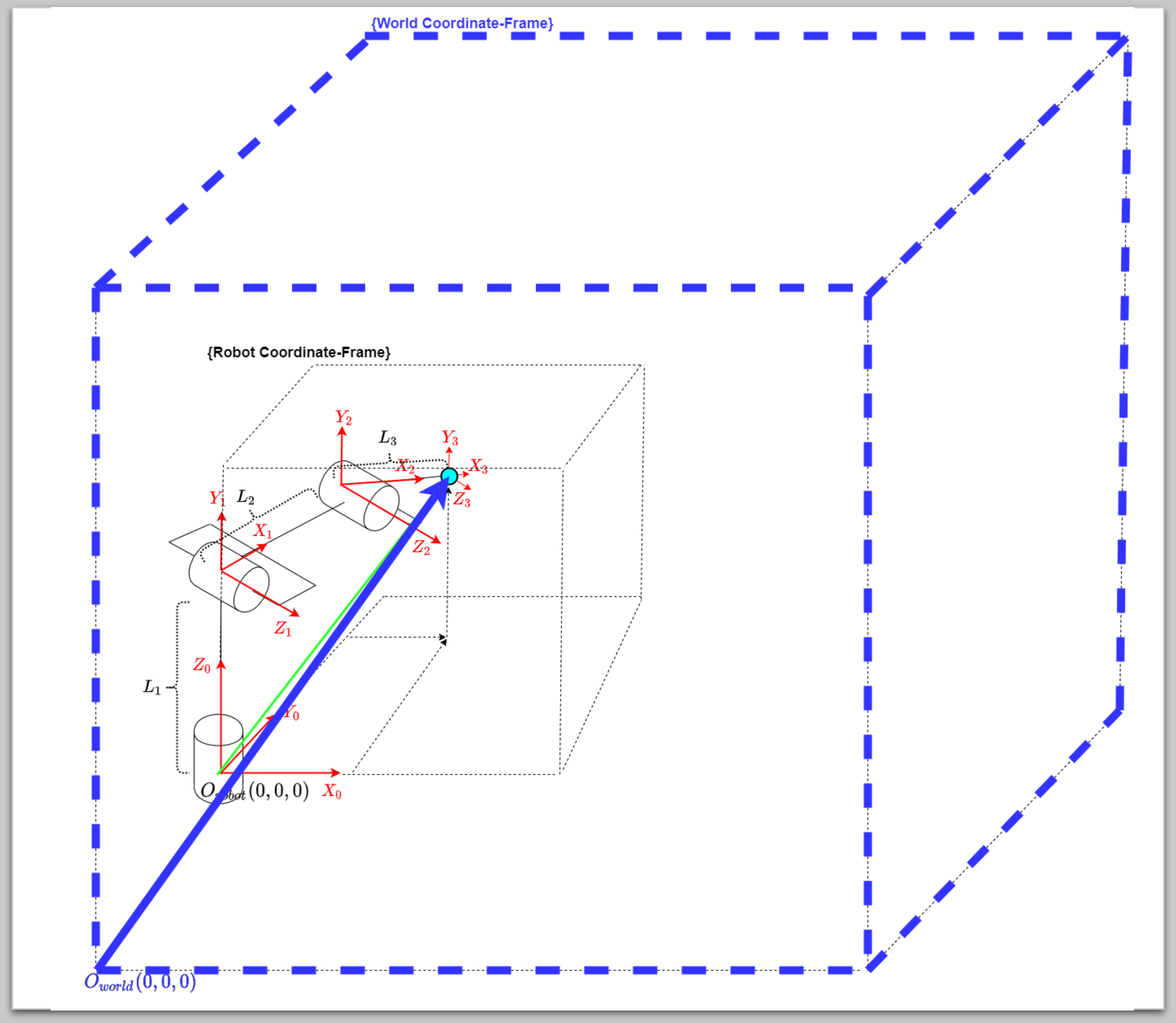
Robot Coordinate Frame

- A robot performs task in its own coordinate frame.
- The next lecture will focus on modeling a robotic manipulator using forward-kinematics

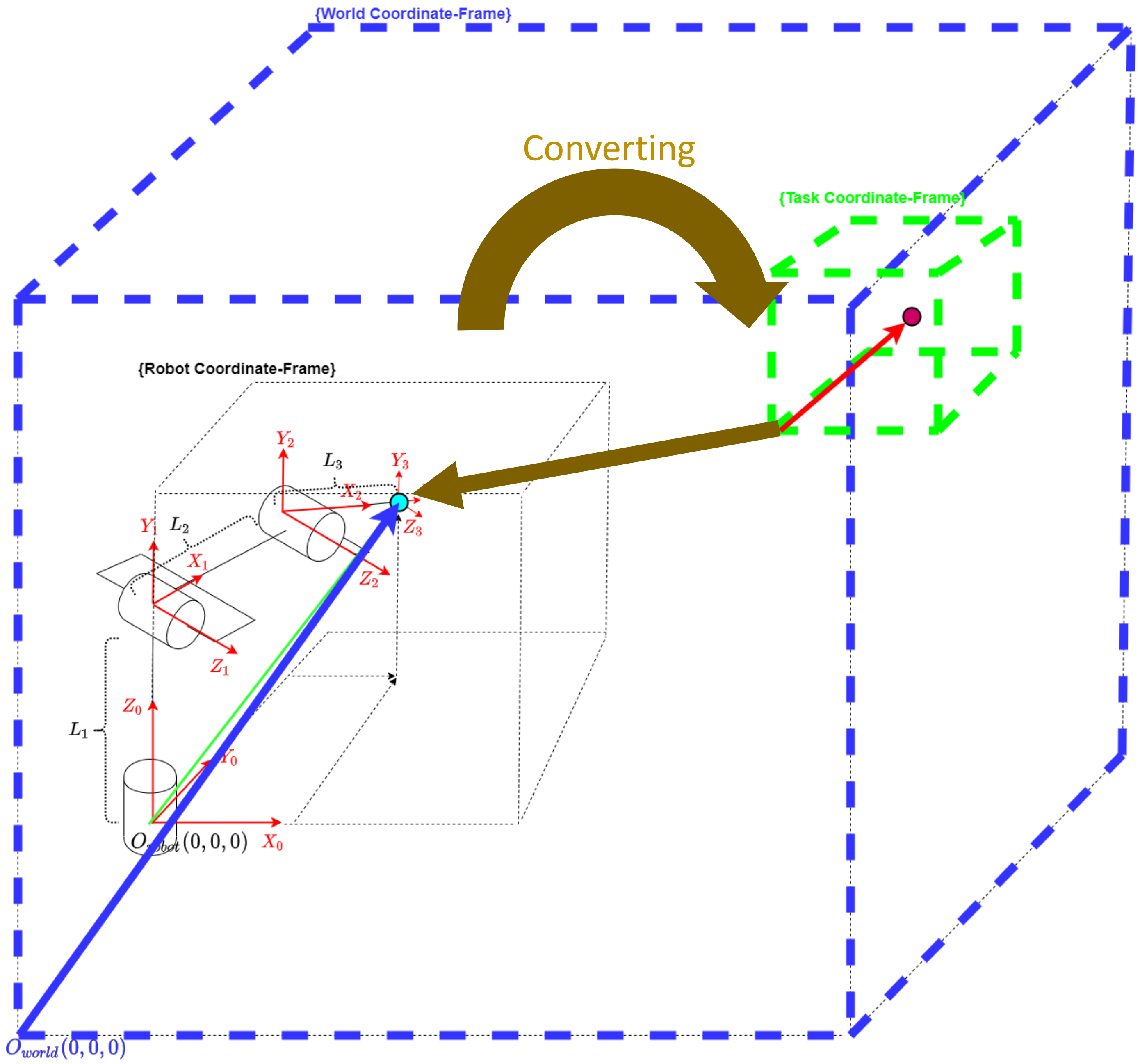


Two coordinate frames

- A robot is placed on another coordinate frame. Let's call this world coordinate frame.



Three Coordinate frames
- With homogeneous transforms we can represent the points amongst the different coordinate frames



Find H^R_T

One way:

Find H^{World}_{Robot}

Remember

$$H^1_0 = (H^0_1)^{-1}$$

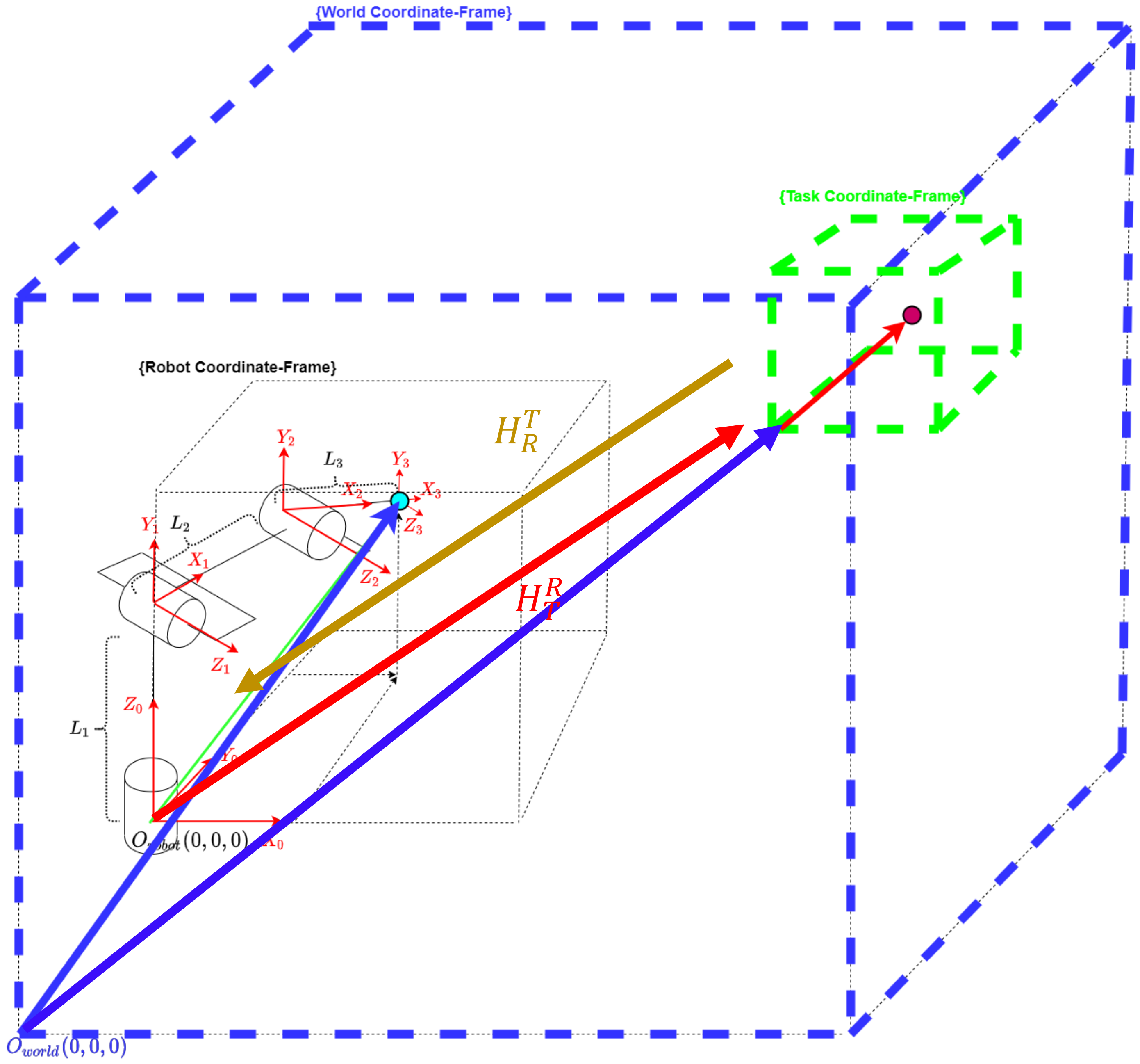
Find H^{World}_{Task}

$$H^R_T = (H^W_R)^{-1}\, H^W_T = H^R_W H^W_T$$

$$= H^R_{\cancel{W}} H^{\cancel{W}}_T$$

$$= H^R_T$$

$$H^T_R = (H^R_T)^{-1} = \begin{bmatrix} {R^R_T}^T & -{R^R_T}^T d^R_T \\ 0 & 1 \end{bmatrix}$$

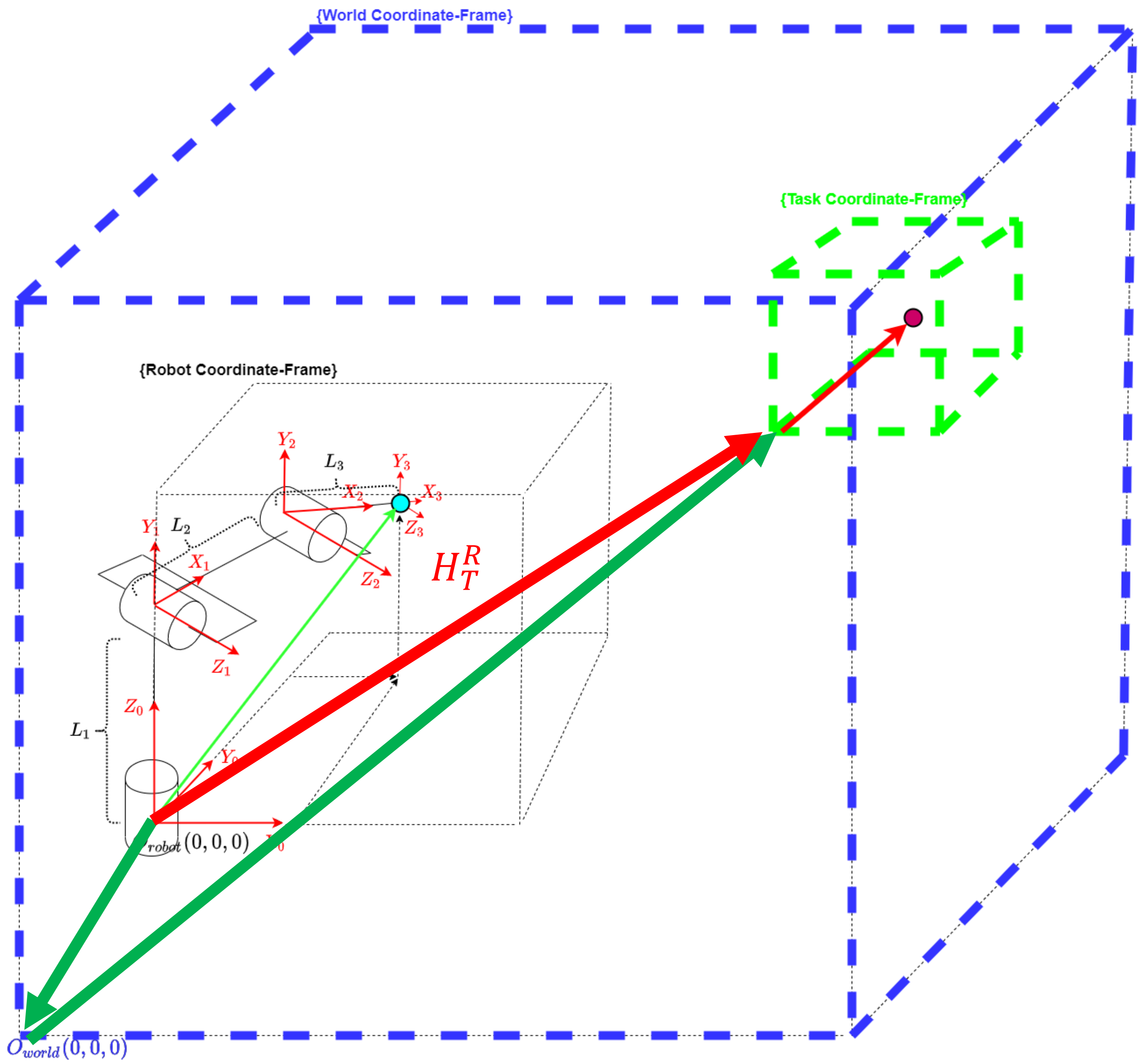


Second way:

Go directly between Robot and Task

- The distance between the coordinate frames are known

- Axes must be known

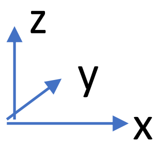


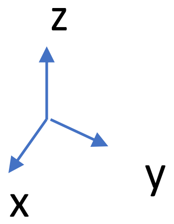
2.10

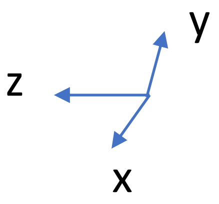
- R = $R_{y,\varphi}\ R_{x,\phi}\ R_{z,\Theta}$
- Fixed, premultiply
- Current, post-multiply
- Read 2.4.3 for a more detailed explanation

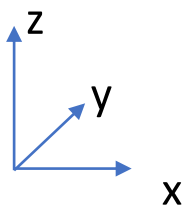
Assume we are operating only in SO(3) so the rotation-matrices are not commutative

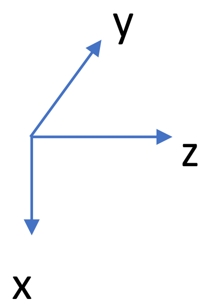
R = $R_{y,\varphi} R_{x,\phi}\ R_{z,\Theta} \neq\ R_{x,\phi} R_{z,\Theta} R_{y,\varphi}$











2.11

- R = $R_{z,\Theta}\ R_{x,\phi}\ R_{x,\Psi}$
- Fixed, premultiply
- Current, post-multiply
- Read 2.4.3 for a more detailed explanation

BREAK!

2.38

Find H_1^0, H_2^0, H_2^1

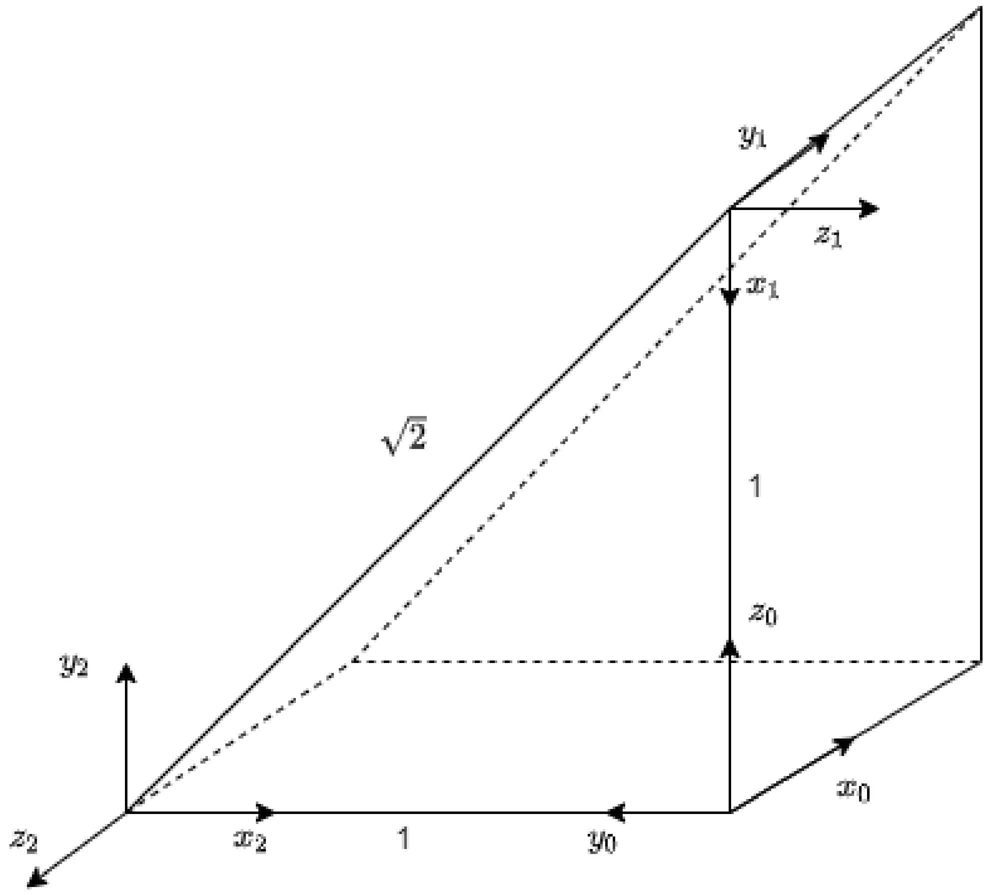
H=$\begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$, R is rotation matrix (3x3),

d is translation vector (1x3)

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



2.38 cont

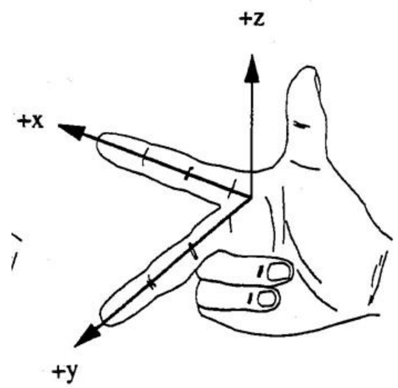
Let us start by finding the H_1^0

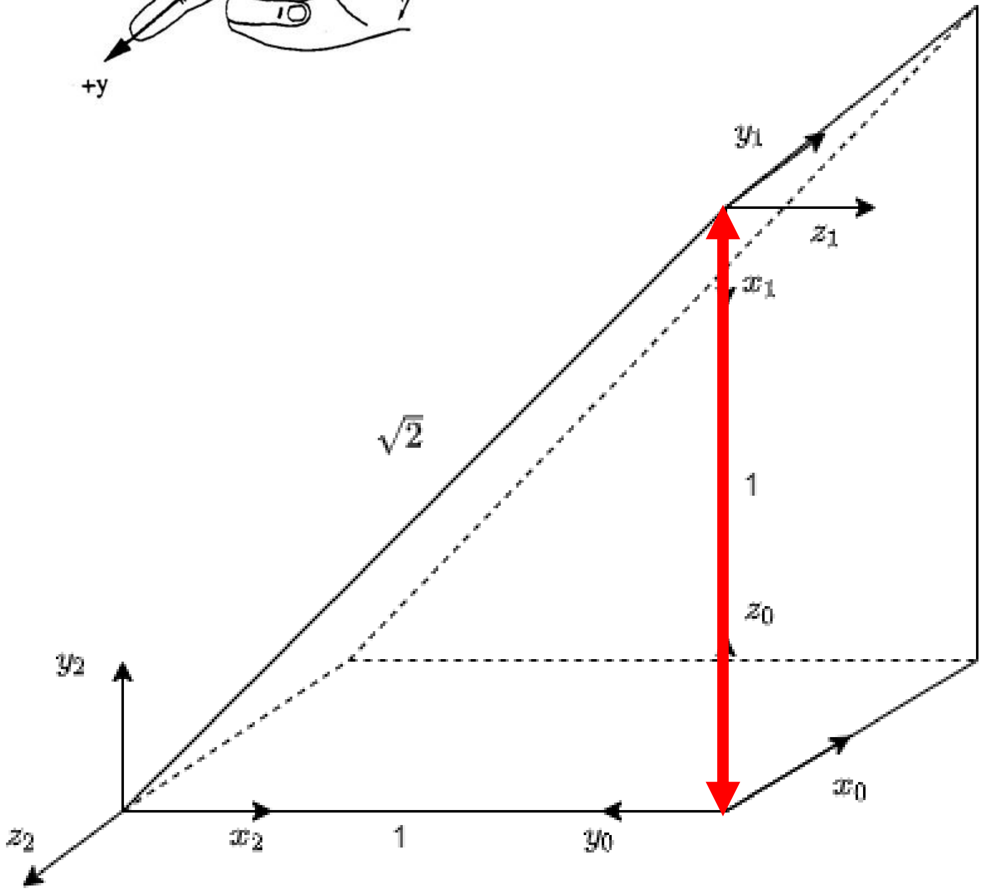
$R = R_{x,90}\, R_{z,270}$ $\qquad\qquad$ $(R_{z,-90})$

t = [0 0 1]

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_1^0 = \begin{bmatrix} R_{x,90}R_{z,270} & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





2.38 cont

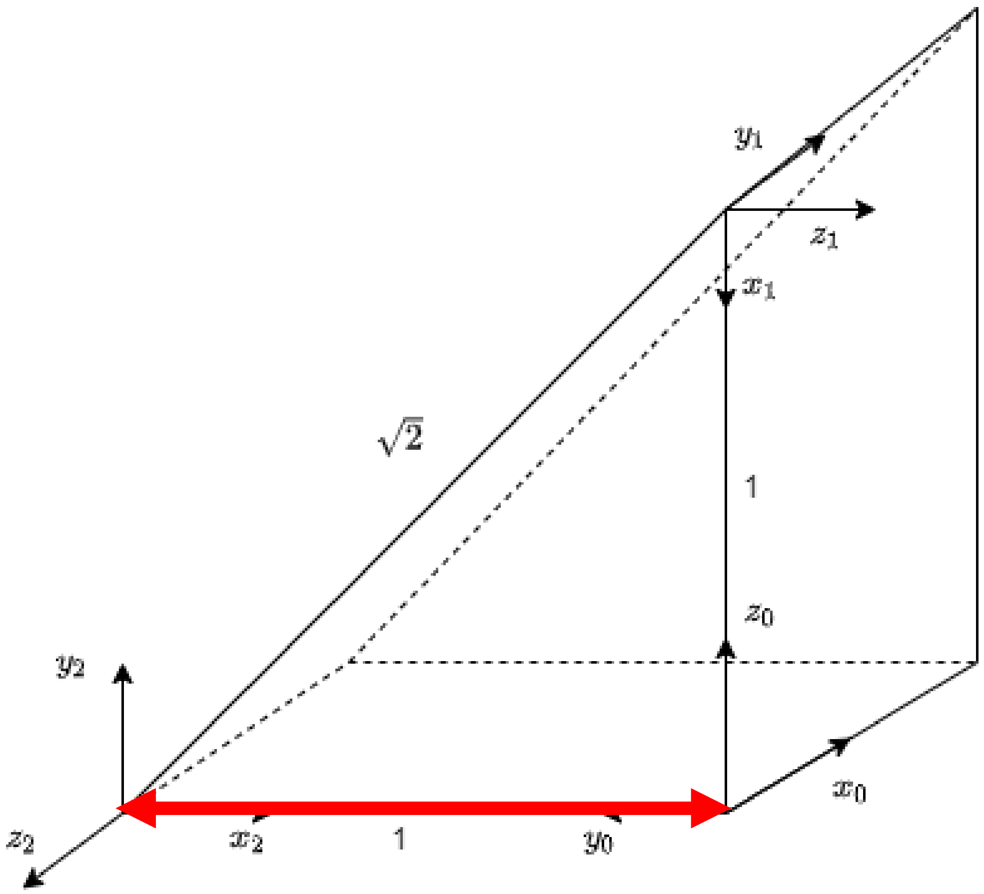
Finding the H_2^0

$R = \ R_{y,270}\ R_{x,270}$ $\qquad\qquad\qquad\qquad (R_{y,-90}R_{x,-90})$

t = [0 1 0]

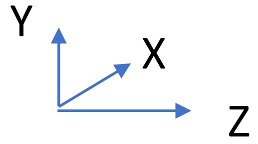
$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$H_2^0 = \begin{bmatrix} R_{y,270}\ R_{x,270} & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 1 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

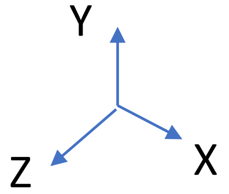


2.38 cont

First rotation



Second rotation



Finding the H_2^1

$R = \; R_{z,90}\, R_{y,-90}$

$(R_{z,-270} R_{y,270})$

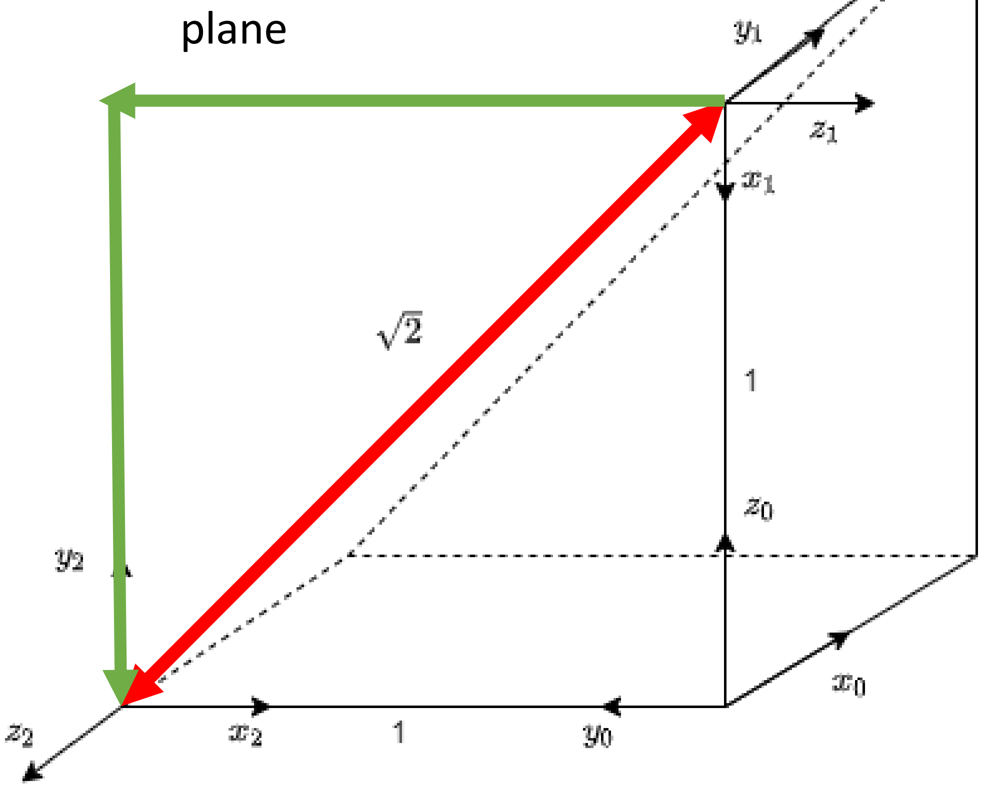
t = [1 0 -1]

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} R_{z,90}\, R_{y,-90} & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 1 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The coordinate frames can be considered on the same plane

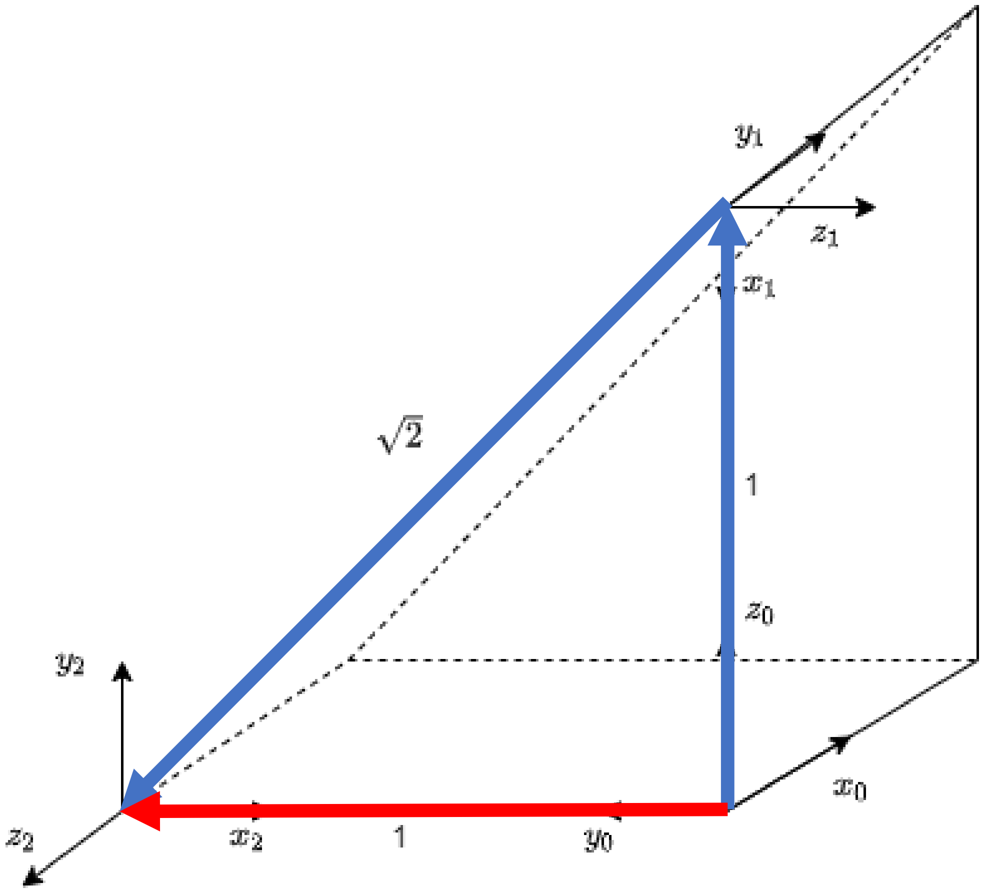


2.38 cont

Show that $H_2^0 = H_1^0\ H_2^1$

$$H_1^0 = \begin{bmatrix} R_{x,90}R_{z,270} & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

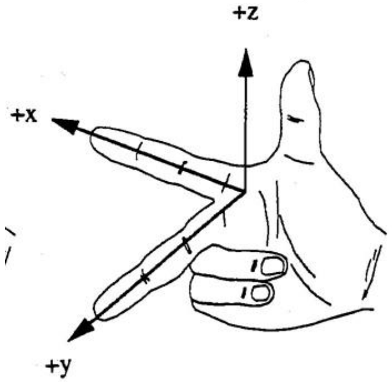
$$H_2^1 = \begin{bmatrix} R_{z,90}\ R_{y,-90} & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 1 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



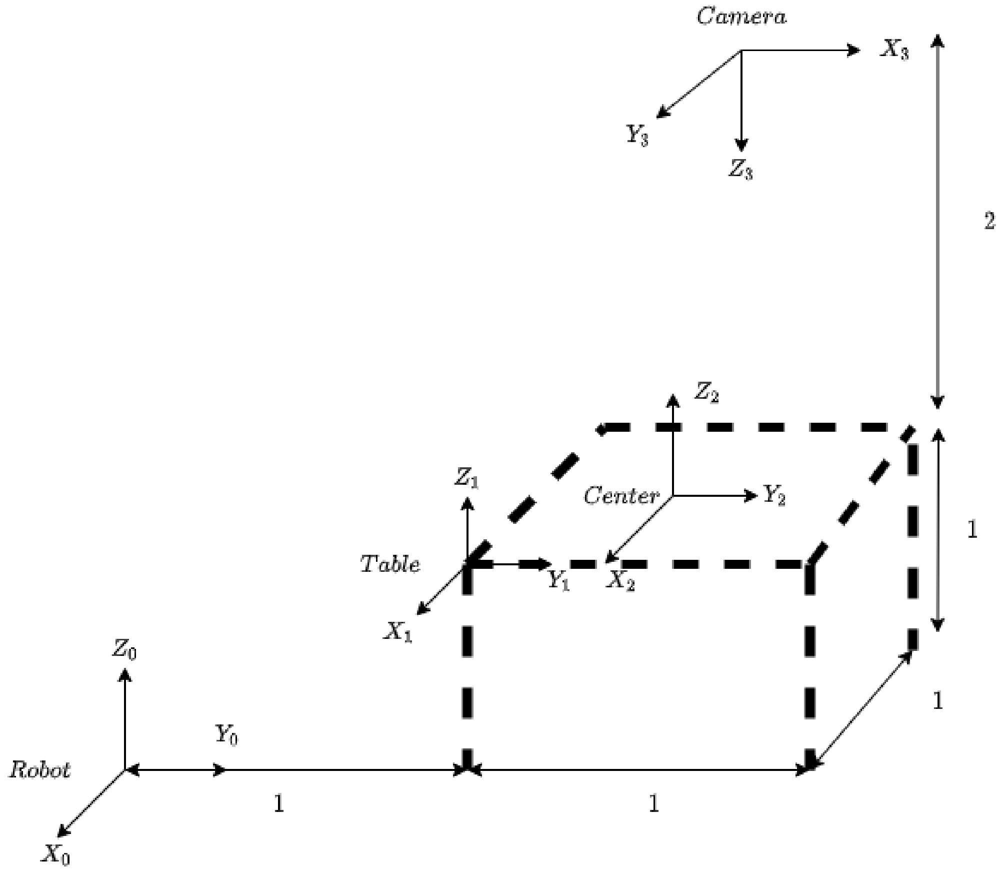
$$H_2^0 = H_1^0\ H_2^1 = \begin{bmatrix} R_{x,90}R_{z,270} & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{z,90}\ R_{y,-90} & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_{y,270}\ R_{x,270} & t \\ 0 & 1 \end{bmatrix}$$

2.39

- Try to solve 2.39 by hand. Get an idea how to rotate the coordinate frames using right hand rule.



Find $\boldsymbol{H}_1^0, \boldsymbol{H}_2^0, \boldsymbol{H}_3^0$ and $\boldsymbol{H}_3^2$



2.39 cont

Find $\boldsymbol{H_1^0}$

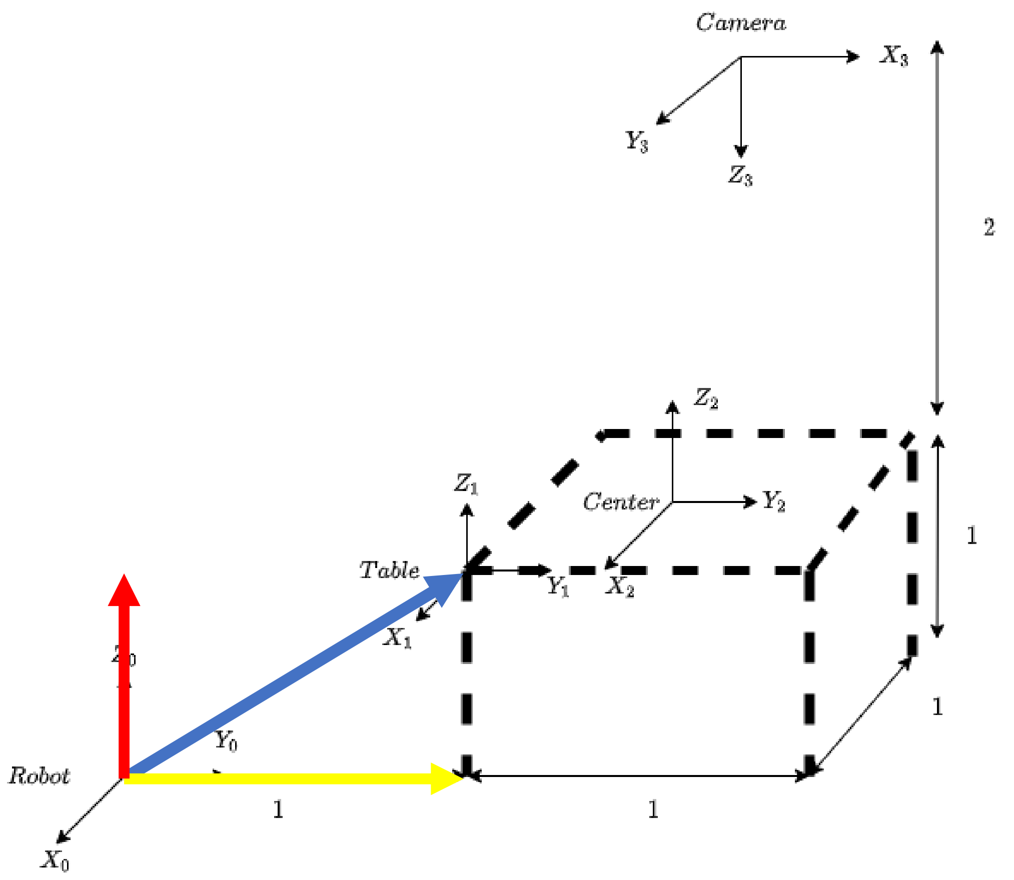
$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

t = [Tx, Ty, Tz] = [0 1 1]

$$H_1^0 = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

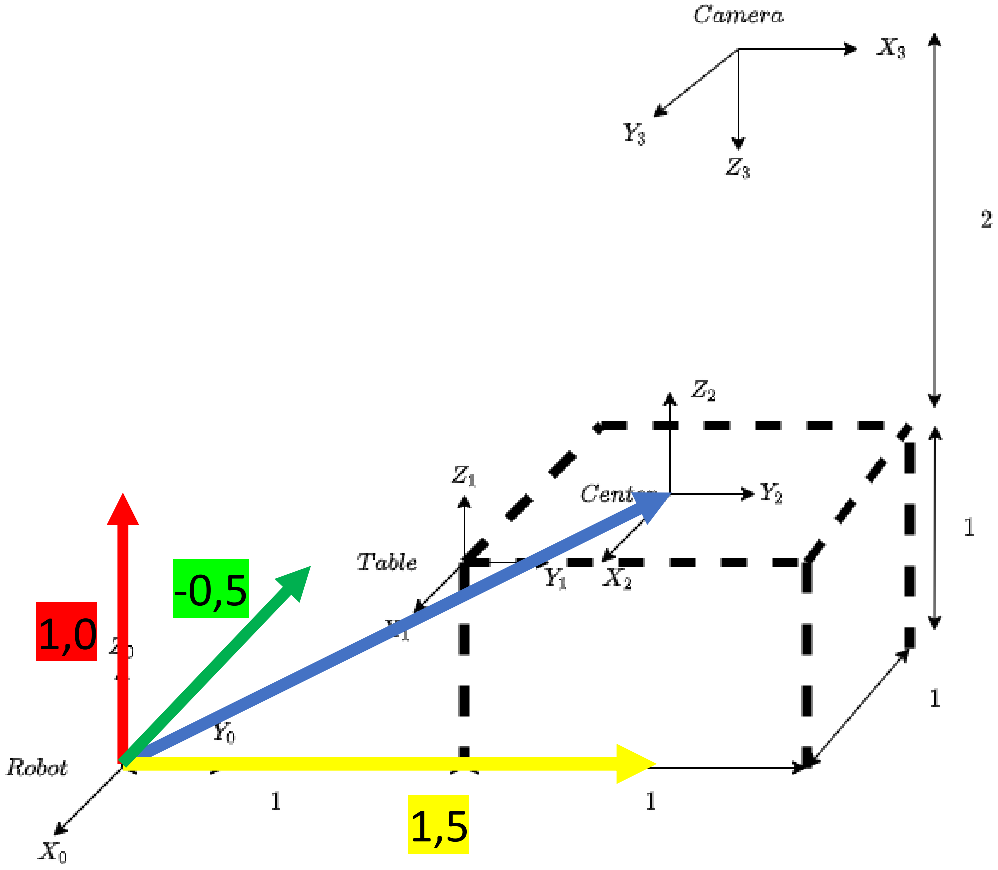


2.39 cont

Find $\boldsymbol{H_2^0}$

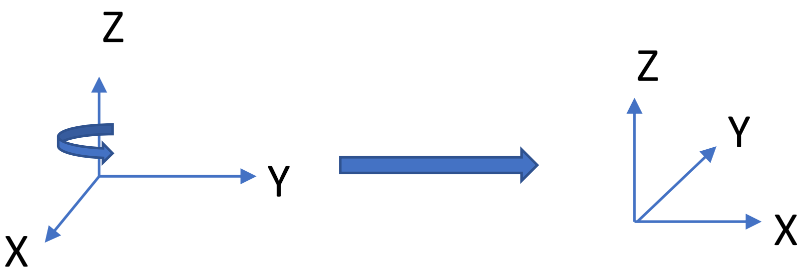
T = [Tx, Ty, Tz] = [-0.5, 1.5, 1.0]

$$H_2^0 = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



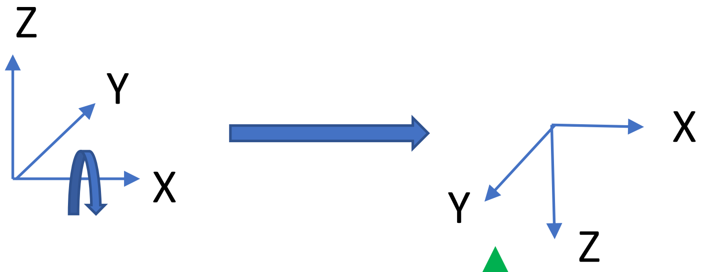
2.39 cont

First rotation



Find $\boldsymbol{H_3^0}$

Second rotation

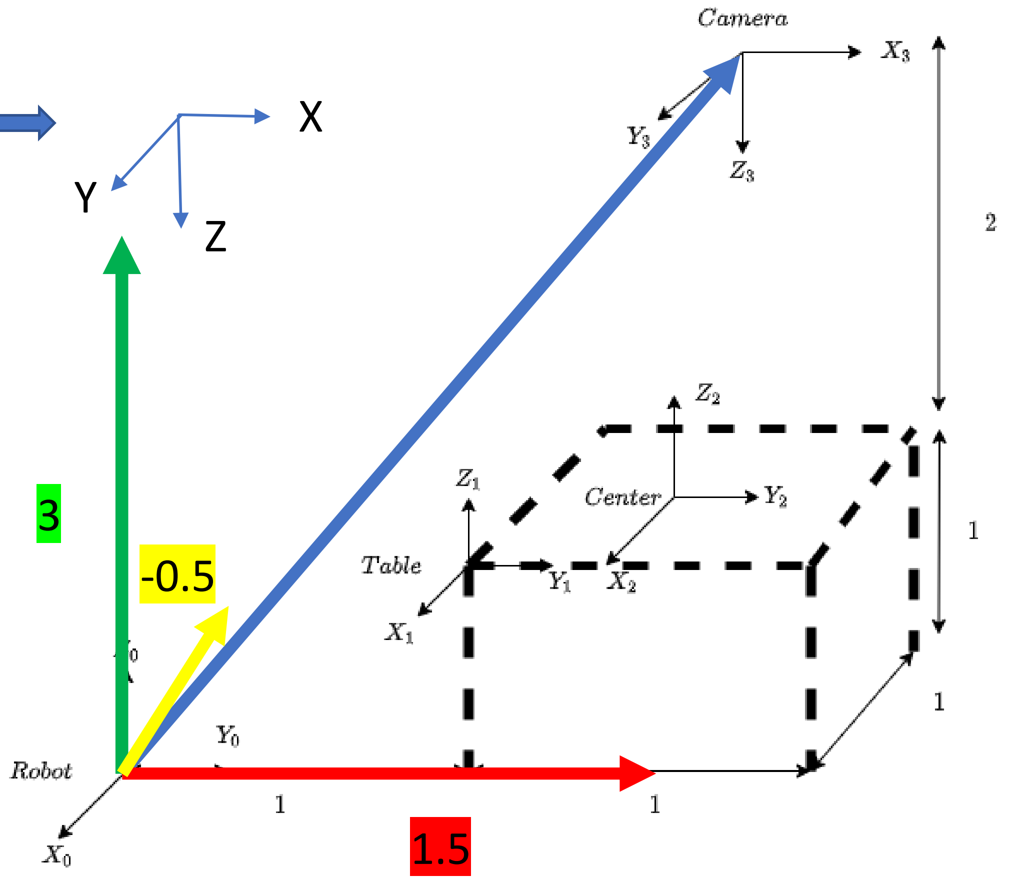


R = $R_{z,90}\,R_{x,180}$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

T = [Tx, Ty, Tz] = [-0.5, 1.5, 3.0]

$$H_3^0 = \begin{bmatrix} R_{z,90}\,R_{X,180} & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & -0.5 \\ r_{21} & r_{22} & r_{23} & 1.5 \\ r_{31} & r_{32} & r_{33} & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



2.39 cont

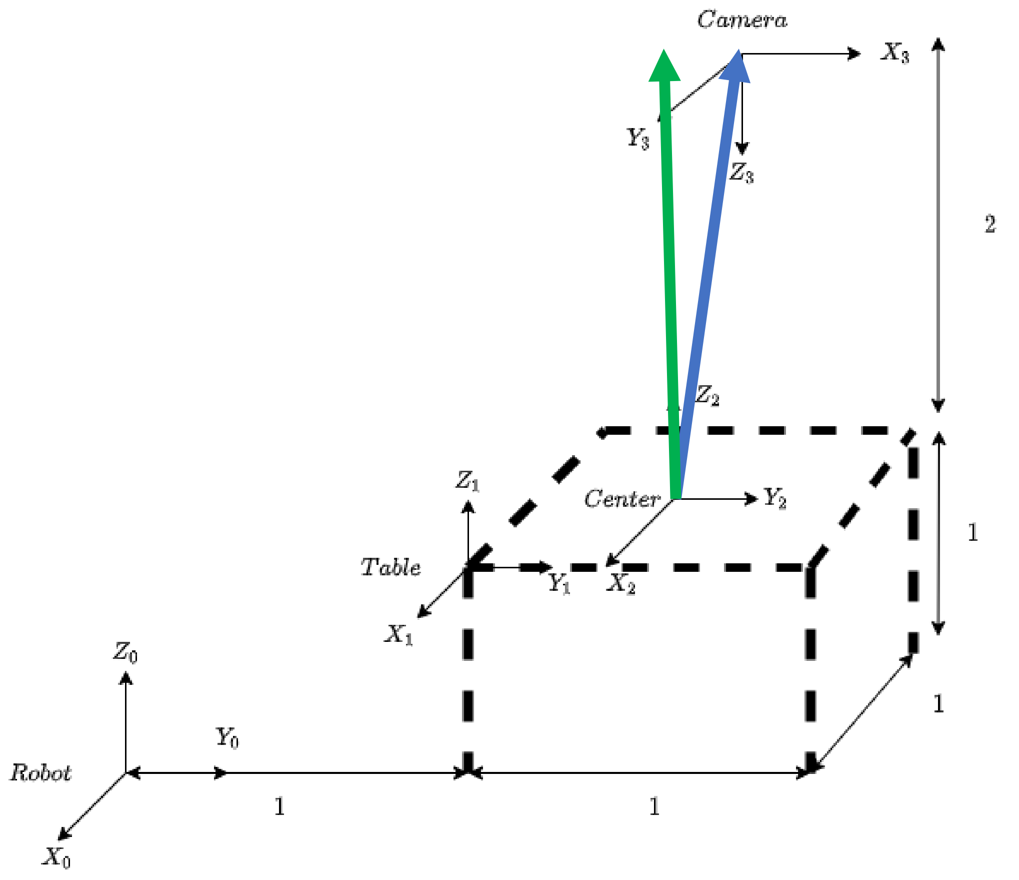
Find $\boldsymbol{H_3^2}$

R = $R_{z,90}\ R_{X,180}$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

T = [Tx Ty Tz] = [0 0 2]

$$H_3^2 = \begin{bmatrix} R_{z,90}\ R_{X,180} & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Next week

- Forward kinematics, Denavit Hartenberg