Group Session 10.03.2021

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General Info

- Deadline for Mandatory Assignment II, this week

- **12.03.2021**

Plan for today,

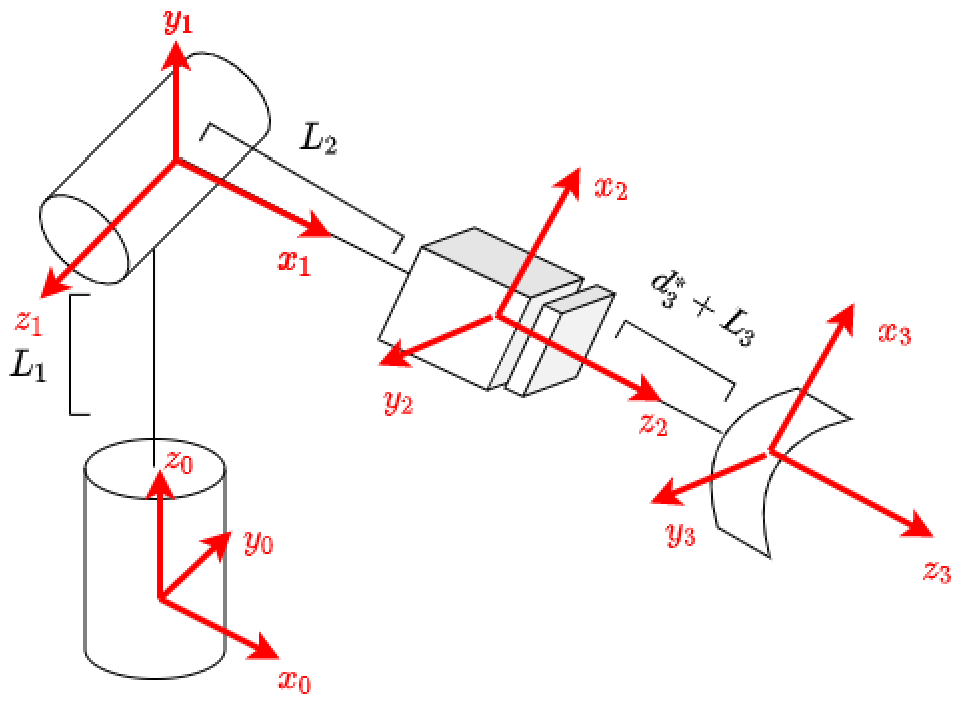
- Solving Jacobian for Kristians robot manipulator
- Finding the singularities for the robot

• Programming the Jacobian
• Programming the determinant

NB!
Remember to comment your code and try to get the print outputs structured

IF there is time
Programming forward and inverse-kinematics

Spherical Robot (RRP)



Rot_{z,θ_i}	$Trans_{z,d_i}$	$Trans_{x,a_i}$	Rot_{x,α_i}
θ_1^*	L_1	0	$\frac{\pi}{2}$
$\theta_2^* + 90$	0	0	$\frac{\pi}{2}$
0	$d_3^*+L_2$	0	0

A-matrices

$$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_{2+\frac{\pi}{2}} & 0 & s_{2+\frac{\pi}{2}} & 0 \\ s_{2+\frac{\pi}{2}} & 0 & -c_{2+\frac{\pi}{2}} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

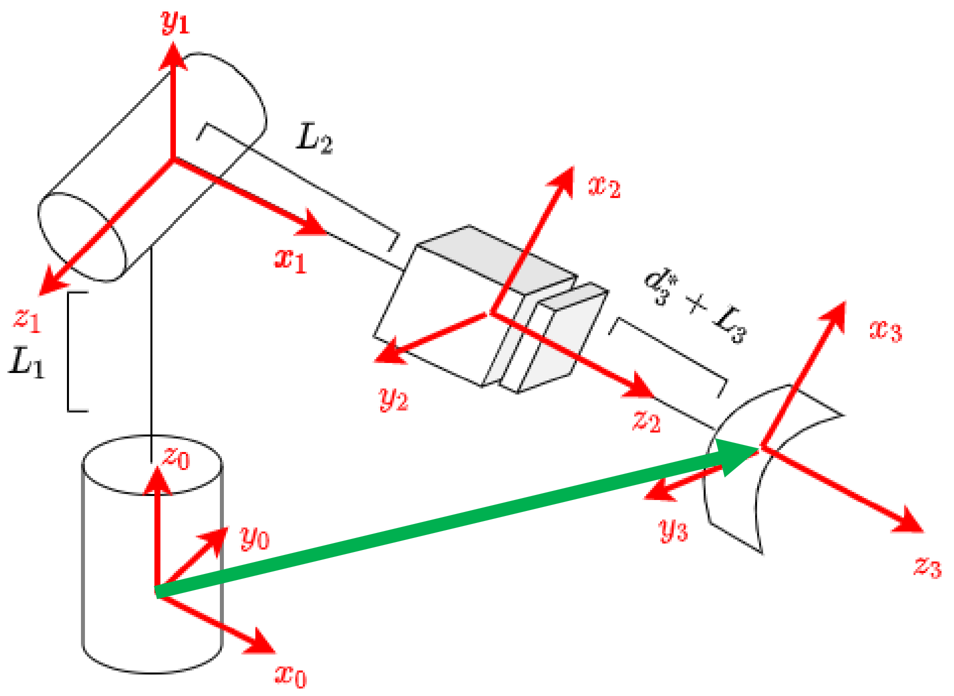
$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* + L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{2+\frac{\pi}{2}} & 0 & s_{2+\frac{\pi}{2}} & 0 \\ s_{2+\frac{\pi}{2}} & 0 & -c_{2+\frac{\pi}{2}} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* + L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -c_1 s_2 & s_1 & c_1 c_2 & c_1 c_2 (d_3^* + L_2) \\ -s_1 s_2 & -c_1 & c_2 s_1 & c_2 s_1 (d_3^* + L_2) \\ c_2 & 0 & s_2 & L_1 + s_2 (d_3^* + L_2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

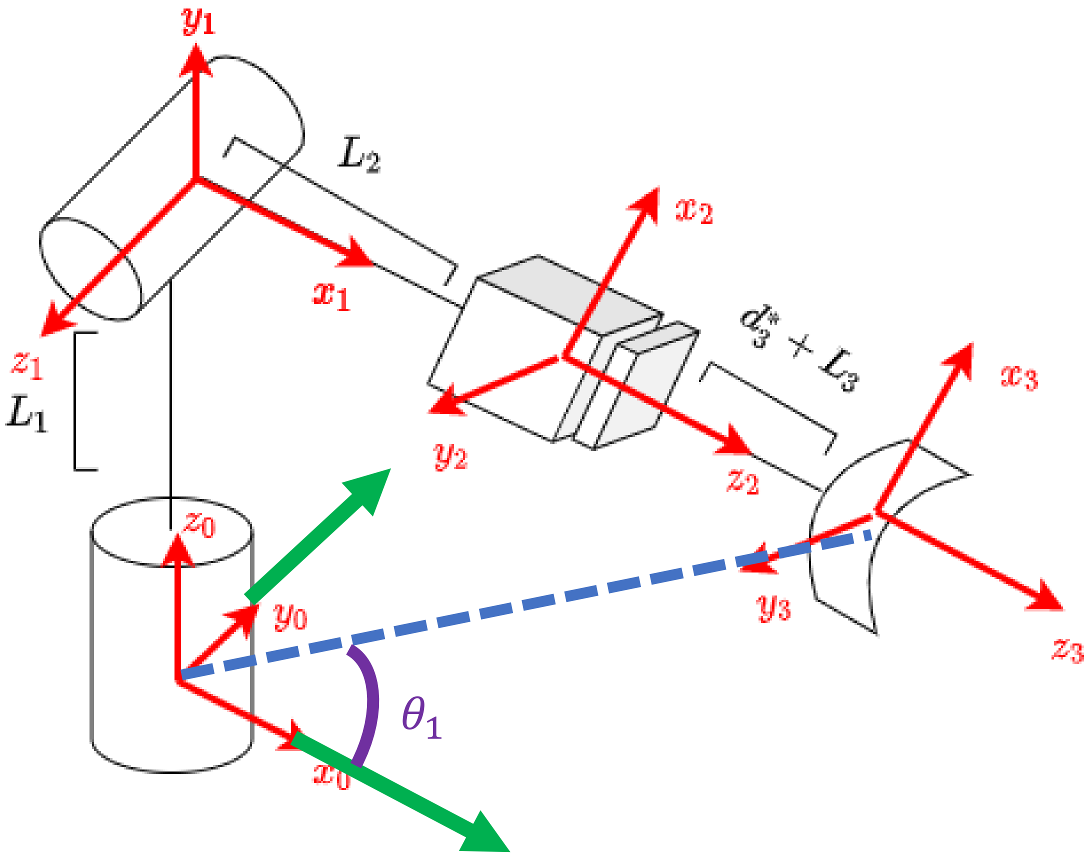
Forward-kinematics

$$T_3^0 = A_1A_2A_3 = \begin{bmatrix} -c_1s_2 & s_1 & c_1c_2 & c_1c_2(d_3^*+L_2) \\ -s_1s_2 & -c_1 & c_2s_1 & c_2s_1(d_3^*+L_2) \\ c_2 & 0 & s_2 & L_1 + s_2(d_3^*+L_2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Inverse-kinematics

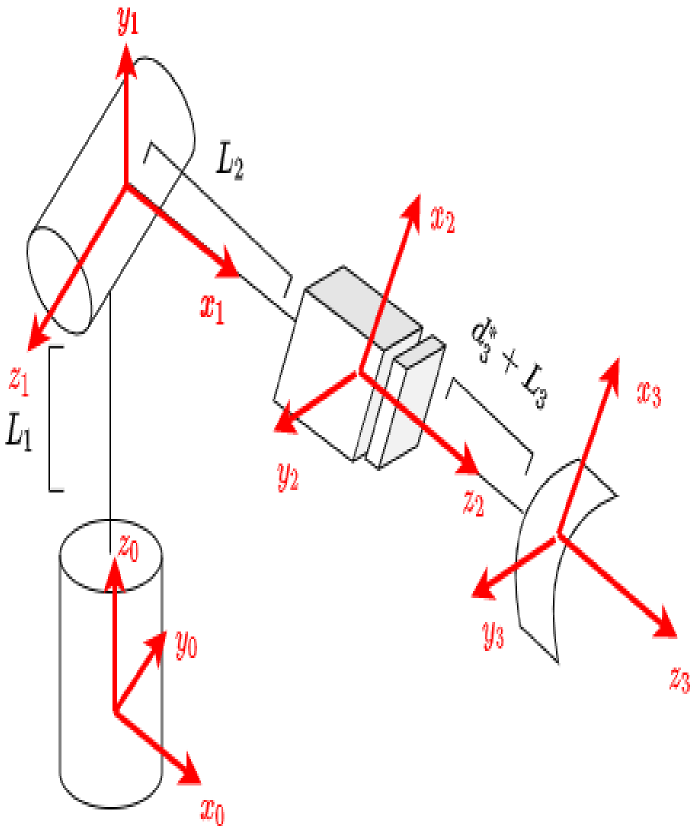
$\theta_1 = atan2(y_c, x_c)$

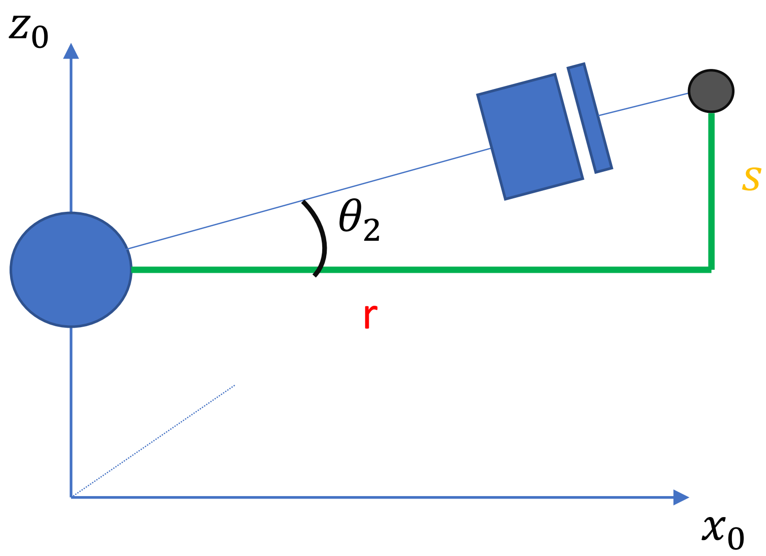


$$s = z_c - L_1$$

$$r = \pm\sqrt{x_0^2 + y_0^2}$$

$$\theta_2 = atan2(s, r)$$





$$r = \pm\sqrt{x_0^2 + y_0^2}$$

$$s = z_c - L_1$$

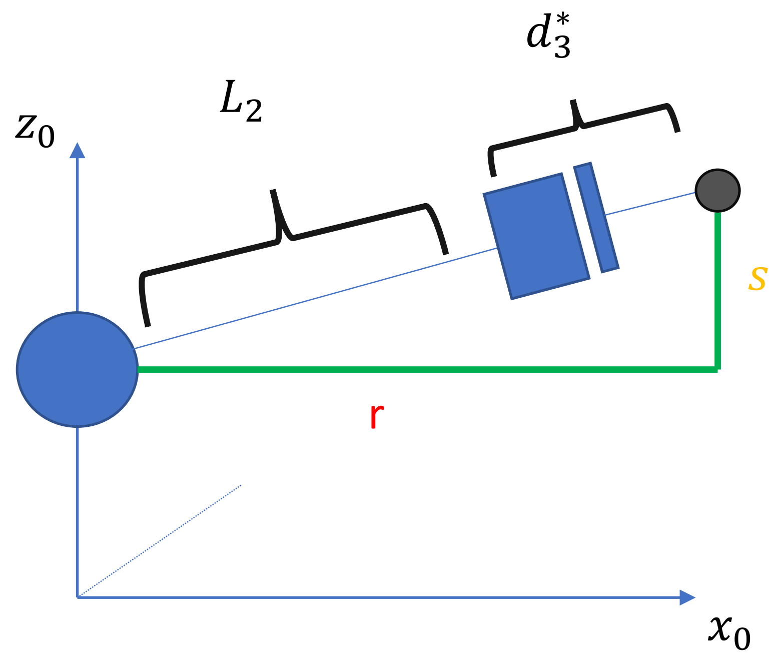
Pythagoras gives us the following solution:

$$(d_3^* + L_2)^2 = r^2 + s^2$$

$$\sqrt{(d_3^* + L_2)^2} = \sqrt{r^2 + s^2}$$

$$(d_3^* + L_2) = \sqrt{r^2 + s^2}$$

$$d_3^* = \sqrt{r^2 + s^2} - L_2$$



$$s = z_c - L_1$$

$$r = \pm\sqrt{x_0^2 + y_0^2}$$

IK-solutions

$$\theta_1 = atan2(y_c, x_c)$$

$$\theta_2 = atan2(s, r)$$

$$d_3^* = \pm\sqrt{r^2 + s^2} - L_2$$

Jacobian for the manipulator

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}, \text{where J} = \text{m} \ * 6 \text{ matrix}$$

$J_{v_i} = z_{i-1} \times (o_n - o_{i-1})$ Revolute

$J_{v_i} = z_{i-1}$ Prismatic Joint

$J_{\omega_i} = z_{i-1}$ Revolute

$J_{\omega_i} = 0$ Prismatic Joint

Jacobian for the manipulator cont.

$$T_1^0 = A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{2+\frac{\pi}{2}} & 0 & s_{2+\frac{\pi}{2}} & 0 \\ s_{2+\frac{\pi}{2}} & 0 & -c_{2+\frac{\pi}{2}} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -c_1 s_2 & s_1 & c_1 c_2 & 0 \\ -s_1 s_2 & -c_1 & c_2 s_1 & 0 \\ c_2 & 0 & s_2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{2+\frac{\pi}{2}} & 0 & s_{2+\frac{\pi}{2}} & 0 \\ s_{2+\frac{\pi}{2}} & 0 & -c_{2+\frac{\pi}{2}} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^*+L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -c_1 s_2 & s_1 & c_1 c_2 & c_1 c_2 (d_3^*+L_2) \\ -s_1 s_2 & -c_1 & c_2 s_1 & c_2 s_1 (d_3^*+L_2) \\ c_2 & 0 & s_2 & L_1 + s_2 (d_3^*+L_2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Helpful variables

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \quad z_2 = \begin{bmatrix} c_1 c_2 \\ c_2 s_1 \\ s_2 \end{bmatrix}$$

$$J_{v_i} = z_{i-1} \times (o_n - o_{i-1})$$
$$J_{v_i} = z_{i-1}$$

$$J_{\omega_i} = z_{i-1}$$
$$J_{\omega_i} = 0$$

$$(o_3 - o_0) = \begin{bmatrix} c_1 c_2 (d_3^* + L_2) \\ c_2 s_1 (d_3^* + L_2) \\ L_1 + s_2 (d_3^* + L_2) \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} c_1 c_2 (d_3^* + L_2) \\ c_2 s_1 (d_3^* + L_2) \\ L_1 + s_2 (d_3^* + L_2) \end{bmatrix}$$

$$(o_3 - o_1) = \begin{bmatrix} c_1 c_2 (d_3^* + L_2) \\ c_2 s_1 (d_3^* + L_2) \\ L_1 + s_2 (d_3^* + L_2) \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix} = \begin{bmatrix} c_1 c_2 (d_3^* + L_2) \\ c_2 s_1 (d_3^* + L_2) \\ s_2 (d_3^* + L_2) \end{bmatrix}$$

$$J_{v_1} = z_{1-1} \times (o_3 - o_{1-1}) = z_0 \times (o_3 - o_0) = \begin{bmatrix} -c_2 s_1 (d_3^* + L_2) \\ c_1 c_2 (d_3^* + L_2) \\ 0 \end{bmatrix}$$

$$J_{v_2} = z_{2-1} \times (o_3 - o_{2-1}) = z_1 \times (o_3 - o_1) = \begin{bmatrix} -c_1 s_2 (d_3^* + L_2) \\ -s_1 s_2 (d_3^* + L_2) \\ c_2 (d_3^* + L_2) \end{bmatrix}$$

$$J_{v_3} = z_{3-1} = z_2 = \begin{bmatrix} c_1 c_2 \\ c_2 s_1 \\ s_2 \end{bmatrix}$$

$$J_{\omega_1} = z_{1-1} = z_0$$
$$J_{\omega_2} = z_{2-1} = z_1$$
$$J_{\omega_3} = 0$$

Jacobian

Putting all of it together yields the following Jacobian matrix

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} J_{v_1} & J_{v_2} & J_{v_3} \\ J_{\omega_1} & J_{\omega_2} & J_{\omega_3} \end{bmatrix} = \begin{bmatrix} -c_2 s_1 (d_3^* + L_2) & -c_2 s_1 (d_3^* + L_2) & c_1 c_2 \\ c_1 c_2 (d_3^* + L_2) & -s_1 s_2 (d_3^* + L_2) & c_2 s_1 \\ 0 & c_2 (d_3^* + L_2) & s_2 \\ 0 & s_1 & 0 \\ 0 & -c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

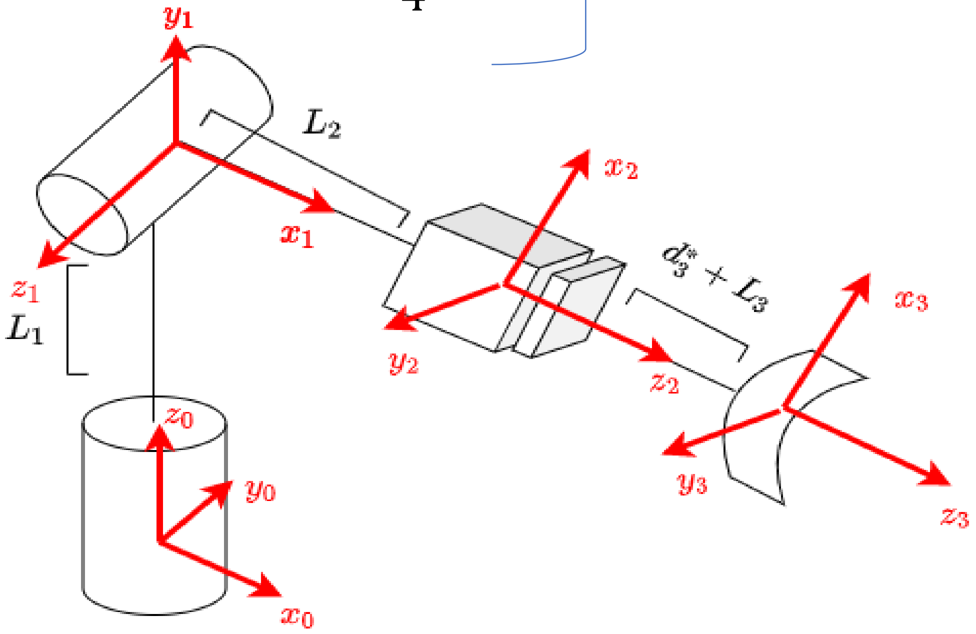
Singularities

These two angles causes singularities for spherical robot manipulator

Configuration where motion in some directions is impossible.
- Minor movement of the tool could require infinite force in the joints.
- - "Does not mean that joints will fly off into the stratosphere at superluminal speeds, but the joints might strain and break."
- - Mathematically: rang J(q) < max
- - Appears when the manipulator loses a degree of freedom-
  |J| = 0
  |Jv| = 0 (arm singularities)

$$\theta_2 = \frac{\pi}{4}$$

$$\theta_2 = -\frac{\pi}{4}$$



We can find the singularities in two ways, by observing or solving analytically

Determinant of the robotic manipulator

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} J_{v_1} & J_{v_2} & J_{v_3} \\ J_{\omega_1} & J_{\omega_2} & J_{\omega_3} \end{bmatrix} = \begin{bmatrix} -c_2 s_1(d_3^*+L_2) & -c_2 s_1(d_3^*+L_2) & c_1 c_2 \\ c_1 c_2(d_3^*+L_2) & -s_1 s_2(d_3^*+L_2) & c_2 s_1 \\ 0 & c_2(d_3^*+L_2) & s_2 \\ 0 & s_1 & 0 \\ 0 & -c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Can only find determinant of square matrices, so we decouple the J_v component from J by itself.

$$J_v = [J_v] = [J_{v_1} \quad J_{v_2} \quad J_{v_3}] = \begin{bmatrix} -c_2 s_1(d_3^*+L_2) & -c_2 s_1(d_3^*+L_2) & c_1 c_2 \\ c_1 c_2(d_3^*+L_2) & -s_1 s_2(d_3^*+L_2) & c_2 s_1 \\ 0 & c_2(d_3^*+L_2) & s_2 \end{bmatrix}$$

$$J = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\det(J) = \det(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix})$$

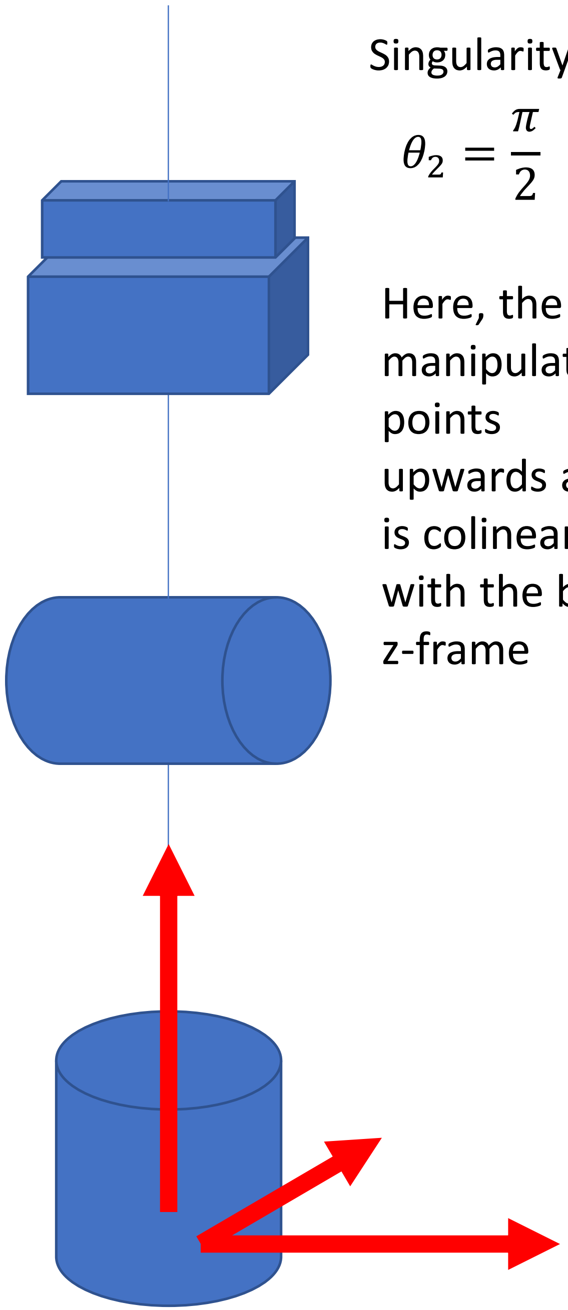
$$\det(J) = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - \mathrm{b} \begin{vmatrix} d & f \\ g & i \end{vmatrix} + \mathrm{c} \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\det(J) = a\,(e * i\; - f * h) - \mathrm{b}(d * i\; - f * g) + \mathrm{c}(\mathrm{d} * \mathrm{h}\; - \mathrm{e} * \mathrm{g})$$

$$c_2(L_2 + d_3)(L_2 + d_3 + L_2 c_2^2 - d_3 c_2^2) = 0$$

Setting

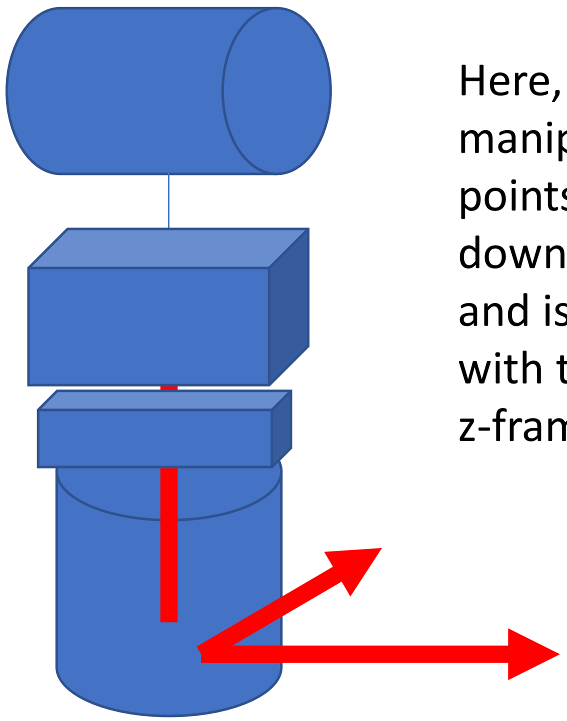
$$\theta_2 = \frac{\pi}{2}\;, or\; \theta_2 = -\frac{\pi}{2}$$



Singularity 1

$$\theta_2 = \frac{\pi}{2}$$

Here, the manipulator points upwards and is colinear with the base z-frame



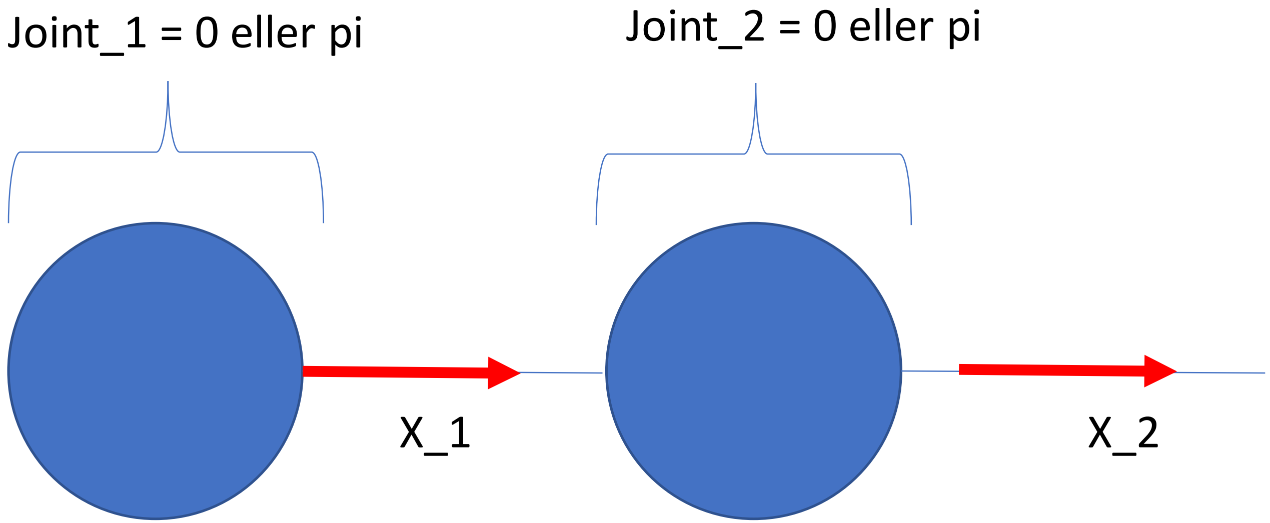
Singularity 1

$$\theta_2 = -\frac{\pi}{2}$$

Here, the manipulator points downwards and is colinear with the base z-frame

Singularities for planar robot-arm **through observation**

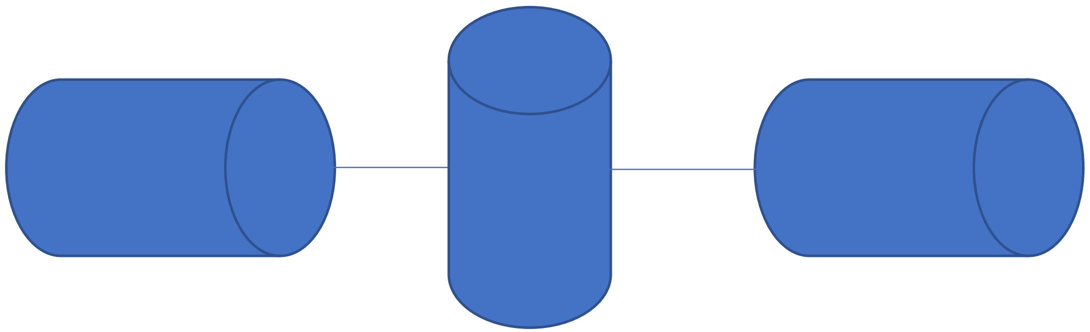
Colinear-axes causes singularity



Singularities for wrist arms

Colinear-axes causes singularity for joint 1 and joint 3

Theta_2 = 0



Theta_2 = pi