Group Session 12.03.2021

Hpnguyen

E-mail: hpnguyen@ifi.uio.no

General Info

- Deadline for Mandatory Assignment II, this week

- **15.03.2021**

- Reason why

Plan for today,

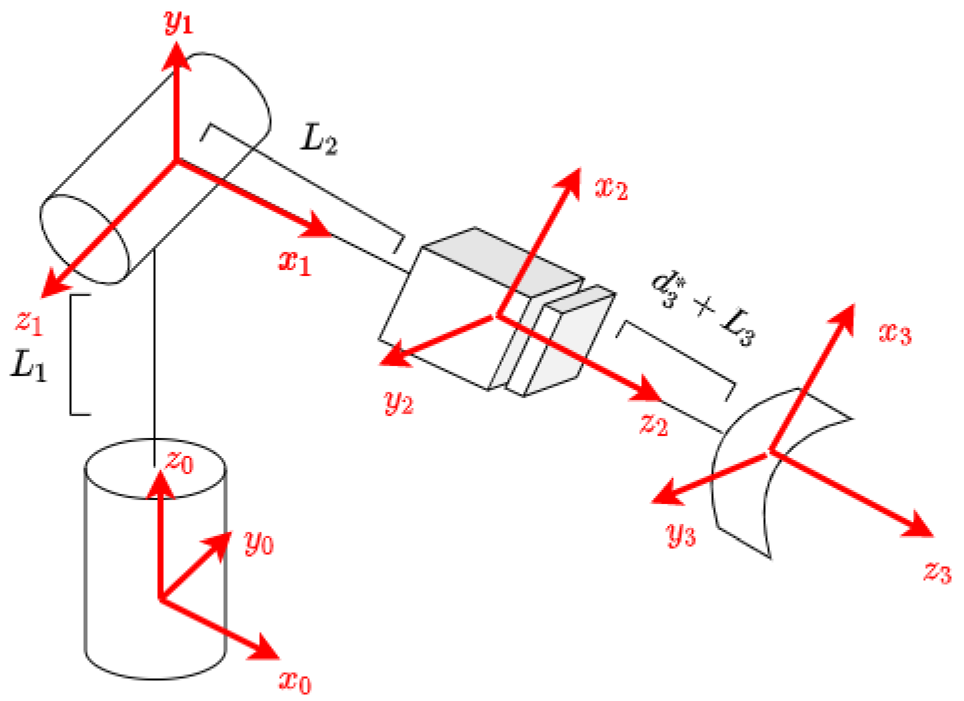
- Solving Jacobian for Kristians robot manipulator
- Finding the singularities for the robot

- Programming the Jacobian
- Programming the determinant

NB!
Remember to comment your code and try to get the print outputs structured

IF there is time
- Programming forward and inverse-kinematics

Spherical Robot (RRP)



Rot_{z,θ_i}	$Trans_{z,d_i}$	$Trans_{x,a_i}$	Rot_{x,α_i}
θ_1^*	L_1	0	$\frac{\pi}{2}$
$\theta_2^* + 90$	0	0	$\frac{\pi}{2}$
0	d_3^*+L_2	0	0

A-matrices

$$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_{2+\frac{\pi}{2}} & 0 & s_{2+\frac{\pi}{2}} & 0 \\ s_{2+\frac{\pi}{2}} & 0 & -c_{2+\frac{\pi}{2}} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

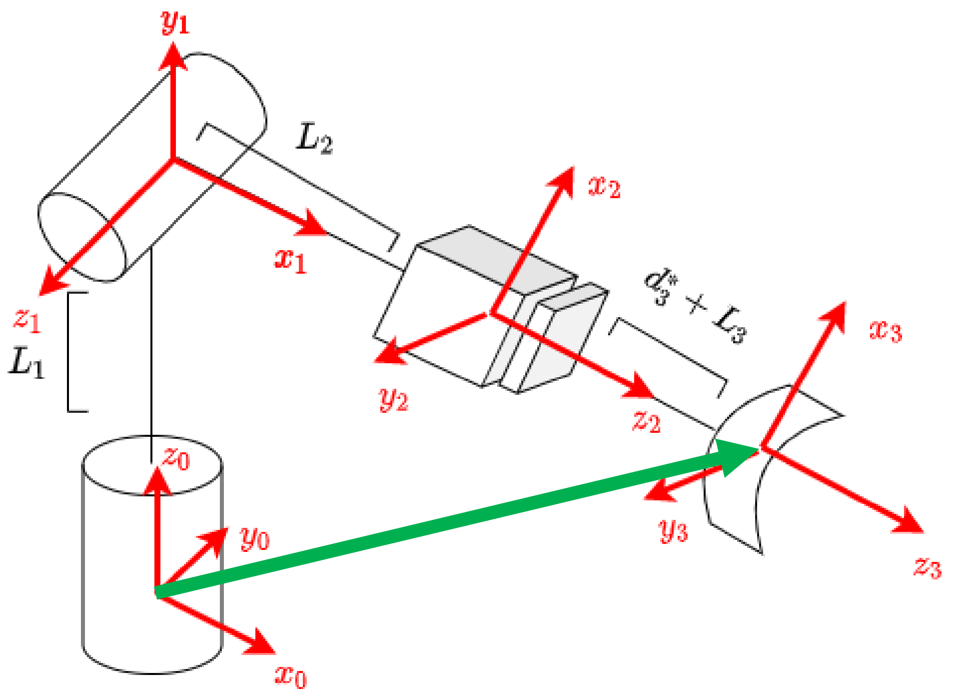
$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* + L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{2+\frac{\pi}{2}} & 0 & s_{2+\frac{\pi}{2}} & 0 \\ s_{2+\frac{\pi}{2}} & 0 & -c_{2+\frac{\pi}{2}} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* + L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -c_1 s_2 & s_1 & c_1 c_2 & c_1 c_2 (d_3^* + L_2) \\ -s_1 s_2 & -c_1 & c_2 s_1 & c_2 s_1 (d_3^* + L_2) \\ c_2 & 0 & s_2 & L_1 + s_2 (d_3^* + L_2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

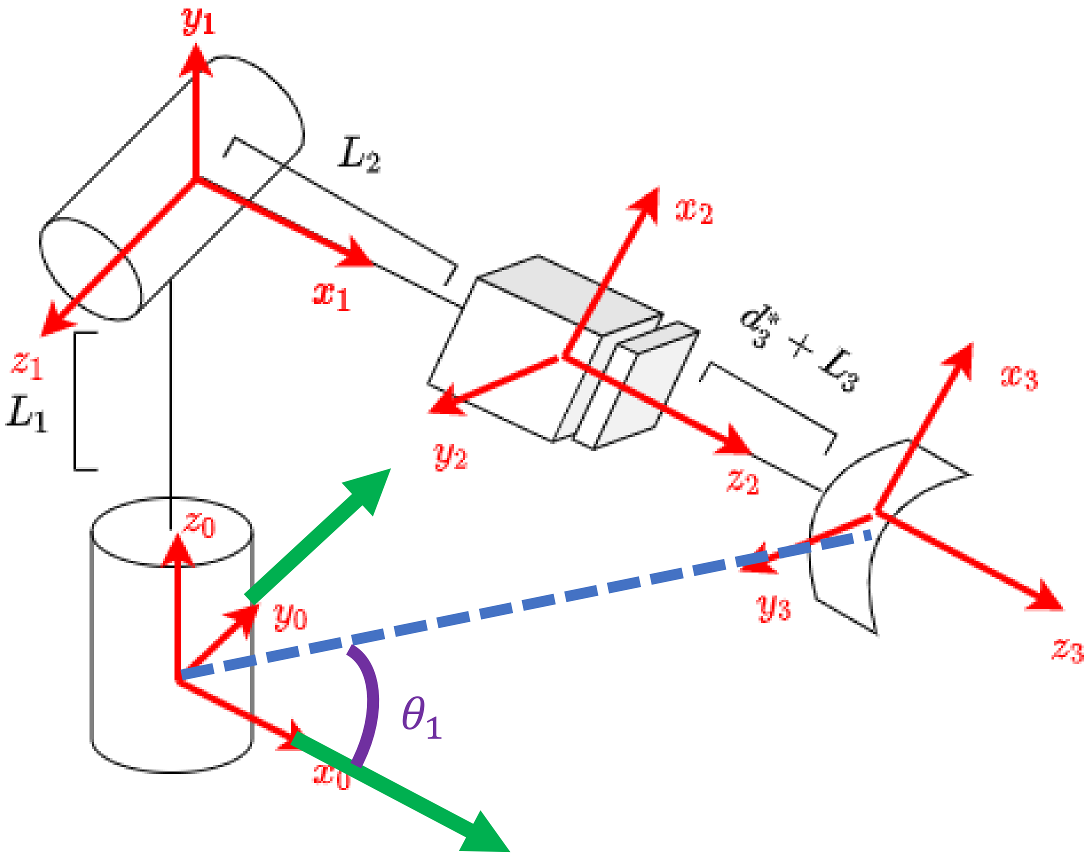
Forward-kinematics

$$T_3^0 = A_1A_2A_3 = \begin{bmatrix} -c_1s_2 & s_1 & c_1c_2 & c_1c_2(d_3^*{+}L_2) \\ -s_1s_2 & -c_1 & c_2s_1 & c_2s_1(d_3^*{+}L_2) \\ c_2 & 0 & s_2 & L_1 + s_2(d_3^*{+}L_2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Inverse-kinematics

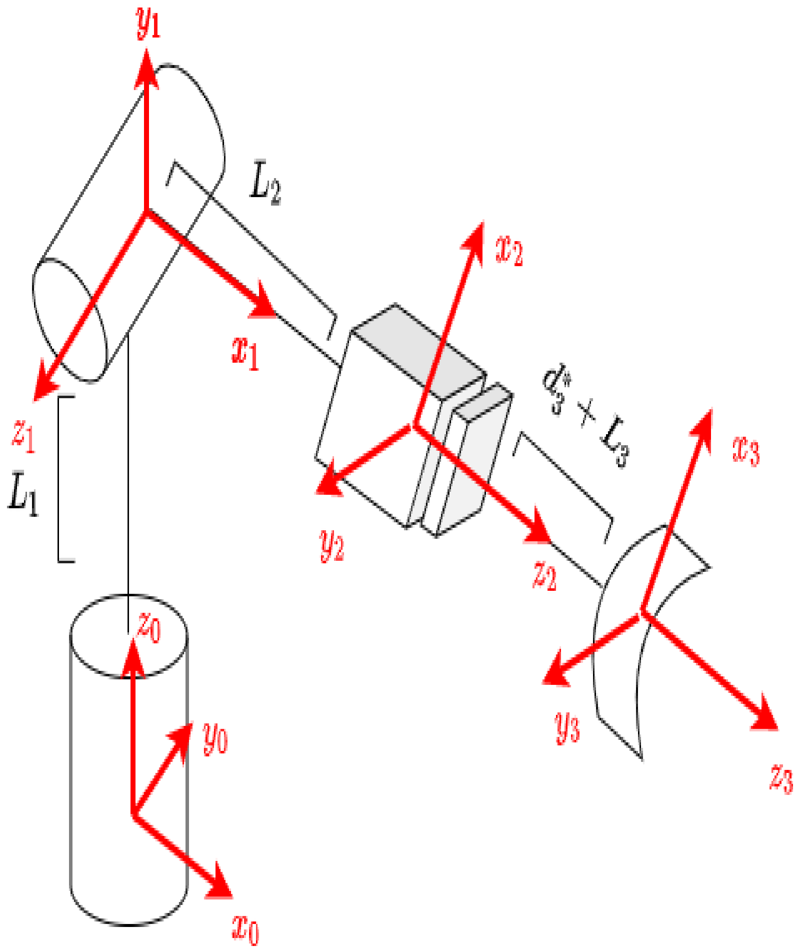
$\theta_1 = atan2(y_c, x_c)$

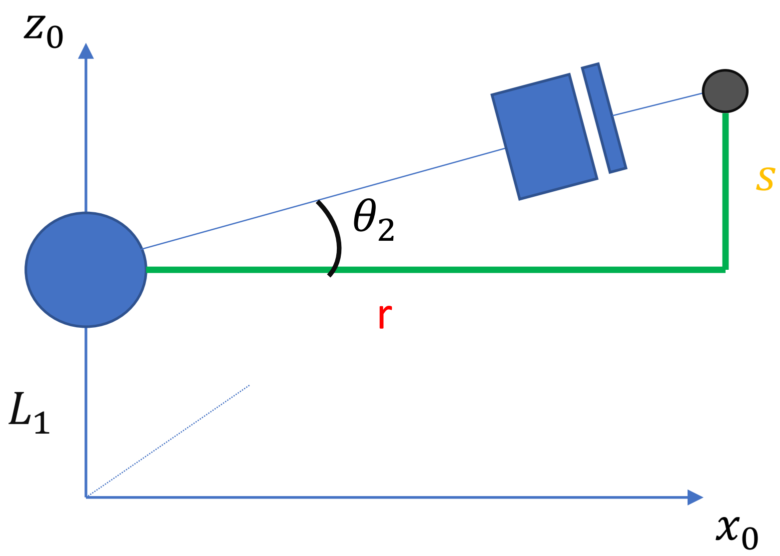


$s = z_c - L_1$

$r = \pm\sqrt{x_0^2 + y_0^2}$

$\theta_2 = atan2(s, r)$





$$r = \pm\sqrt{x_0^2 + y_0^2}$$

$$s = z_c - L_1$$

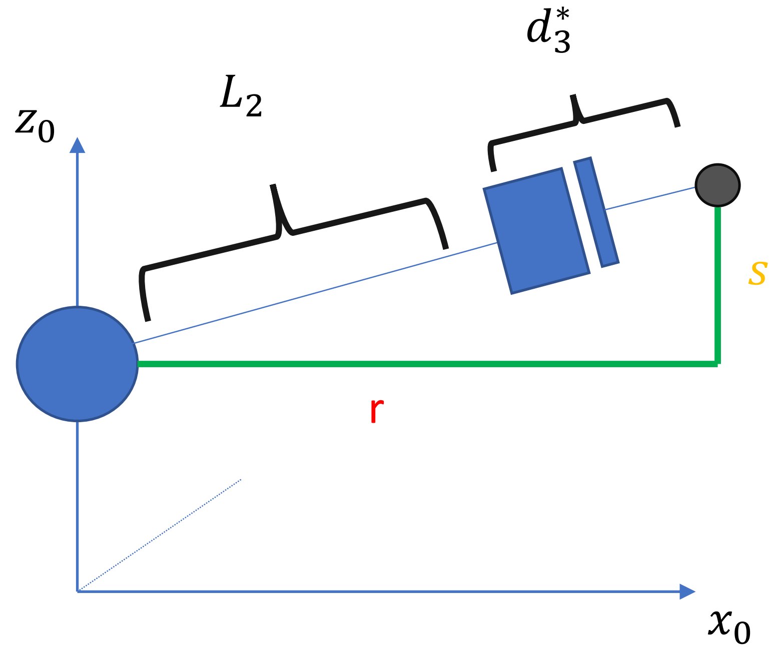
Pythagoras gives us the following solution:

$$(d_3^* + L_2)^2 = r^2 + s^2$$

$$\sqrt{(d_3^* + L_2)^2} = \sqrt{r^2 + s^2}$$

$$(d_3^* + L_2) = \sqrt{r^2 + s^2}$$

$$d_3^* = \sqrt{r^2 + s^2} - L_2$$



$$s = z_c - L_1$$

$$r = \pm\sqrt{x_0^2 + y_0^2}$$

IK-solutions

$$\theta_1 = atan2(y_c, x_c)$$

$$\theta_2 = atan2(s, r)$$

$$d_3^* = \pm\sqrt{r^2 + s^2} - L_2$$

Jacobian for the manipulator

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}, \text{where J} = \text{m} \ast 6 \text{ matrix}$$

$J_{v_i} = z_{i-1} \times (o_n - o_{i-1})$ Revolute

$J_{v_i} = z_{i-1}$ Prismatic Joint

$J_{\omega_i} = z_{i-1}$ Revolute

$J_{\omega_i} = 0$ Prismatic Joint

Jacobian for the manipulator cont.

$$T_1^0 = A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{2+\frac{\pi}{2}} & 0 & s_{2+\frac{\pi}{2}} & 0 \\ s_{2+\frac{\pi}{2}} & 0 & -c_{2+\frac{\pi}{2}} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -c_1 s_2 & s_1 & c_1 c_2 & 0 \\ -s_1 s_2 & -c_1 & c_2 s_1 & 0 \\ c_2 & 0 & s_2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{2+\frac{\pi}{2}} & 0 & s_{2+\frac{\pi}{2}} & 0 \\ s_{2+\frac{\pi}{2}} & 0 & -c_{2+\frac{\pi}{2}} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* + L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -c_1 s_2 & s_1 & c_1 c_2 & c_1 c_2 (d_3^* + L_2) \\ -s_1 s_2 & -c_1 & c_2 s_1 & c_2 s_1 (d_3^* + L_2) \\ c_2 & 0 & s_2 & L_1 + s_2 (d_3^* + L_2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Helpful variables

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \quad z_2 = \begin{bmatrix} c_1 c_2 \\ c_2 s_1 \\ s_2 \end{bmatrix}$$

$$J_{v_i} = z_{i-1} \times (o_n - o_{i-1})$$
$$J_{v_i} = z_{i-1}$$

$$J_{\omega_i} = z_{i-1}$$
$$J_{\omega_i} = 0$$

$$(o_3 - o_0) = \begin{bmatrix} c_1 c_2 (d_3^* + L_2) \\ c_2 s_1 (d_3^* + L_2) \\ L_1 + s_2 (d_3^* + L_2) \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} c_1 c_2 (d_3^* + L_2) \\ c_2 s_1 (d_3^* + L_2) \\ L_1 + s_2 (d_3^* + L_2) \end{bmatrix}$$

$$(o_3 - o_1) = \begin{bmatrix} c_1 c_2 (d_3^* + L_2) \\ c_2 s_1 (d_3^* + L_2) \\ L_1 + s_2 (d_3^* + L_2) \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix} = \begin{bmatrix} c_1 c_2 (d_3^* + L_2) \\ c_2 s_1 (d_3^* + L_2) \\ s_2 (d_3^* + L_2) \end{bmatrix}$$

$$J_{v_1} = z_{1-1} \times (o_3 - o_{1-1}) = z_0 \times (o_3 - o_0) = \begin{bmatrix} -c_2 s_1 (d_3^* + L_2) \\ c_1 c_2 (d_3^* + L_2) \\ 0 \end{bmatrix}$$

$$J_{v_2} = z_{2-1} \times (o_3 - o_{2-1}) = z_1 \times (o_3 - o_1) = \begin{bmatrix} -c_1 s_2 (d_3^* + L_2) \\ -s_1 s_2 (d_3^* + L_2) \\ c_2 (d_3^* + L_2) \end{bmatrix}$$

$$J_{v_3} = z_{3-1} = z_2 = \begin{bmatrix} c_1 c_2 \\ c_2 s_1 \\ s_2 \end{bmatrix}$$

$$J_{\omega_1} = z_{1-1} = z_0$$

$$J_{\omega_2} = z_{2-1} = z_1$$

$$J_{\omega_3} = 0$$

Jacobian

Putting all of it together yields the following Jacobian matrix

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} J_{v_1} & J_{v_2} & J_{v_3} \\ J_{\omega_1} & J_{\omega_2} & J_{\omega_3} \end{bmatrix} = \begin{bmatrix} -c_2 s_1 (d_3^* + L_2) & -c_2 s_1 (d_3^* + L_2) & c_1 c_2 \\ c_1 c_2 (d_3^* + L_2) & -s_1 s_2 (d_3^* + L_2) & c_2 s_1 \\ 0 & c_2 (d_3^* + L_2) & s_2 \\ 0 & s_1 & 0 \\ 0 & -c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Jv

Jw

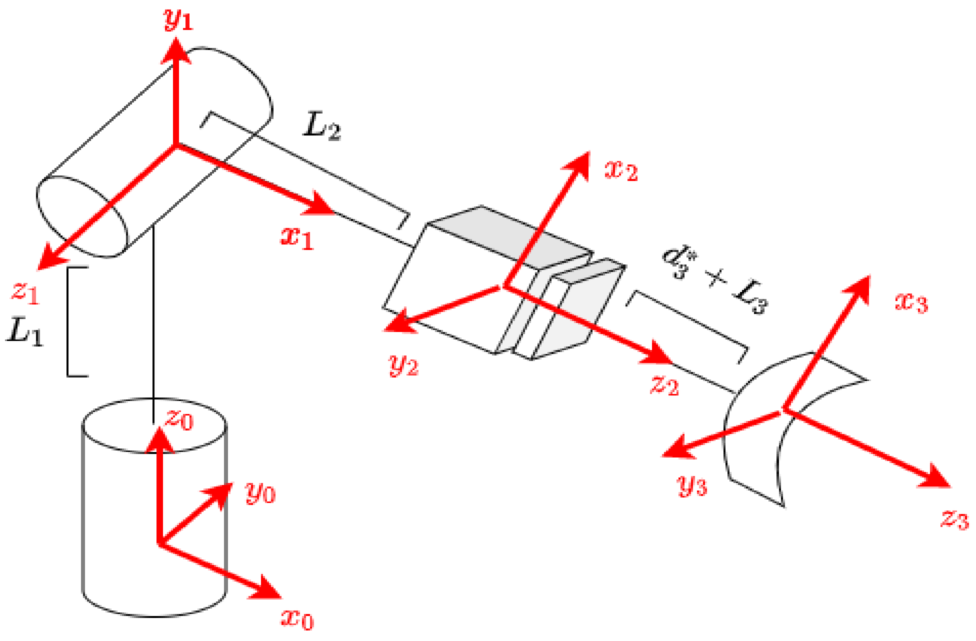
Singularities

Configuration where motion in some directions is impossible.

- Minor movement of the tool could require infinite force in the joints.
- - "Does not mean that joints will fly off into the stratosphere at superluminal speeds, but the joints might strain and break."
- Mathematically: rang J(q) < max
- Appears when the manipulator loses a degree of freedom- (gimbal lock)
- $|J| = 0$

  $|J_v| = 0$ (arm singularities)

We can find the singularities in two ways, by observing or solving analytically



Determinant of the robotic manipulator

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} J_{v_1} & J_{v_2} & J_{v_3} \\ J_{\omega_1} & J_{\omega_2} & J_{\omega_3} \end{bmatrix} = \begin{bmatrix} -c_2 s_1 (d_3^* + L_2) & -c_2 s_1 (d_3^* + L_2) & c_1 c_2 \\ c_1 c_2 (d_3^* + L_2) & -s_1 s_2 (d_3^* + L_2) & c_2 s_1 \\ 0 & c_2 (d_3^* + L_2) & s_2 \\ 0 & s_1 & 0 \\ 0 & -c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Can only find determinant of square matrices, so we decouple the J_v component from J by itself.

$$J_v = [J_v] = [J_{v_1} \quad J_{v_2} \quad J_{v_3}] = \begin{bmatrix} -c_2 s_1 (d_3^* + L_2) & -c_2 s_1 (d_3^* + L_2) & c_1 c_2 \\ c_1 c_2 (d_3^* + L_2) & -s_1 s_2 (d_3^* + L_2) & c_2 s_1 \\ 0 & c_2 (d_3^* + L_2) & s_2 \end{bmatrix}$$

$$J = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\det(J) = \det(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix})$$

$$\det(J) = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - \mathrm{b} \begin{vmatrix} d & f \\ g & i \end{vmatrix} + \mathrm{c} \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\det(J) = a\ (e * i\ - f * h) - \mathrm{b}(d * i\ - f * g) + \mathrm{c}(\mathrm{d} * \mathrm{h}\ - \mathrm{e} * \mathrm{g})$$

$$\det(J) = a\ (e * i\ - f * h) - \mathrm{b}(d * i\ - f * g) + \mathrm{c}(\mathrm{d} * \mathrm{h}\ - \mathrm{e} * \mathrm{g}) = 0$$

Program here and plot the results from matlab

$$c_2(L_2 d_3)^2$$

$$c_2(L_2 d_3)^2 = 0$$

$$c_2(L_2d_3)^2$$

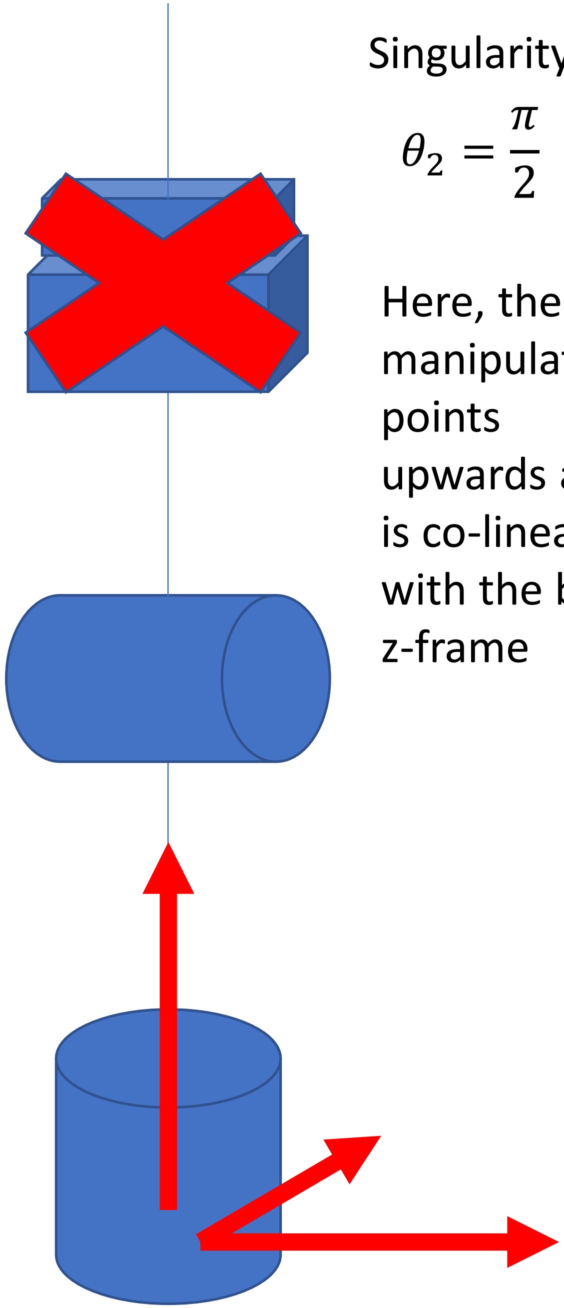
$$c_2(L_2d_3)^2 = 0$$

Remember:

$$\cos\frac{\pi}{2} = 0$$

Putting $\theta_2 = \frac{\pi}{2}$ and $\theta_2 = -\frac{\pi}{2}$

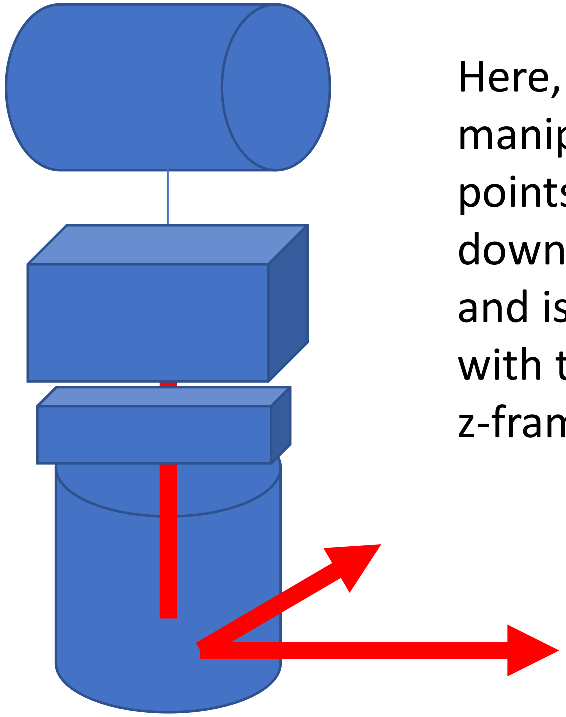
$$0 * (L_2d_3)^2 = 0$$



Singularity 1

$$\theta_2 = \frac{\pi}{2}$$

Here, the manipulator points upwards and is co-linear with the base z-frame



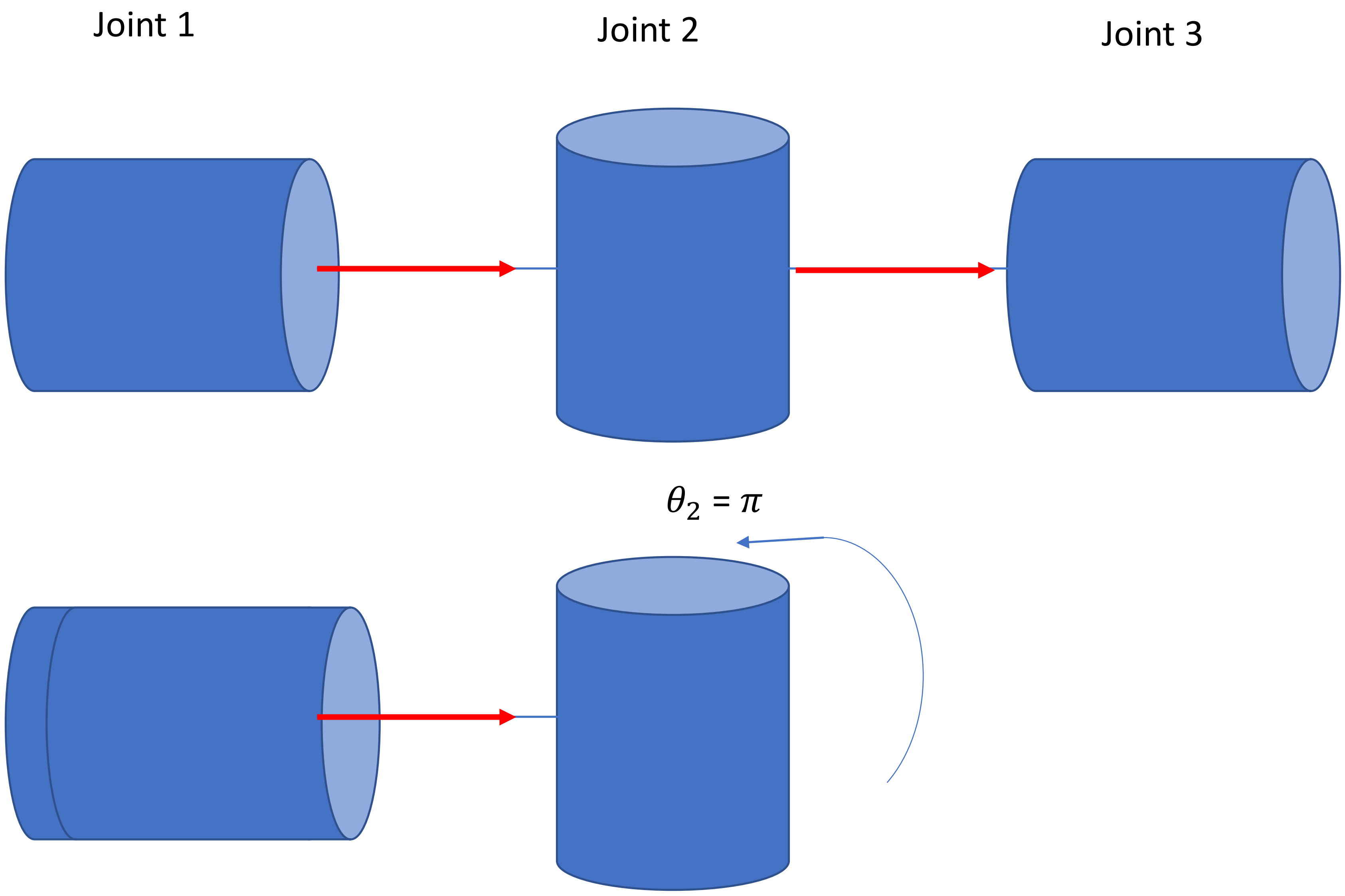
Singularity 1

$$\theta_2 = -\frac{\pi}{2}$$

Here, the manipulator points downwards and is colinear with the base z-frame

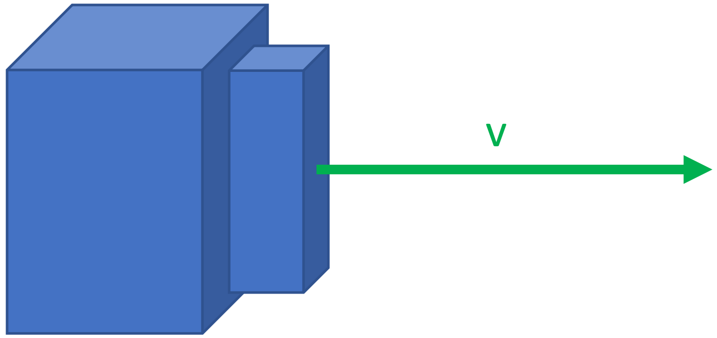
Wrist singularities

Putting θ_2 = 0 and $\theta_2 = \pi$

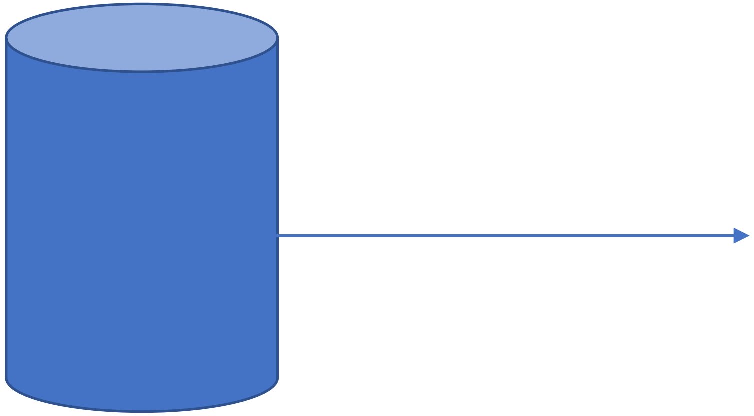


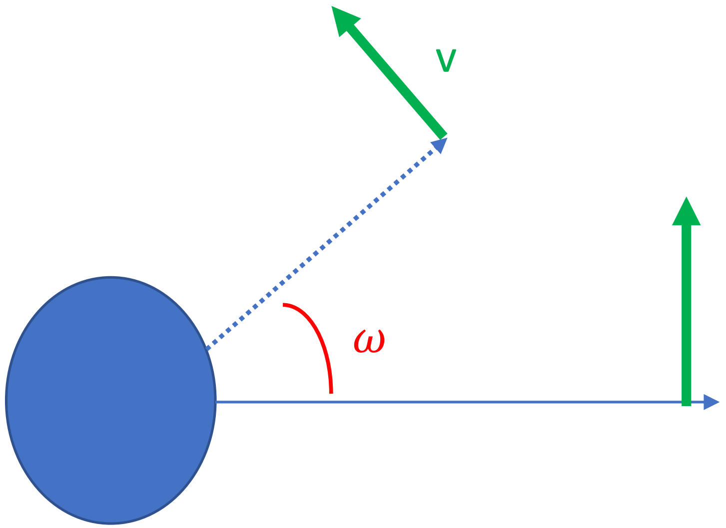
Explanation of j_ω

No rotation



Revolute joints rotate along its own z-axis





Finding the velocity

$$v = J_v(q)\dot{q}$$