ØV8 — DFT

Innleveringsfrist: **23. oktober** 2020.

Ukeoppgavene skal løses selvstendig og vurderes i øvingstimene. Det forventes at alle har satt seg inn i fagets øvingsopplegg og godkjenningskrav for øvinger. Dette er beskrevet påhjemmesiden til IN3190:
`http://www.uio.no/studier/emner/matnat/ifi/IN3190/h20/informasjon-om-ovingsopplegget/`

Oppgave 1 — Oppgave 8.1 fra Ambardar: DFT fra definisjon Vekt: 4

Compute the DFT from its definition, for the following signals:

a) $x(n) = \{1, 2, 1, 2\}$ b) $x(n) = \{2, 1, 3, 0, 4\}$
c) $x(n) = \{2, 2, 2, 2\}$ d) $x(n) = \{1, 0, 0, 0, 0, 0, 0, 0\}$

- Hint: $e^{-j\pi/2} = -j$, $e^{-j\pi} = -1$, $e^{-j\pi/2} = -j$, $e^{-j3\pi/2} = j$
- Hint: The DFT exhibits conjugate symmetry around $k = N/2$. Hence, $X_{\mathrm{DFT}}(N-k) = X^{\star}_{\mathrm{DFT}}(k)$. This way, one can save some calculation effort if only calculating the value for indices $k \le N/2$, and then use the conjugate symmetry relation for the rest of the indices.

a) $\boxed{X_{\mathrm{DFT}}(k) = \{6, 0, -2, 0\}}$ b) $\boxed{X_{\mathrm{DFT}}(k) = \{10, 1.12 + j1.09, -1.12 + j4.62, -1.12 - j4.62, 1.12 - j1.09\}}$
c) $\boxed{X_{\mathrm{DFT}}(k) = \{8, 0, 0, 0\}}$ d) $\boxed{X_{\mathrm{DFT}}(k) = \{1, 1, 1, 1, 1, 1, 1, 1\}}$

Oppgave 2 — Tema: DFT.
Exercise 7.5 from Manolakis & Ingle: 2 Points

Determine the N-point DFT of the following sequences, which are all defined over $0 \le n \le (N-1)$:

a) $x_1(n) = 4 - n$, $N = 8$.
   Solution: $4 - 6j\sin(k\pi/4) - 4j\sin(k\pi/2) - 2j\sin(3k\pi/4)$, which was composed from the equal expression $4 + 3(e^{-j2\pi k/8} - e^{j2\pi k/8}) + 2(e^{-j2\pi k2/8} - e^{j2\pi k2/8}) + (e^{-j2\pi k3/8} - e^{j2\pi k3/8})$

b) $x_2(n) = 4\sin(0.2\pi n)$, $N = 10$.   Solution: $-20j\delta(k-1) + 20j\delta(k-9)$

c) $x_3(n) = 6\cos^2(0.2\pi n)$, $N = 10$.   Solution: $3\delta(k) + 15\delta(k-2) + 15\delta(k-8)$

d) $x_4(n) = 5(0.8)^n$, $N = 16$.

Oppgave 3— Tema: Sampling og aliasing.
Oppgave 6.01 fra Manolakis & Ingle 4 Poeng

The periodic signal $x_c(t) = 5\cos(200\pi t + \pi/6) + 4\sin(300\pi t)$ is sampled at a rate of $F_s = 1$ kHz to obtain the discrete-time signal $x(n)$.

(a) Determine the spectrum $X(e^{j\omega})$ of $x(n)$.

Plot its magnitude as a function of normalized angular frequency ω in $\frac{\text{rad}}{\text{sample}}$ and as a function of frequency F in Hz. Plot the spectrum for $-2.5 \le \omega/\pi \le 2.5$ and $-2F_s \le F \le 2F_s$.

Explain whether the original signal $x_c(t)$ can be recovered from $x(n)$.

Hints:

- Finn først $X_c(j\Omega)$ og så deretter $X(e^{j\omega})$, som er en skalert og periodisert versjon av $X_c(j\Omega)$. Husk at $\omega = \Omega T = 2\pi F/F_s$.
- Hvis du dekomponerer $x_c(t)$ på formen $x_c(t) = A\left(e^{j\Omega_1}e^{j\phi} + e^{-j\Omega_1}e^{-j\phi}\right) + B\left(e^{j\Omega_2} - e^{-j\Omega_2}\right)$, så har du allerede funnet $X_c(j\Omega)$ "by inspection."
- Husk at sampling i tid gir periodisering / "kopiering" i Fourierdomenet.
- Husk at $e^{iy} = \cos(y) + j\sin(y)$ som gir at $\cos(y) = \dfrac{e^{jy} + e^{-jy}}{2}$ og $\sin(y) = \dfrac{e^{jy} - e^{-jy}}{2j}$.
- Det kan også være nyttig å bruke at $e^{j(y+\phi)} = e^{jy}e^{j\phi}$.

(b) Repeat part (a) for $F_s = 500$ Hz.

(c) Repeat part (a) for $F_s = 100$ Hz.

(d) Comment on your results: For what sampling frequencies can the original continuous signal be reconstructed from the sampled signal? What happens when the sampling frequency is too low?