IN5060

Quantitative Performance Analysis

User studies (cntd)

UiO : University of Oslo

Does blur hide asynchrony?

study by Ragnhild Eg (Simula) et al., 2011

Perception of synchrony

Sensitivity for perceptual synchrony is subjective and depends on the content

Spoken sentences (Grant et al., 2003)

- Discrimination thresholds: ≈50 ms audio lead, ≈200 ms audio lag

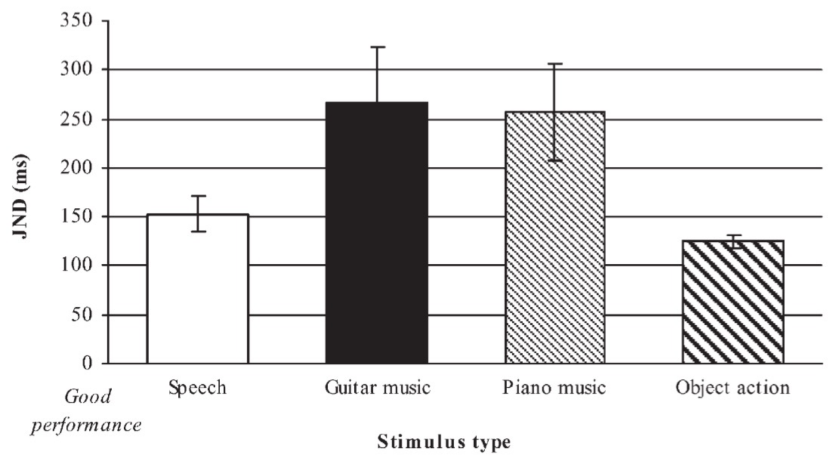
Hitting table with wand (Levitin et al., 2000)

- Synchrony thresholds set to 75 %: 41 ms Alead to 45 ms Alag

Music, baseball, speech

(Vatakis & Spence, 2006)

- Temporal order judgements (audio/video first)



Stimuli

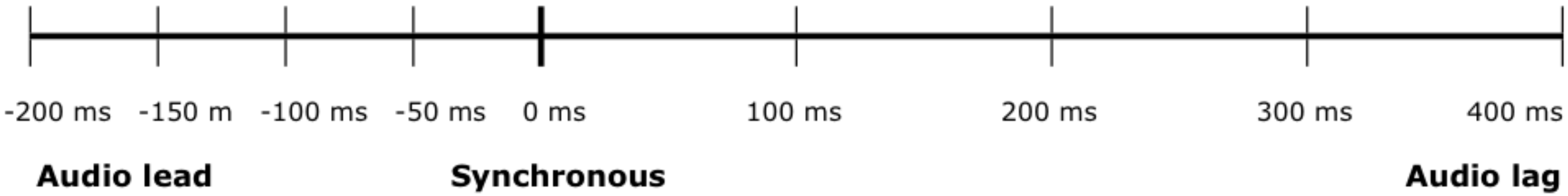
3 content types

Chess game

News broadcast

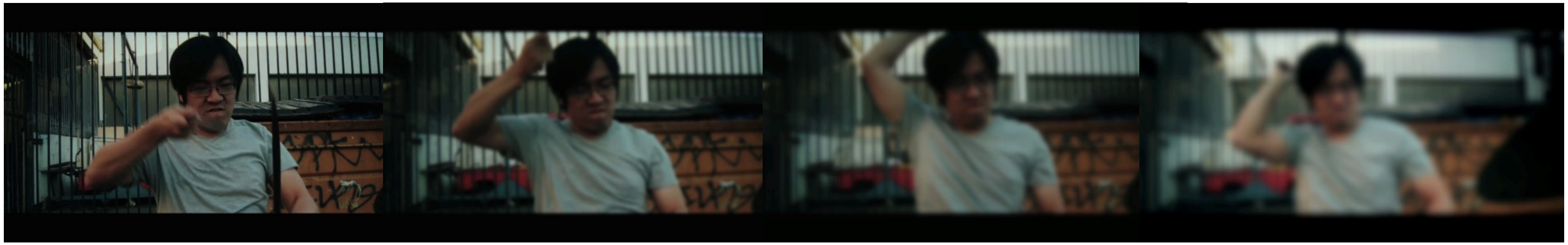
Drummer

9 asynchrony levels



Stimuli

Visual distortion, 4 levels, Gaussian blur filter



Undistorted | Blur 2x2 pixels | Blur 4x4 pixels | Blur 6x6 pixels

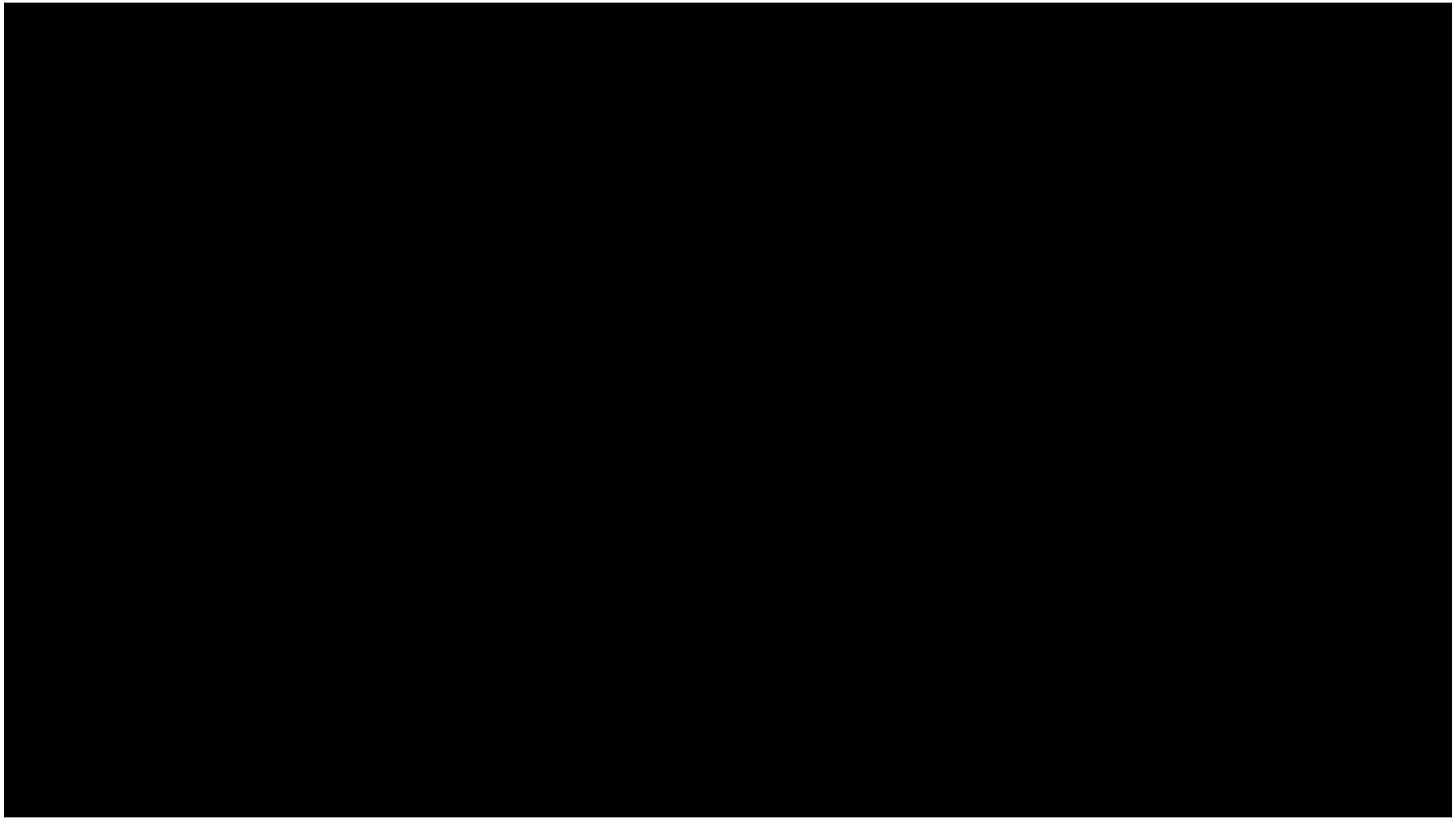
Procedure

- Carried out at the Speech Lab, NTNU

Cue 2 sec | Video presentation - 13 sec | Response 0.2 - 2 sec

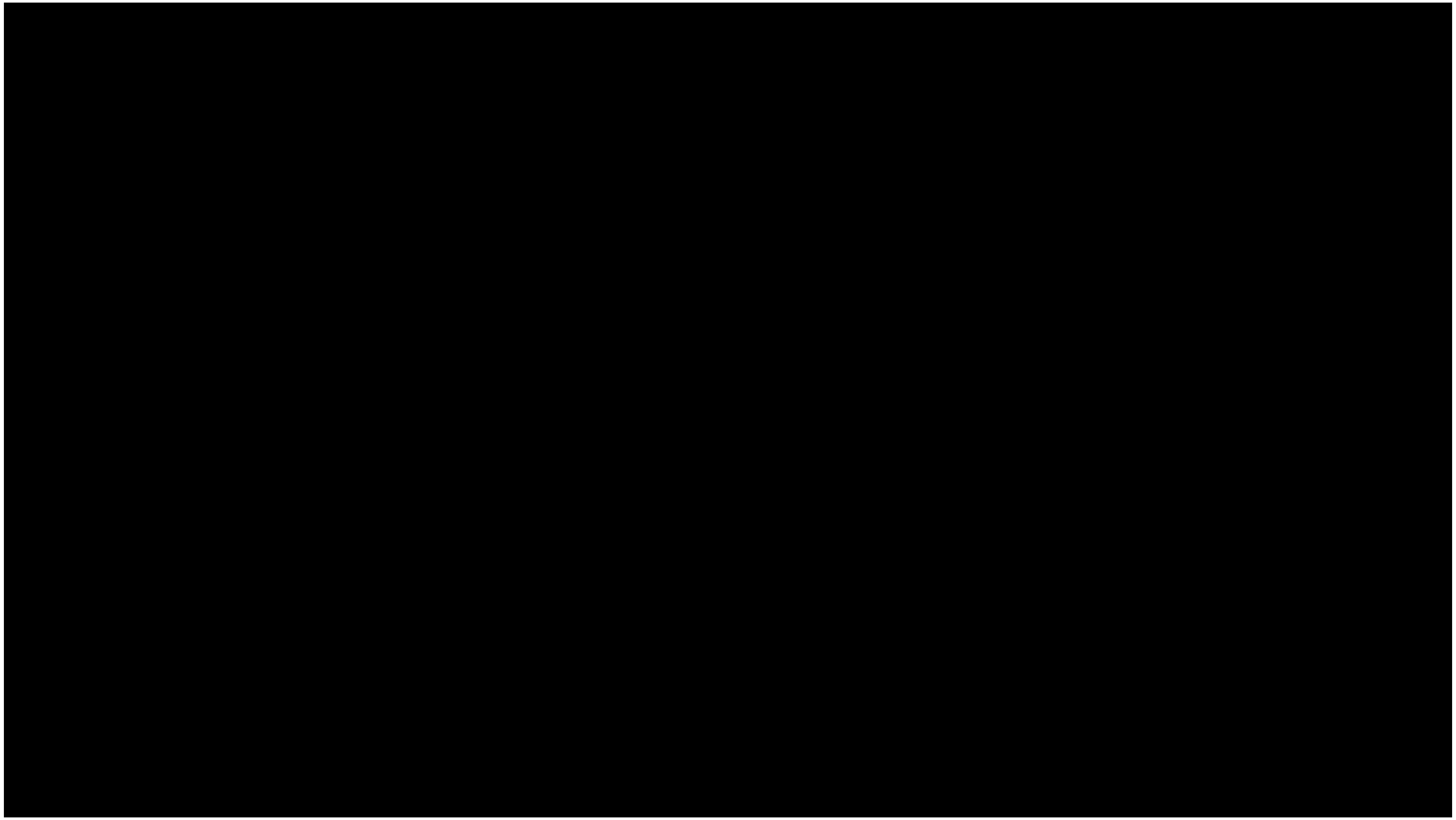
Chess content - 200 ms audio lead

Chess content - 200 ms audio lag, blurred



News - 300 ms audio lag, blurred

Drums - 100 ms audio lag, blurred



Drums - 150 ms audio lead, slightly blurred

Audio streaming from PPT in Zoom is really bad.

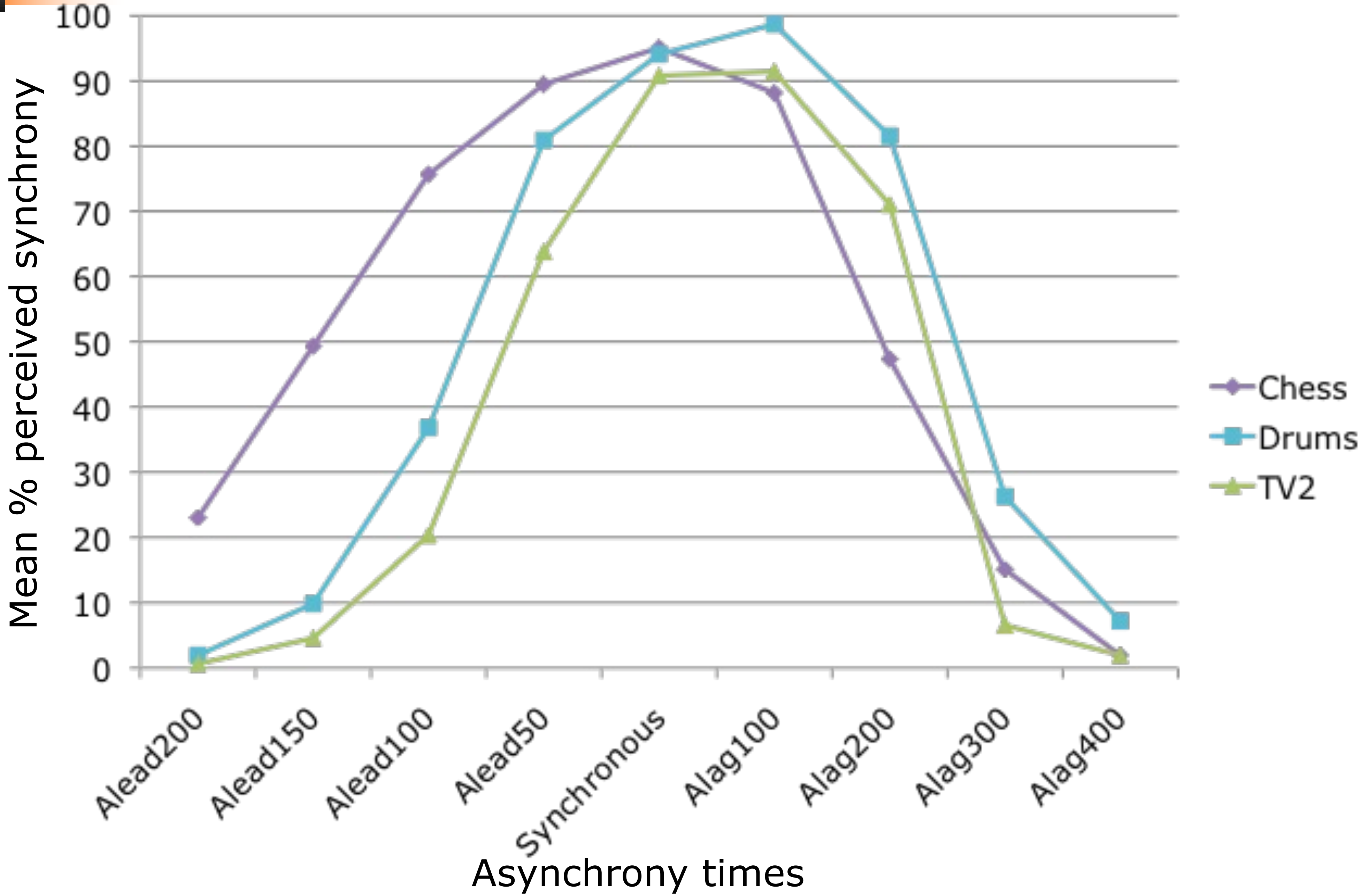
See the examples here:

https://drive.google.com/drive/folders/1hxXFdh5xCeN1pMril2kZzNwPC3ZmuL-u?usp=sharing

Design & Analysis

- 2 independent studies
- Full-factorial design
- 2 repetitions of each condition
- Binomial responses converted to percentages
- Repeated-measures ANOVAs
- Separate analyses for:
  – Audio lag and audio lead (different scales)
  – Content types (different response patterns)

Mean perceived synchrony, averaged across **blur** levels



When to use One-way ANOVA

		Alternatives			
	Measure-ments	Un-blurred	Blur 2x2	Blur 4x4	Blur 6x6
Chess	Audio lead 200	*% noticed*	*% noticed*	*% noticed*	*% noticed*
	Audio lead 150	*% noticed*	*% noticed*	*% noticed*	*% noticed*
	Audio lead 100	*% noticed*	*% noticed*	*% noticed*	*% noticed*
	Audio lead 50	*% noticed*	*% noticed*	*% noticed*	*% noticed*

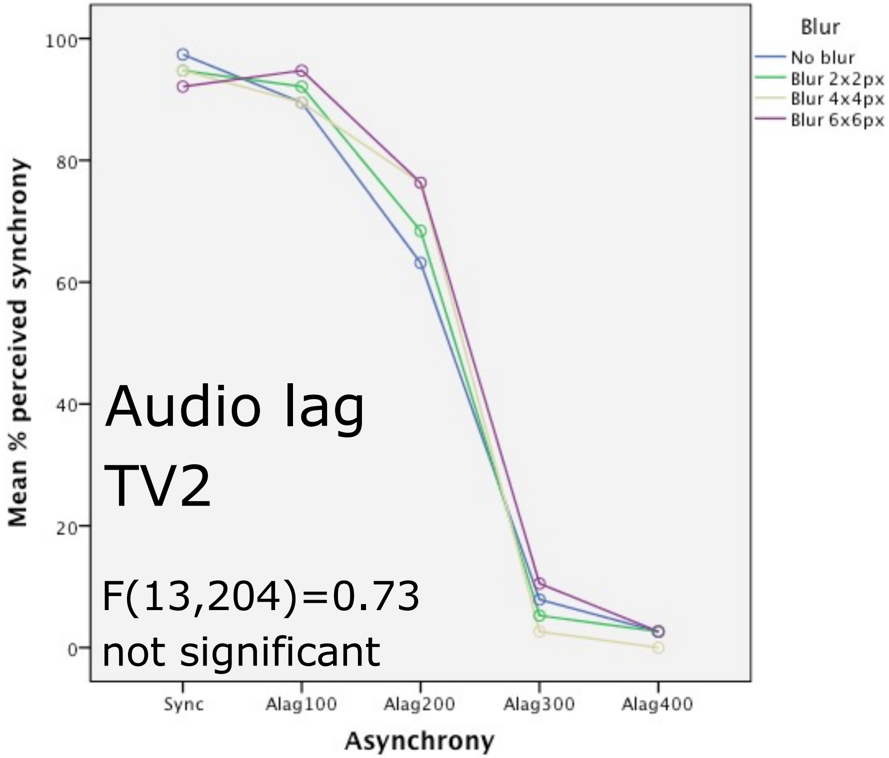
Assessment of relevance

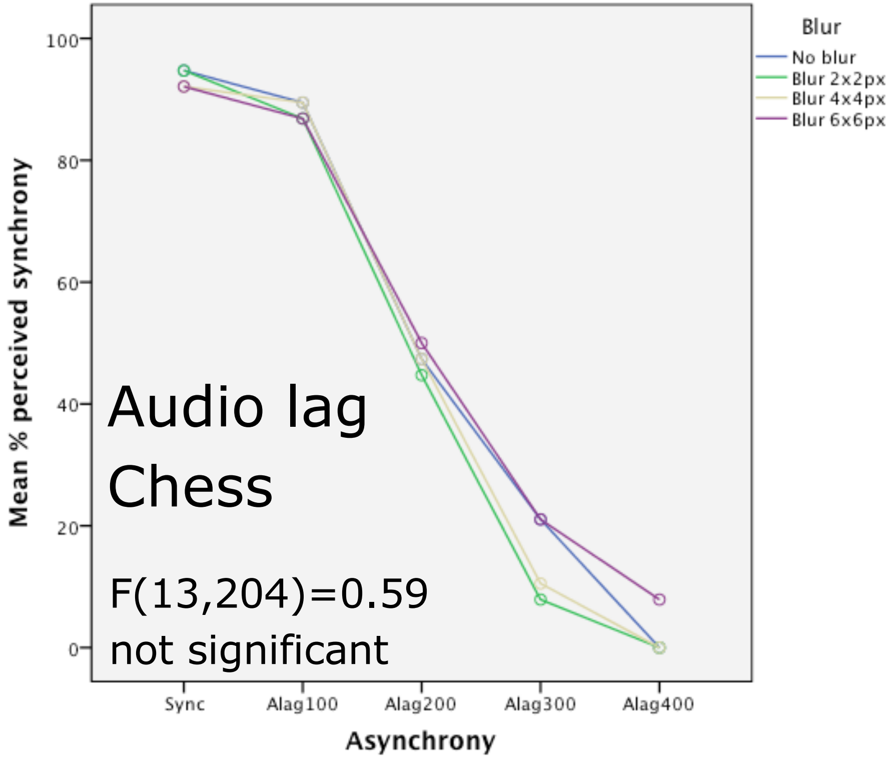
	Visual distortion	
	Content	F-statistics
Audio lag	Chess	$F(4,85)=88.79$, $p<.001$
	TV2	$F(4,85)=232.54$, $p<.001$
	Drums	$F(4,85)=197.57$, $p<.001$
Audio lead	Chess	$F(4,85)=71.77$, $p<.001$
	TV2	$F(4,85)=100.26$, $p<.001$
	Drums	$F(4,85)=126.31$, $p<.001$

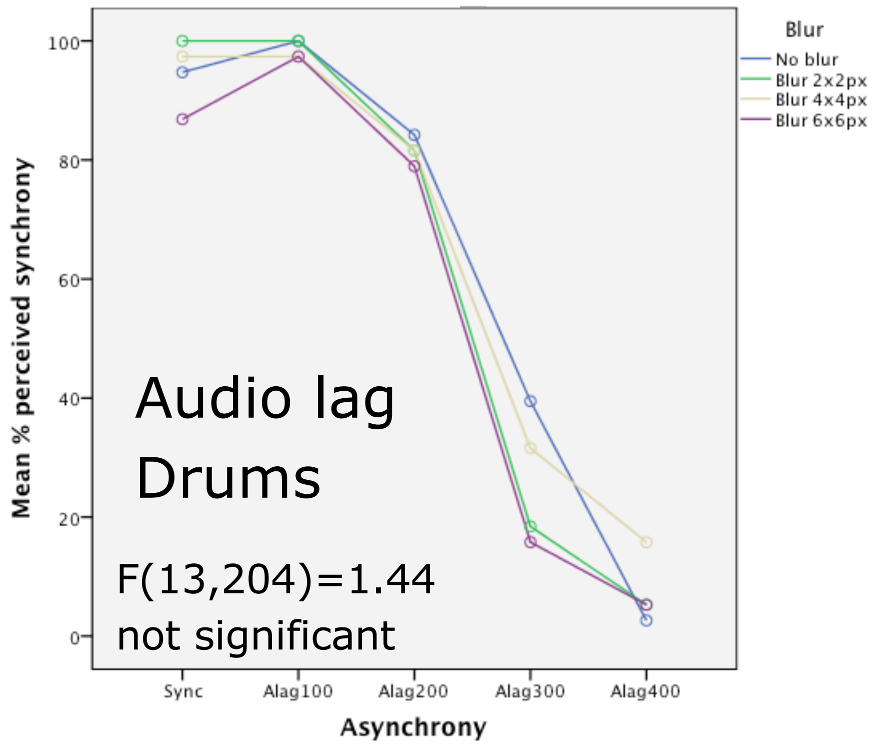
5 settings

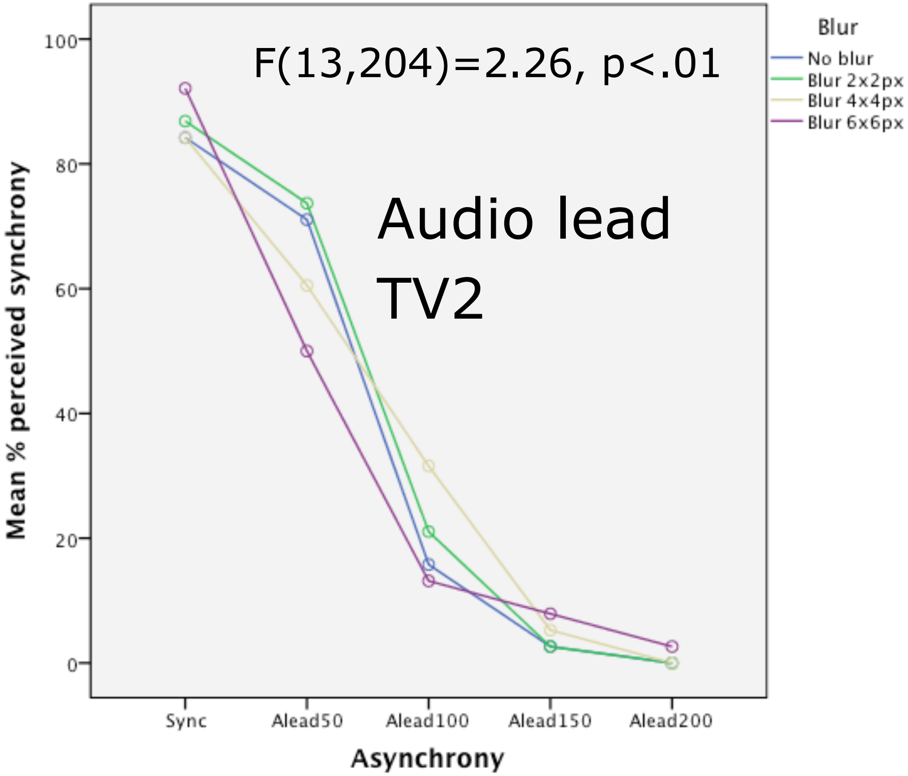
18 participants

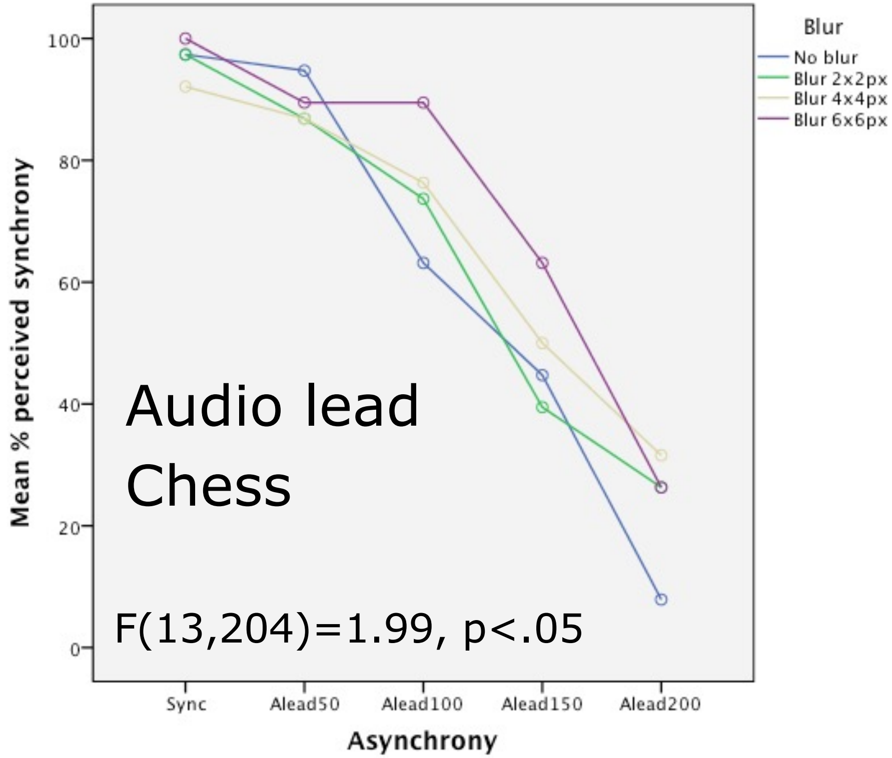
Blur distortion



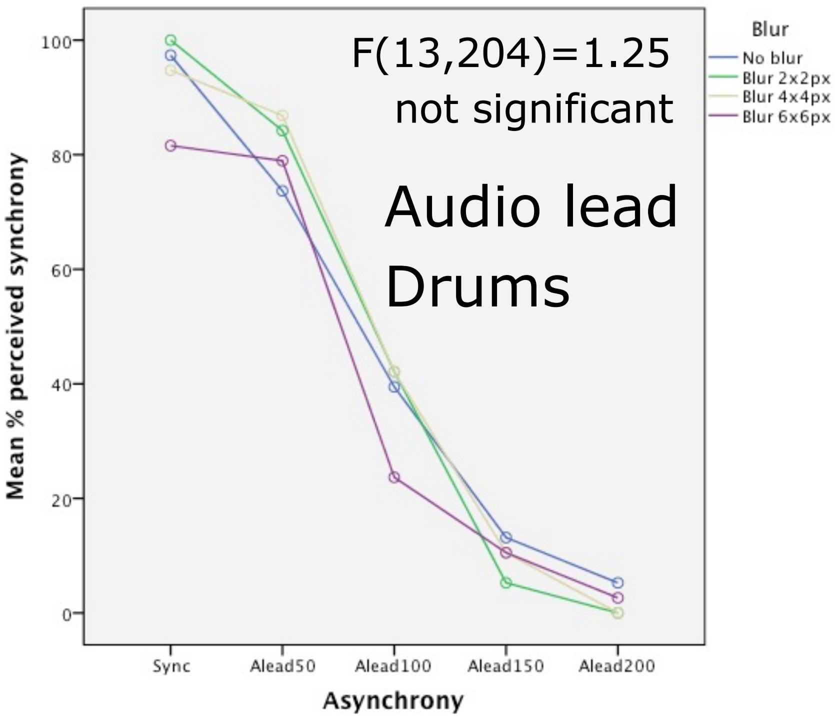








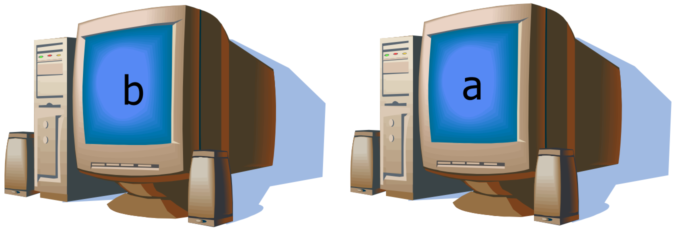
Blur distortion



ANOVA

Analysis of Variance

A-B Comparison



Candidate (i)	Audio lag (b_i)	Audio in sync (a_i)	Difference ($d_i = b_i - a_i$)
1	5	6	-1
2	3	8	-5
3	14	10	4
4	10	15	-5
5	8	11	-3
6	7	3	4

Mean of differences $\bar{d} = -1$, Standard deviation $\sigma_d = 4.15$

A-B Comparison

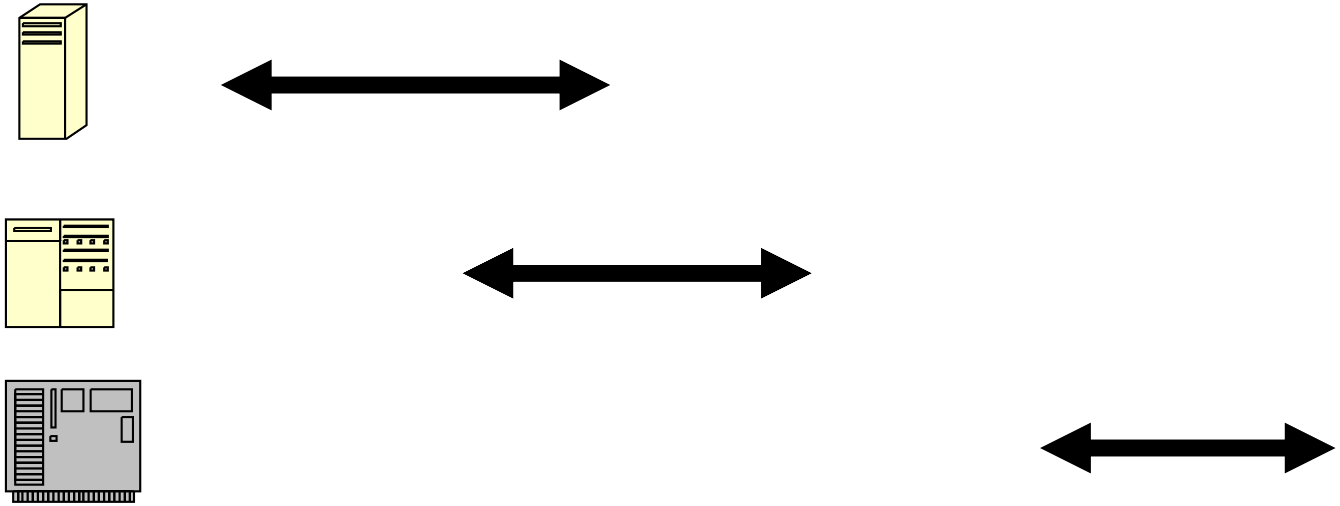
Mean of differences $\bar{d} = -1$

Standard deviation $\sigma_d = 4.15$

- From mean of differences, appears that audio lag reduced performance
- However, standard deviation is large
- Is the variation between the two alternatives greater than the variation (error) in the measurements?
- Confidence intervals can work, but what if there are more than two alternatives?

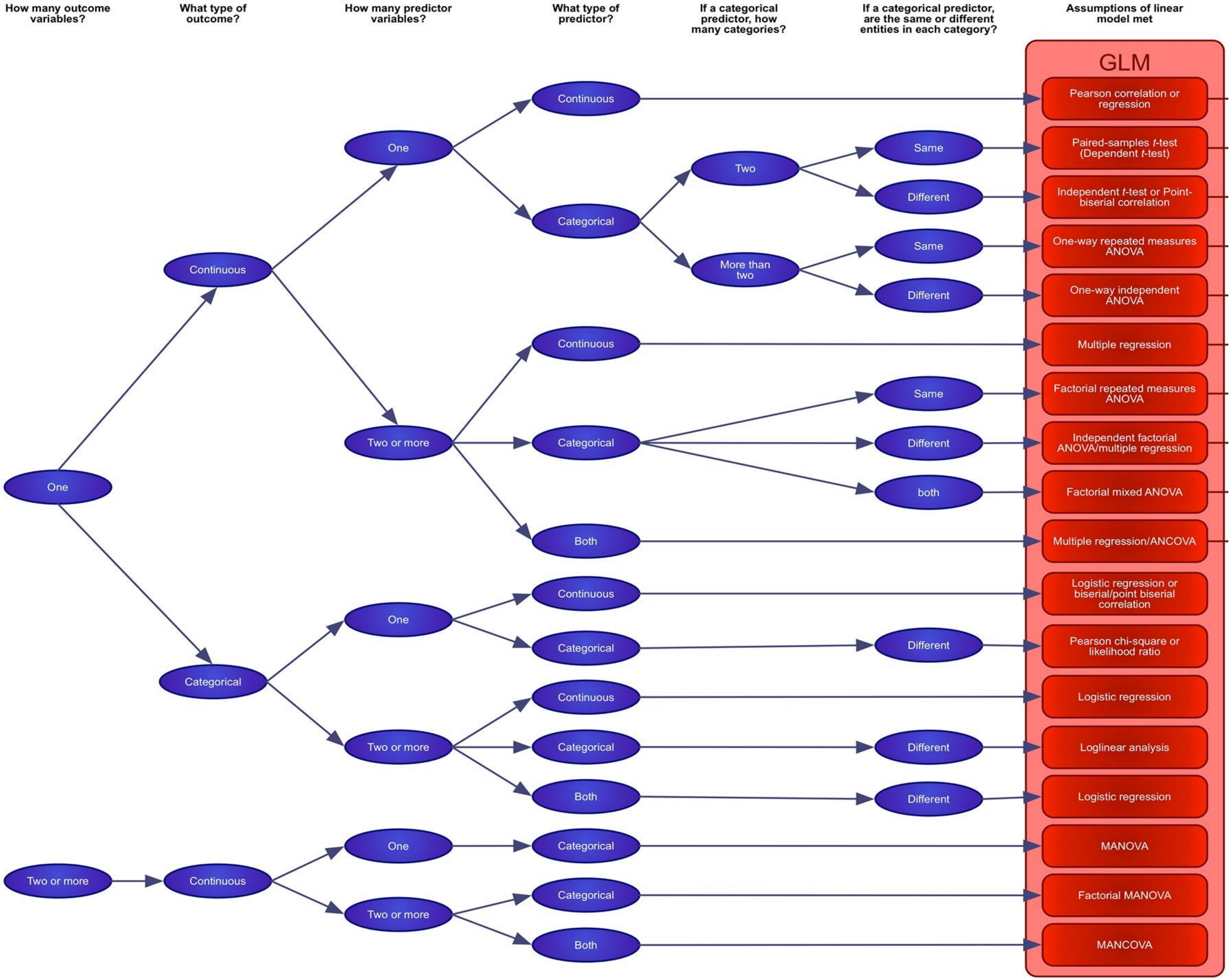
Comparing more than two alternatives

- Naïve approach
  - Compare confidence intervals



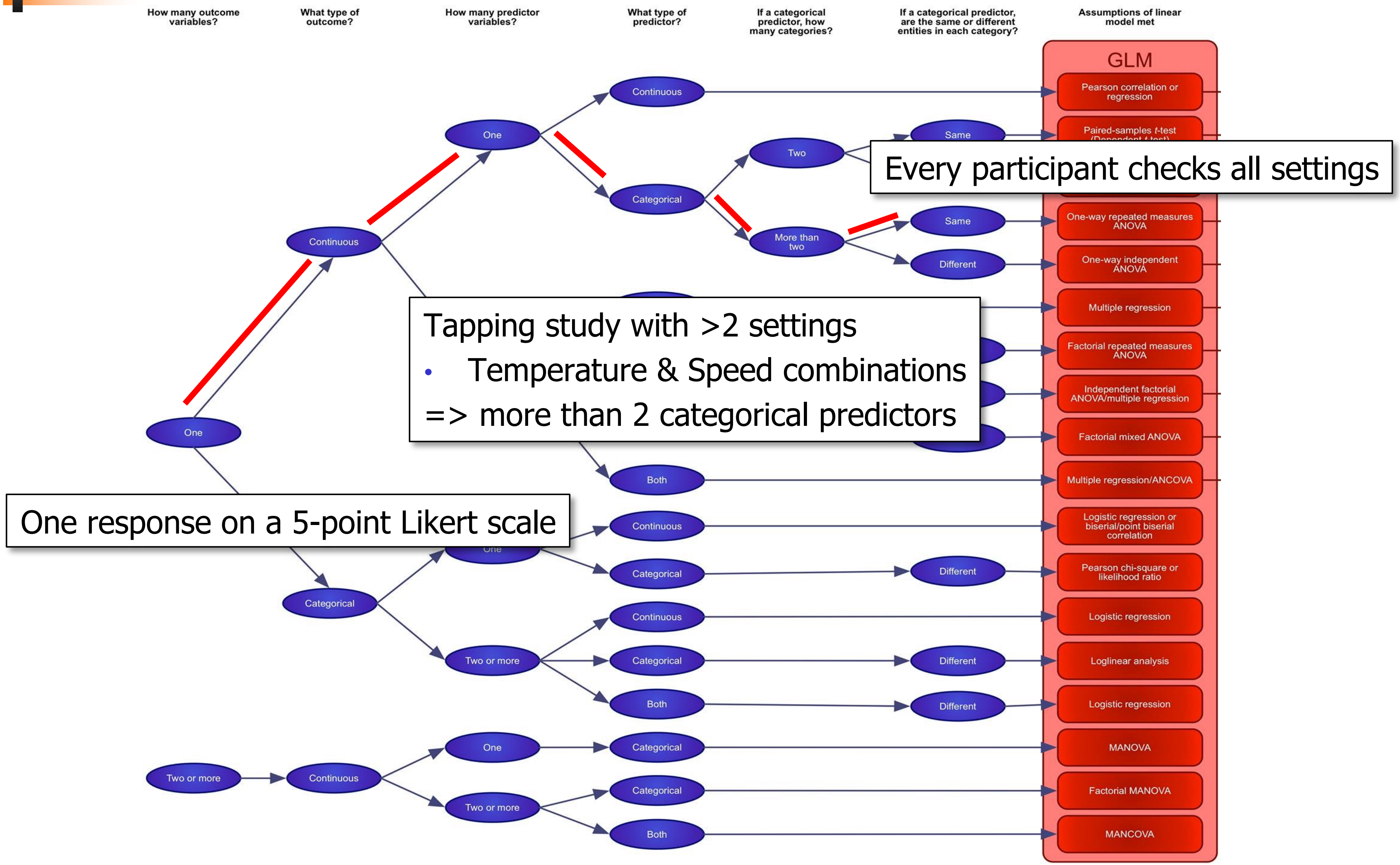
  - Need to do for all ***pairs***. This grows very quickly.
  - Example: 7 alternatives would require 21 pair-wise comparisons
    - possible combinations: $\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 1}$
    - for our case: $\binom{7}{2} = \frac{7*6}{2*1} = \frac{42}{2} = 21$
  - Would not be surprising to find 1 pair differed (at 95%)

SPSS Chart of methods



From Field, A. P. (2013). *Discovering statistics using IBM SPSS Statistics: And sex and drugs and rock 'n' roll* (4th ed.). London: Sage.

SPSS Chart of methods



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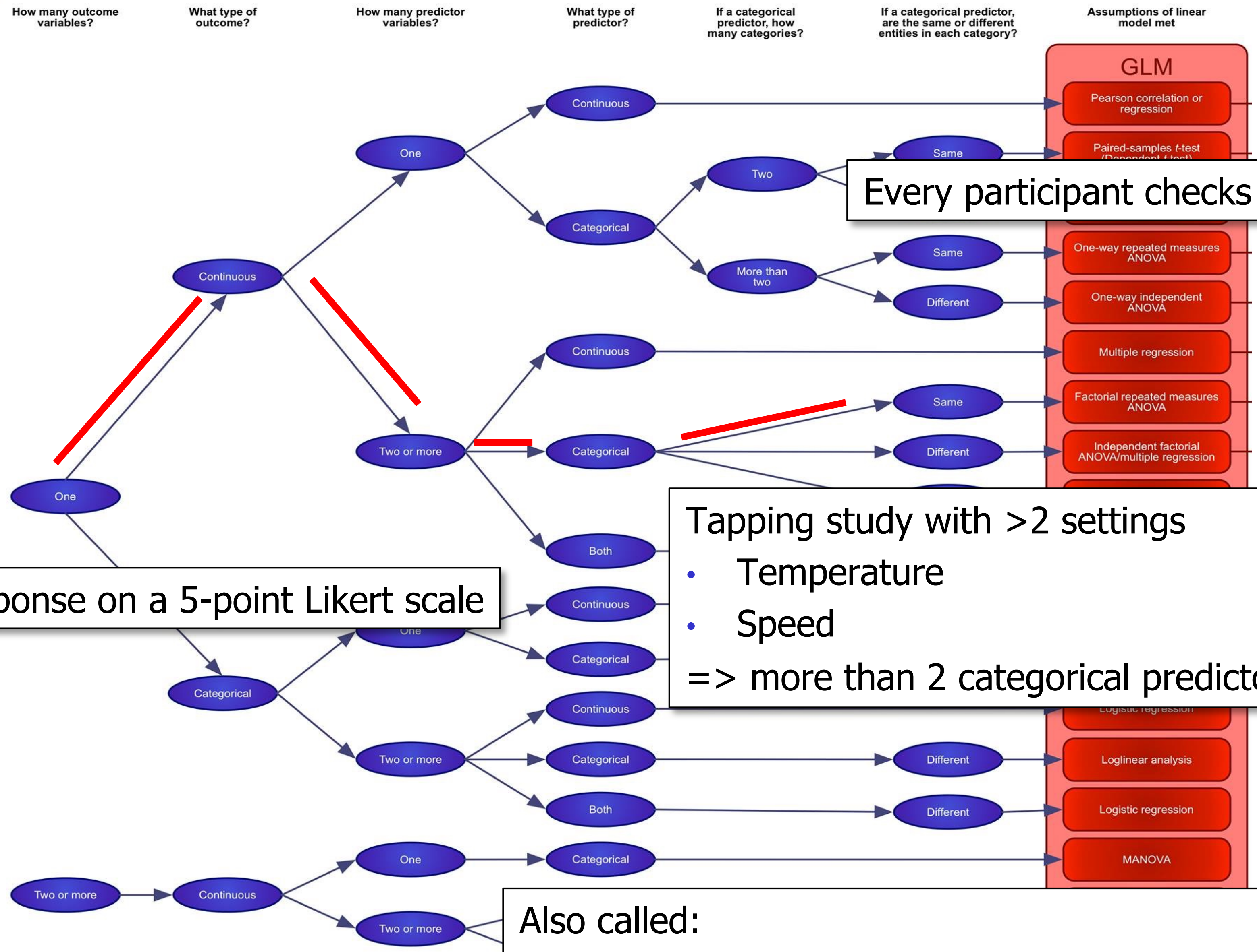
SPSS Chart of methods



Every participant checks all settings

Tapping study with >2 settings

- Temperature
- Speed

=> more than 2 categorical predictors

One response on a 5-point Likert scale

Also called:

Three-Way Repeated Measures ANOVA within subjects

From Field, A. P. (2013). *Discovering statistics using IBM SPSS Statistics: And sex and drugs and rock 'n' roll* (4th ed.). London: Sage.

ANOVA – Analysis of Variance

- Partitioning *variation* (not variance) into the part that can be explained and the part that cannot be explained

- Separates total variation observed in a set of measurements into:
  1. Variation within one system
     due to uncontrolled measurement errors
  2. Variation between systems
     due to real differences + random error

- Is variation (2) statistically greater than variation (1)?

One-way repeated measures ANOVA

- Make n measurements of k alternatives
- $y_{ij} = i$-th measurement on j-th alternative

- Assumes errors are
  - independent
  - normally distributed

- In user studies, each measurement is the set of responses by one participant

When to use One-way ANOVA

Independent variable: categorical

	Alternatives					
Measure-ments	1	2	…	*j*	…	*k*
1	y_{11}	y_{12}	…	y_{1j}	…	y_{k1}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2k}
…	…	…	…	…	…	…
i	y_{i1}	y_{i2}	…	y_{ij}	…	y_{ik}
…	…	…	…	…	…	…
n	y_{n1}	y_{n2}	…	y_{nj}	…	y_{nk}

When to use One-way ANOVA

Observations: independent

	Alternatives					
Measure-ments	1	2	…	*j*	…	*k*
1	y_{11}	y_{12}	…	y_{1j}	…	y_{k1}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2k}
…	…	…	…	…	…	…
i	y_{i1}	y_{i2}	…	y_{ij}	…	y_{ik}
…	…	…	…	…	…	…
n	y_{n1}	y_{n2}	…	y_{nj}	…	y_{nk}

When to use One-way ANOVA

Dependent variable:

- **continuous**
- **interval or ratio terms**

	Alternatives					
Measure-ments	1	2	…	*j*	…	*k*
1	y_{11}	y_{12}	…	y_{1j}	…	y_{k1}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2k}
…	…	…	…	…	…	…
i	y_{i1}	y_{i2}	…	y_{ij}	…	y_{ik}
…	…	…	…	…	…	…
n	y_{n1}	y_{n2}	…	y_{nj}	…	y_{nk}

When to use One-way ANOVA

Dependent variable:

- **approximately normally distributed**
- **in each category of the indep. variable**

	Alternatives					
Measure-ments	1	2	…	*j*	…	*k*
1	y_{11}	y_{12}	…	y_{1j}	…	y_{k1}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2k}
…	…	…	…	…	…	…
i	y_{i1}	y_{i2}	…	y_{ij}	…	y_{ik}
…	…	…	…	…	…	…
					…	y_{nk}

Three main options:

1. More than 25 observations? No test required !
2. Visual confirmation by plotting a histogram of the value
3. Shapiro-Wilk test

All Measurements for All Alternatives

	Alternatives					
Measure-ments	1	2	…	*j*	…	*k*
1	y_{11}	y_{12}	…	y_{1j}	…	y_{k1}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2k}
…	…	…	…	…	…	…
i	y_{i1}	y_{i2}	…	y_{ij}	…	y_{ik}
…	…	…	…	…	…	…
n	y_{n1}	y_{n2}	…	y_{nj}	…	y_{nk}

Overall Mean

Average of all measurements made of all alternatives:

$$\bar{y} = \frac{\sum_{j=1}^{k} \sum_{i=1}^{n} y_{ij}}{kn}$$

	Alternatives					
Measure-ments	1	2	…	*j*	…	*k*
1	y_{11}	y_{12}	…	y_{1j}	…	y_{k1}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2k}
…	…	…	…	…	…	…
i	y_{i1}	y_{i2}	…	y_{ij}	…	y_{ik}
…	…	…	…	…	…	…
n	y_{n1}	y_{n2}	…	y_{nj}	…	y_{nk}

Column Means

Column means are average values of all measurements within a single alternative

$$\overline{y_{.j}} = \frac{\sum_{i=1}^{n} y_{ij}}{n}$$

- average performance of a single alternative

	Alternatives					
Measure-ments	1	2	…	*j*	…	*k*
1	y_{11}	y_{12}	…	y_{1j}	…	y_{k1}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2k}
…	…	…	…	…	…	…
i	y_{i1}	y_{i2}	…	y_{ij}	…	y_{ik}
…	…	…	…	…	…	…
n	y_{n1}	y_{n2}	…	y_{nj}	…	y_{nk}
Column mean	$y_{.1}$	$y_{.2}$	…	$y_{.j}$	…	$y_{.k}$

Effect = Deviation From Overall Mean

- α_j: effect of alternative j = deviation of column mean from overall mean: $\alpha_j = \overline{y_{.j}} - \bar{y}$

	Alternatives					
Measure-ments	1	2	…	*j*	…	*k*
1	y_{11}	y_{12}	…	y_{1j}	…	y_{k1}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2k}
…	…	…	…	…	…	…
i	y_{i1}	y_{i2}	…	y_{ij}	…	y_{ik}
…	…	…	…	…	…	…
n	y_{n1}	y_{n2}	…	y_{nj}	…	y_{nk}
Column mean	$y_{.1}$	$y_{.2}$	…	$y_{.j}$	…	$y_{.k}$
Effect	α_1	α_2	…	α_j	…	α_k

Error = Deviation From Column Mean

- e_{ij}: error of each measurement = deviation from column mean: $e_{ij} = y_{ij} - \overline{y_{.j}}$

	Alternatives					
Measure-ments	1	2	...	*j*	...	*k*
1	y_{11}	y_{12}	...	y_{1j}	...	y_{k1}
2	y_{21}	y_{22}	...	y_{2j}	...	y_{2k}
...	...	...	...	...	...	...
i	y_{i1}	y_{i2}	...	y_{ij}	...	y_{ik}
...	...	...	...	...	...	...
n	y_{n1}	y_{n2}	...	y_{nj}	...	y_{nk}
Column mean	$y_{.1}$	$y_{.2}$	...	$y_{.j}$	...	$y_{.k}$

Effects and Errors

- Effect is distance of column mean from overall mean
  - Horizontally across alternatives

- Error is distance of sample from column mean
  - Vertically within one alternative
  - Error across alternatives, too

- Note that neither Effect nor Error are absolute values, they can be positive of negative
- Individual measurements are then:

$$y_{ij} = \bar{y} + \alpha_j + e_{ij}$$

Sum of Squares of Differences

- SST = differences between each measurement and overall mean

$$SST = \sum_{j=1}^{k} \sum_{i=1}^{n} (y_{ij} - \bar{y})^2$$

- SSA = variation due to effects of alternatives

$$SSA = n \sum_{j=1}^{k} \alpha_j^2 = n \sum_{j=1}^{k} (\overline{y_{.j}} - \bar{y})^2$$

- SSE = variation due to errors in measurements

$$SSE = \sum_{j=1}^{k} \sum_{i=1}^{n} e_{ij}^2 = \sum_{j=1}^{k} \sum_{i=1}^{n} (y_{ij} - \overline{y_{.j}})^2$$

- $SSE = SST - SSA \Leftrightarrow SST = SSE + SSA$

ANOVA

Separates variation in measured values into:

1. variation due to effects of alternatives
   - SSA – variation across column averages
2. variation due to errors
   - SSE – variation within a single column

If differences among alternatives are due to real differences:

→ SSA statistically greater than SSE

Comparing SSE and SSA

- Simple approach
  - $\frac{SSA}{SST}$ = fraction of total variation explained by differences among alternatives
  - $\frac{SSE}{SST} = \frac{SST-SSA}{SST}$ = fraction of total variation due to experimental error
- But is it statistically significant?

Comparing SSE and SSA

- Is it statistically significant?

- variance = mean square values

  = total variation / degrees of freedom

$$\sigma_x^2 = \frac{SSx}{df(SSx)}$$

- df(SSx):
  - degrees of freedom
  - this is the number of independent terms in sum

Degrees of Freedom for Effects

$df(SSA) = k - 1$, since k alternatives

	Alternatives					
Measure-ments	1	2	…	*j*	…	*k*
1	y_{11}	y_{12}	…	y_{1j}	…	y_{k1}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2k}
…	…	…	…	…	…	…
i	y_{i1}	y_{i2}	…	y_{ij}	…	y_{ik}
…	…	…	…	…	…	…
n	y_{n1}	y_{n2}	…	y_{nj}	…	y_{nk}
Column mean	$y_{.1}$	$y_{.2}$	…	$y_{.j}$	…	$y_{.k}$
Effect	α_1	α_2	…	α_j	…	α_k

Degrees of Freedom for Errors

$df(SSE) = k \cdot (n - 1)$, since k alternatives, each with $(n - 1)$ degrees of freedom

	Alternatives					
Measure-ments	1	2	…	j	…	k
1	y_{11}	y_{12}	…	y_{1j}	…	y_{k1}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2k}
…	…	…	…	…	…	…
i	y_{i1}	y_{i2}	…	y_{ij}	…	y_{ik}
…	…	…	…	…	…	…
n	y_{n1}	y_{n2}	…	y_{nj}	…	y_{nk}
Column mean	$y_{.1}$	$y_{.2}$	…	$y_{.j}$	…	$y_{.k}$
Effect	α_1	α_2	…	α_j	…	α_k

Degrees of Freedom for Total

$df(SST) = df(SSA) + df(SSE) = k \cdot n - 1$, since we consider all kn alternatives as independent experiments, thus fixing only 1 pair

	Alternatives					
Measure-ments	1	2	…	*j*	…	*k*
1	y_{11}	y_{12}	…	y_{1j}	…	y_{k1}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2k}
…	…	…	…	…	…	…
i	y_{i1}	y_{i2}	…	y_{ij}	…	y_{ik}
…	…	…	…	…	…	…
n	y_{n1}	y_{n2}	…	y_{nj}	…	y_{nk}
Column mean	$y_{.1}$	$y_{.2}$	…	$y_{.j}$	…	$y_{.k}$
Effect	α_1	α_2	…	α_j	…	α_k

Note: $k(n-1) + k - 1 = kn - 1$

Variances from Sum of Squares (Mean Square Value)

Variation between sample means

$$\sigma_a^2 = \frac{SSA}{k-1}$$

in user studies, this is the variance of
the average scores for the different alternatives

Variation within the samples

$$\sigma_e^2 = \frac{SSE}{k(n-1)}$$

in user studies, this is the variance of
the score differences between participants

Comparing Variances

Use F-test to compare ratio of variances

- an F-test is used to test if the standard deviations of two populations are equal
  - $F = \frac{\sigma_a^2}{\sigma_e^2}$
  - $F_{[1-\alpha;df(num),df(denum)]} = F_{[1-\alpha,k-1,k(n-1)]} = tabulated\ critical\ values$
- table for $p = 0.001$ at:
  http://www.tutor-homework.com/statistics_tables/f-table-0.001.html

if $F_{computed} > F_{table}$ for a given α

→ we have $(1-\alpha)100\%$ **confidence** that
variation due to actual differences in alternatives, SSA,
is statistically greater than variation due to errors, SSE

Comparing Variances

Table of F-statistics P=0.001

t-statistics
F-statistics with other P-values: P=0.05 | P=0.01
Chi-square statistics

df(num)

df(denum)

df2\df1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	22	24	26	28	30
3	167.03	148.50	141.11	137.10	134.58	132.85	131.59	130.62	129.86	129.25	128.74	128.32	127.96	127.65	127.38	127.14	126.93	126.74	126.57	126.42	126.16	125.94	125.75	125.59	125.45
4	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.48	48.05	47.71	47.41	47.16	46.95	46.76	46.60	46.45	46.32	46.21	46.10	45.92	45.77	45.64	45.53	45.43
5	47.18	37.12	33.20	31.09	29.75	28.84	28.16	27.65	27.25	26.92	26.65	26.42	26.22	26.06	25.91	25.78	25.67	25.57	25.48	25.40	25.25	25.13	25.03	24.95	24.87
6	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69	18.41	18.18	17.99	17.83	17.68	17.56	17.45	17.35	17.27	17.19	17.12	17.00	16.90	16.81	16.74	16.67
7	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33	14.08	13.88	13.71	13.56	13.43	13.32	13.23	13.14	13.06	12.99	12.93	12.82	12.73	12.66	12.59	12.53
8	25.42	18.49	15.83	14.39	13.49	12.86	12.40	12.05	11.77	11.54	11.35	11.20	11.06	10.94	10.84	10.75	10.67	10.60	10.54	10.48	10.38	10.30	10.22	10.16	10.11
9	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11	9.89	9.72	9.57	9.44	9.33	9.24	9.15	9.08	9.01	8.95	8.90	8.80	8.72	8.66	8.60	8.55
10	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96	8.75	8.59	8.45	8.33	8.22	8.13	8.05	7.98	7.91	7.86	7.80	7.71	7.64	7.57	7.52	7.47
11	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.36	8.12	7.92	7.76	7.63	7.51	7.41	7.32	7.24	7.18	7.11	7.06	7.01	6.92	6.85	6.79	6.73	6.68
12	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48	7.29	7.14	7.01	6.89	6.79	6.71	6.63	6.57	6.51	6.45	6.41	6.32	6.25	6.19	6.14	6.09
13	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98	6.80	6.65	6.52	6.41	6.31	6.23	6.16	6.09	6.03	5.98	5.93	5.85	5.78	5.72	5.67	5.63
14	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58	6.40	6.26	6.13	6.02	5.93	5.85	5.78	5.71	5.66	5.60	5.56	5.48	5.41	5.35	5.30	5.25
15	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26	6.08	5.94	5.81	5.71	5.62	5.54	5.46	5.40	5.35	5.29	5.25	5.17	5.10	5.04	4.99	4.95
16	16.12	10.97	9.01	7.94	7.27	6.81	6.46	6.20	5.98	5.81	5.67	5.55	5.44	5.35	5.27	5.21	5.14	5.09	5.04	4.99	4.91	4.85	4.79	4.74	4.70
17	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75	5.58	5.44	5.32	5.22	5.13	5.05	4.99	4.92	4.87	4.82	4.78	4.70	4.63	4.58	4.53	4.48
18	15.38	10.39	8.49	7.46	6.81	6.36	6.02	5.76	5.56	5.39	5.25	5.13	5.03	4.94	4.87	4.80	4.74	4.68	4.63	4.59	4.51	4.45	4.39	4.34	4.30
19	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39	5.22	5.08	4.97	4.87	4.78	4.70	4.64	4.58	4.52	4.47	4.43	4.35	4.29	4.23	4.19	4.14
20	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	5.08	4.94	4.82	4.72	4.64	4.56	4.50	4.44	4.38	4.33	4.29	4.21	4.15	4.09	4.05	4.01
22	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	4.83	4.70	4.58	4.49	4.40	4.33	4.26	4.20	4.15	4.10	4.06	3.98	3.92	3.86	3.82	3.78
24	14.03	9.34	7.55	6.59	5.98	5.55	5.24	4.99	4.80	4.64	4.51	4.39	4.30	4.21	4.14	4.07	4.02	3.96	3.92	3.87	3.80	3.74	3.68	3.63	3.59
26	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	4.48	4.35	4.24	4.14	4.06	3.99	3.92	3.86	3.81	3.77	3.72	3.65	3.59	3.53	3.49	3.45
28	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.70	4.51	4.35	4.22	4.11	4.01	3.93	3.86	3.80	3.74	3.69	3.64	3.60	3.52	3.46	3.41	3.36	3.32
30	13.29	8.77	7.05	6.13	5.53	5.12	4.82	4.58	4.39	4.24	4.11	4.00	3.91	3.83	3.75	3.69	3.63	3.58	3.54	3.49	3.42	3.36	3.30	3.26	3.22

Comparing Variances: α

- Probability that x is in the interval $[c_1, c_2]$
  - formally written:
    $$P(c_1 \leq x \leq c_2) = 1 - \alpha$$
  - (c1, c2) *confidence interval*
  - α *significance level*
  - 100(1- α) *confidence level*

typical confidence levels are 90%, 95%, 99%

significance level $\alpha = 0.1 \equiv$ confidence level 90%

significance level $\alpha = 0.05 \equiv$ confidence level 95%

significance level $\alpha = 0.01 \equiv$ confidence level 99%

One-way repeated measures ANOVA Summary

Variation	Alternatives	Error	Total
Sum of squares	SSA	SSE	SST
Deg freedom	k-1	k(n-1)	kn-1
Mean square	$\sigma_a^2 = \frac{SSA}{k-1}$	$\sigma_e^2 = \frac{SSE}{k(n-1)}$	
Computed F	$\frac{\sigma_a^2}{\sigma_e^2}$		
Tabulated F	$F_{[1-\alpha;k-1,k(n-1)]}$		

ANOVA Example

	Alternatives			
Measurements	1	2	3	Overall mean
1	0.0972	0.1382	0.7966	
2	0.0971	0.1432	0.5300	
3	0.0969	0.1382	0.5152	
4	0.1954	0.1730	0.6675	
5	0.0974	0.1383	0.5298	
Column mean				
Effects				

ANOVA Example

	Alternatives			
Measurements	1	2	3	Overall mean
1	0.0972	0.1382	0.7966	
2	0.0971	0.1432	0.5300	
3	0.0969	0.1382	0.5152	
4	0.1954	0.1730	0.6675	
5	0.0974	0.1383	0.5298	
Column mean	$\overline{y_{.1}} = 0.1168$	$\overline{y_{.2}} = 0.1462$	$\overline{y_{.3}} = 0.6078$	$\bar{y} = 0.2903$
Effects				
Column sum	0.5840	0.7309	3.0391	

ANOVA Example

	Alternatives			
Measurements	1	2	3	Overall mean
1	0.0972	0.1382	0.7966	
2	0.0971	0.1432	0.5300	
3	0.0969	0.1382	0.5152	
4	0.1954	0.1730	0.6675	
5	0.0974	0.1383	0.5298	
Column mean	$\overline{y_{\cdot 1}} = 0.1168$	$\overline{y_{\cdot 2}} = 0.1462$	$\overline{y_{\cdot 3}} = 0.6078$	$\bar{y} = 0.2903$
Effects $\alpha_j = \overline{y_{\cdot j}} - \bar{y}$				

ANOVA Example

	Alternatives			
Measurements	1	2	3	Overall mean
1	0.0972	0.1382	0.7966	
2	0.0971	0.1432	0.5300	
3	0.0969	0.1382	0.5152	
4	0.1954	0.1730	0.6675	
5	0.0974	0.1383	0.5298	
Column mean	$\overline{y_{\cdot 1}} = 0.1168$	$\overline{y_{\cdot 2}} = 0.1462$	$\overline{y_{\cdot 3}} = 0.6078$	$\bar{y} = 0.2903$
Effects	$\overline{\alpha_1} = -0.1735$	$\overline{\alpha_2} = -0.1441$	$\overline{\alpha_3} = 0.3175$	
SSA	$5 \cdot \sum_{j=1}^{3} \alpha_j^2 =$			0.7585

ANOVA Example

	Errors			
Measurements	$e_{i1} = y_{i1} - \overline{y_{\cdot 1}}$	$e_{i2} = y_{i2} - \overline{y_{\cdot 2}}$	$e_{i3} = y_{i3} - \overline{y_{\cdot 3}}$	SSE
1	-0.0196	-0.0080	0.1888	$\sum_{j=1}^{3}\sum_{i=1}^{5} e_{ij}^2$
2	-0.0197	-0.0030	-0.0778	
3	-0.0199	-0.0080	-0.0926	
4	0.0786	0.0268	0.0597	
5	-0.0194	-0.0079	-0.0780	
Column mean	$\overline{y_{\cdot 1}} = 0.1168$	$\overline{y_{\cdot 2}} = 0.1462$	$\overline{y_{\cdot 3}} = 0.6078$	0.0685
SST=SSA+SSE	0.8270			

ANOVA Example

Variation	Alternatives	Error	Total
Sum of squares	$SSA = 0.7585$	$SSE = 0.0685$	$SST = 0.8270$
Deg freedom	$k - 1 = 2$	$k(N-1) = 12$	$kn - 1 = 14$
Mean square	$\sigma_a^2 = 0.3793$	$\sigma_3^2 = 0.0057$	
Computed F	$\frac{0.3793}{0.0057} = 66.4$		
Tabulated F	$F_{[0.95;2,12]} = 3.89$ $F_{[0.99;2,12]} = 6.93$ $F_{[0.999;2,12]} = 12.97$		

SSA/SST = 0.7585/0.8270 = 0.917

→ 91.7% of total variation in measurements is due to differences among alternatives

SSE/SST = 0.0685/0.8270 = 0.083

→ 8.3% of total variation in measurements is due to noise in measurements

ANOVA Example

Variation	Alternatives	Error	Total
Sum of squares	$SSA = 0.7585$	$SSE = 0.0685$	$SST = 0.8270$
Deg freedom	$k - 1 = 2$	$k(N - 1) = 12$	$kn - 1 = 14$
Mean square	$\sigma_a^2 = 0.3793$	$\sigma_3^2 = 0.0057$	
Computed F	$\frac{0.3793}{0.0057} = 66.4$		
Tabulated F	$F_{[0.95;2,12]} = 3.89$ $F_{[0.99;2,12]} = 6.93$ $F_{[0.999;2,12]} = 12.97$		

Computed *F* statistic > tabulated *F* statistic

→ 99.9% confidence that differences among alternatives are statistically significant.

One-way repeated measures ANOVA Summary

- Useful for partitioning total variation into components
  - Experimental error
    - Including differences between human participants – which can be due to actual differences but also due to lack of attention
  - Variation among alternatives
    - Including alternatives comprising a numeric score, a rank or a count
- Compare more than two alternatives
- Note, does not tell you *where* differences may lie
  - Use PostHoc tests

When to use One-way ANOVA

PostHoc testing:

- **If ANOVA indicates that not all population means are equal**
- **Which ones are different?**

Measure-ments	1	2	…	*j*	…	*k*
1	y_{11}	y_{12}	…	y_{1j}	…	y_{k1}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2k}
…	…	…	…	…	…	…

A typical PostHoc option: Tukey's HSD (Honestly Significant Difference)

- Absolute differences of 2 columns' averages: $|\overline{y_{\cdot A}} - \overline{y_{\cdot B}}|$
- Standard error MSE for all columns: average of column variances
- Compute $q_{A,B} = \frac{|\overline{y_{\cdot A}} - \overline{y_{\cdot B}}|}{\sqrt{\frac{MSE}{n}}}$
- Check if $q_{A,B}$ is above the studentized range distribution for a percentage probability
  - https://real-statistics.com/statistics-tables/studentized-range-q-table/