



Chapter 6

Symbolic execution

Course “Model checking”
Volker Stolz, Martin Steffen
Autumn 2019



Section

Targets

Chapter 6 “Symbolic execution”

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Chapter 6

Learning Targets of Chapter “Symbolic execution”.

The chapter gives an not too deep introduction to *symbolic* execution and *concolic* execution.



Chapter 6

Outline of Chapter “Symbolic execution”.

Targets

Introduction

- Testing and path coverage
- Symbolic execution
- Concolic testing



Section

Introduction

Testing and path coverage

Symbolic execution

Concolic testing

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Introduction

- **symbolic** execution: “old” technique [3]
- natural also in the context of **testing**
- **concolic** execution: extension
- used also in compiler
 - code generation
 - optimization



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Testing and path coverage

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Code example

```
1 f(int x, int y) {  
2   if (x*x*x > 0) {  
3     if (x > 0 && y == 10) {  
4       fail();  
5     }  
6   } else {  
7     if (x > 0 && y == 20) {  
8       fail();  
9     }  
10  }  
11  
12  complete();  
13 }
```



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- Testing and path coverage
- Symbolic execution
- Concolic testing

How to analyse a (simple) program like that? :

- testing
- “verification” (whatever that means)
 - could include code review
- model-checking? Hm?
- symbolic and concolic execution (see later)



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Testing

- maybe **the** most used method for ensuring software (and system) “quality”
- broad field
 - many different testing goals, techniques
 - also used in combination, in different phases of software engineering cycle
- here: focus on

“white-box” testing

- AKA structural testing
- program code available (resp. CFG)
- also focus: *unit* testing

Goals

- detect errors
- check corner cases
- provide high (code) **coverage**



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(Code) coverage

- note: typically a non-concurrent setting (unit testing)
- different coverage criteria
 - nodes
 - edges, conditions
 - combinations thereof
 - path coverage
- defined to answer the question

When have I tested “enough”?

path coverage

- ambitious to impossible (loops)
- note: still not *all reachable states*, i.e., not verified yet



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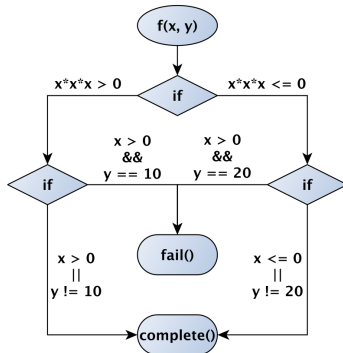
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Path coverage



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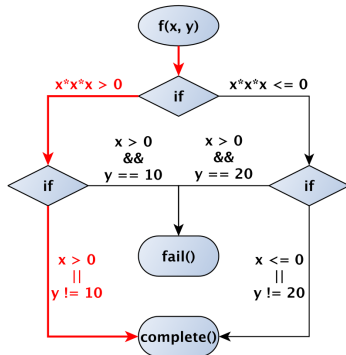
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Path coverage



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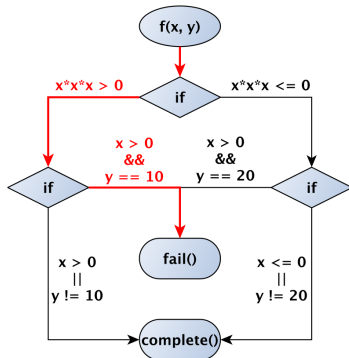
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Path coverage



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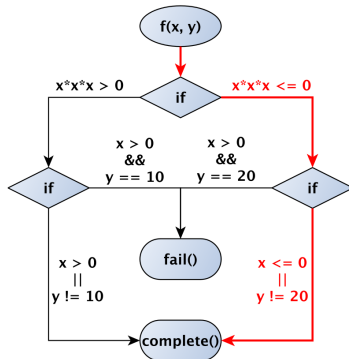
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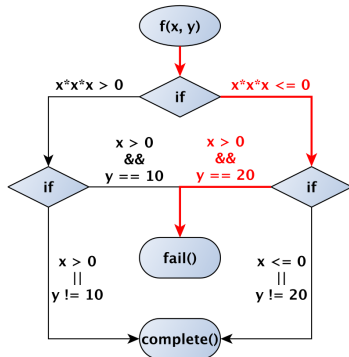
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Path coverage



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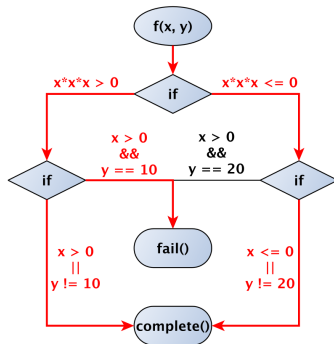
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Path coverage



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- 3 possible exec. path
- corresponding **path conditions**
- “optimal”: cover all path
- **find input set** to run program covering all those paths

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Random testing

- most naive way of testing
- generating random inputs
- **concrete** input values
- **dynamic** executions of programs
- *observe actual* behavior and
- compare it against *expected behavior*



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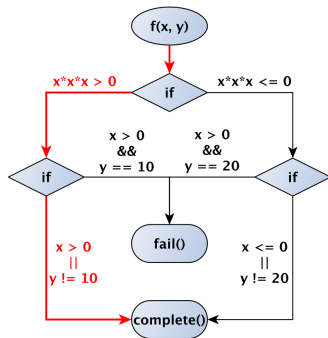
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Random testing

- different inputs, different paths
- maybe
 - $(x, y) = (700, 500)$
 - $(x, y) = (-700, 500)$
 - ...



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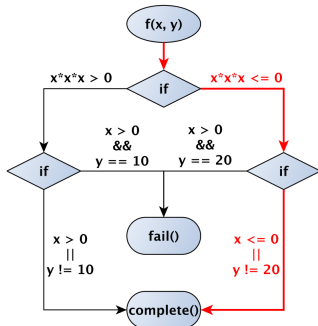
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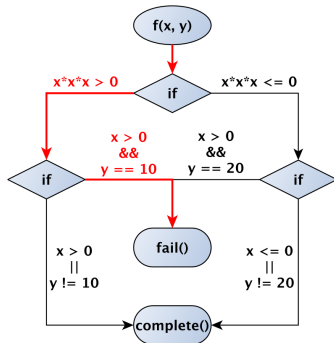
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One path so far missed



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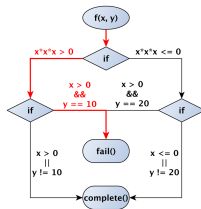
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How to get that path (or others)?



- maybe: $(x, y) = (145, 10)$
- by chance: **very** low probability to randomly get $y = 10$

- **path condition**

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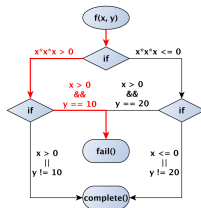
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How to get that path (or others)?



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- maybe: $(x, y) = (145, 10)$
- by chance: **very** low probability to randomly get $y = 10$

Symbolic representation

$$x > 0 \wedge y = 10$$

- **path condition**

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Symbolic execution

- **symbols** instead of concrete value
- use if **path conditions**, aka **path constraints**
- cf. connection to SAT and SMT
- constraint solver computes real values



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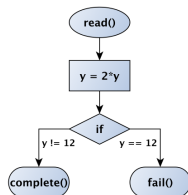
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Simple example

```
1 y = read();
2 y = 2 * y;
3
4 if (y == 12) {
5     fail();
6 }
7
8 complete();
```



- in the code: *assignments* not equations ($y := \text{read}()$)
- introduce variable s for $\text{read}()$
- assignments
 - $y := \text{read}() \Rightarrow y = s$
 - $y := 2 * y \Rightarrow y = 2s$
- branching point in line 4
 - right: $2s = 12$
 - left: $2s \neq 12$



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Which input leads to the error?

```
1 y = read();  
2 y = 2 * y;  
3  
4 if (y == 12) {  
5     fail();  
6 }  
7  
8 complete();
```



Constraint solver

Solve the path constraint $2s = 12$

- child's play: the solution is $s = 6$
- but: requires solver that can do “arithmetic”, including multiplication



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Symbolic execution for dummies

- take the code (resp. the CFG of the code)
 - collect all paths into **path conditions**
 - big conjunctions of all conditions along each the path
 - each condition b will have
 - one positive mention b in one continuation of the path
 - one negated mention $\neg b$ in the other continuation
 - solve the constraints for paths leading to errors with an appropriate SMT solver
-
- works best for loop-free program
 - cf. also SSA
 - but there is another problem as well (see next)

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How about the program we started with?



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Complex condition x^3



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```

- non-linear constraint
- in general **undecidable**
- most constraint solvers throw the towel
- for instance: execution stops, no path covered

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What can one do?

what can one do (beyond accepting the SE won't cover all path)?

- “static analysis”: abstracting
 - cover both path approximately
- theorem proving? one cannot sell that to testers

Concolic testing

Concrete & Symbolic = “concolic”



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Concolic testing

- here following *DART*
- combination of two techniques

Random testing

- concrete values
- dynamic execution

Symbolic execution

- symbols, variables
- static analysis

- other name: **Dynamic symbolic execution (DSE)**



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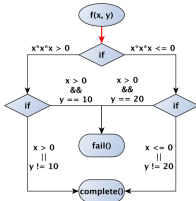
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Dart (1)



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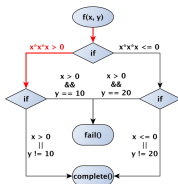
Concolic testing

Dynamic execution

- random input: as in random testing
- concrete
(x, y) = (700, 500)

Symbolic execution

Dart (1)



Dynamic execution

- random input: as in random testing
- concrete
 $(x, y) = (700, 500)$
- $x * x * x > 0$

Symbolic execution

- introduce symbols
 $x_1 = x, y_1 = y$
- constraint $x^3 \leq 0$



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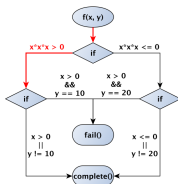
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Dart (1)



Dynamic execution

- random input: as in random testing
- concrete
 $(x, y) = (700, 500)$
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Symbolic execution

- introduce symbols
 $x_1 = x, y_1 = y$
- constraint $x^3 \leq 0$
- non-linear: **fail**



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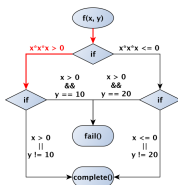
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Dart (1)



Dynamic execution

- random input: as in random testing
- concrete
 $(x, y) = 700, 500)$
- $x * x * x > 0$

Symbolic execution

- introduce symbols
 $x_1 = x, y_1 = y$
- constraint $x^3 \leq 0$
- non-linear: **fail**
- **concrete fall-back:**
 $x_1 = 700$

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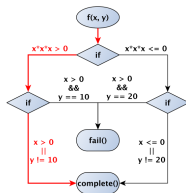
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Dynamic execution

- random input: as in random testing
- concrete
(x, y) = 700, 500)
- $x * x * x > 0$
- $y \neq 10$

Symbolic execution

- introduce symbols
 $x_1 = x, y_1 = y$
- constraint $x^3 \leq 0$
- non-linear: **fail**
- constraint $y_1 = 10$
- solve the constraint:

Dart 2



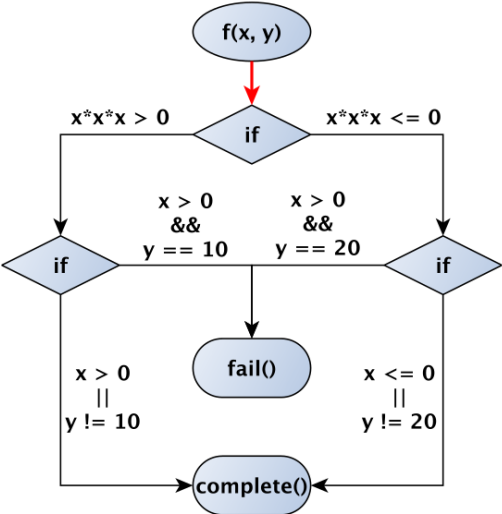
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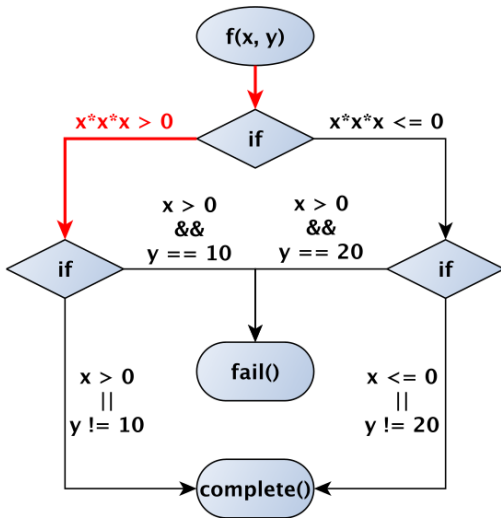
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Dart 2



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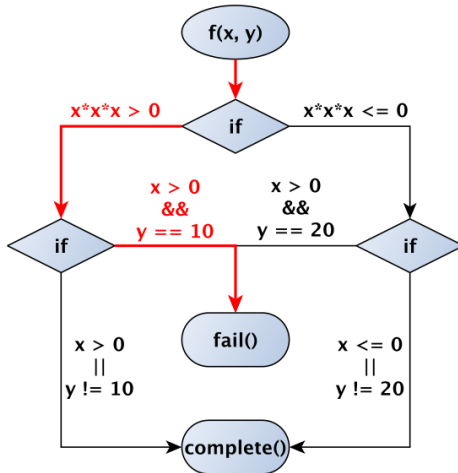
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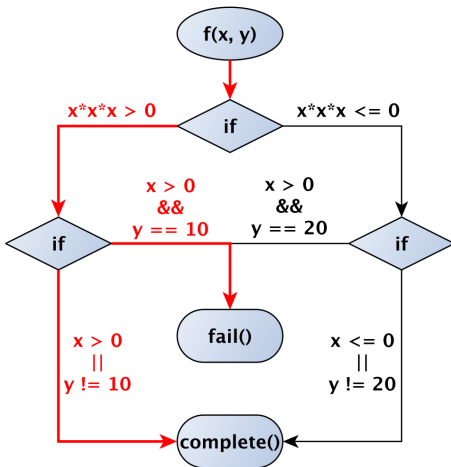
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Dart 3



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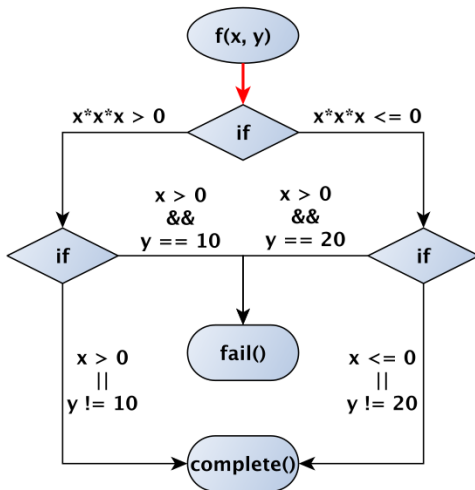
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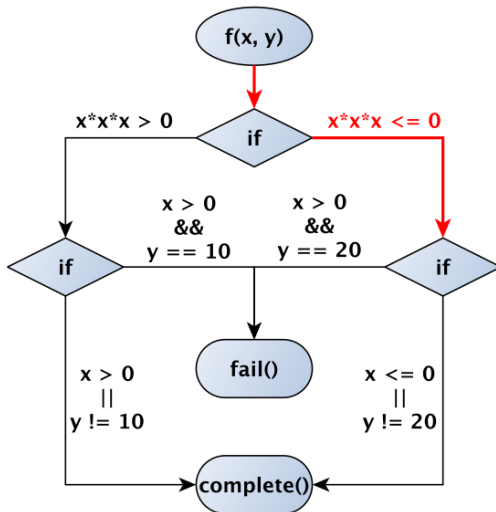
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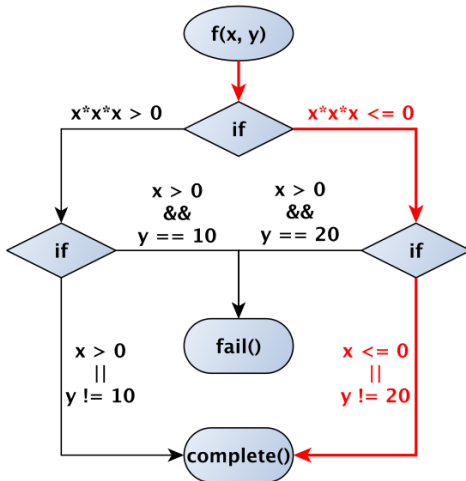
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Dart $n + 1$



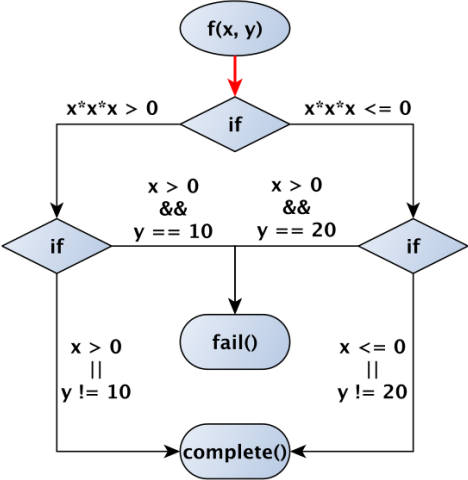
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Dart $n + 1$



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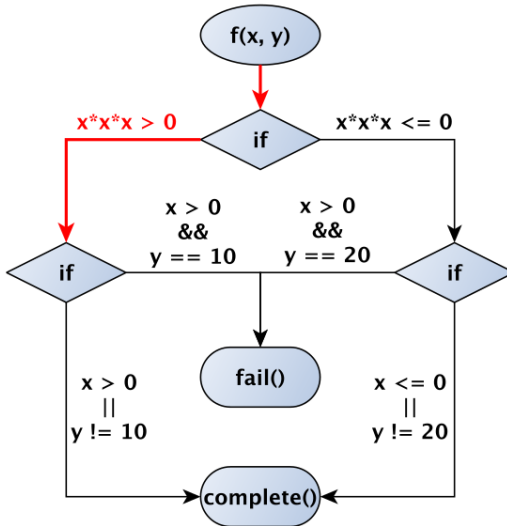
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Dart $n + 1$



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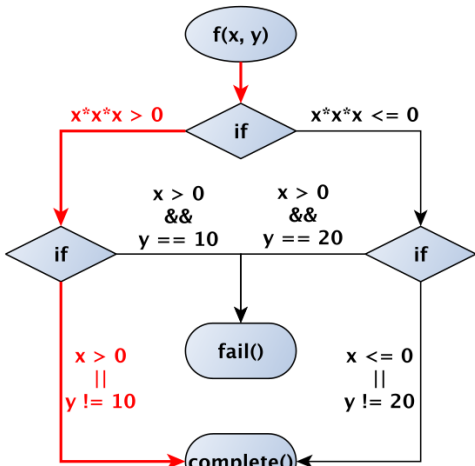
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Dart completed



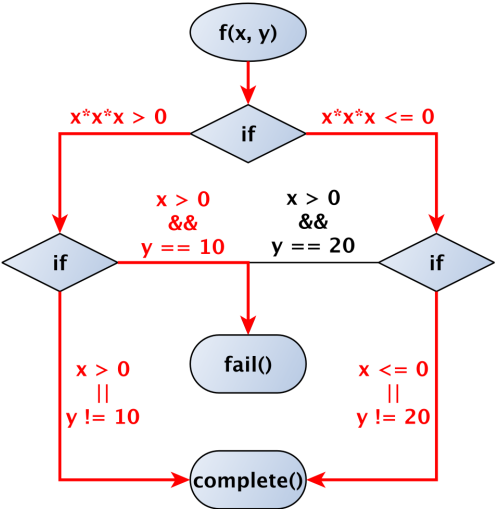
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References I



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Bibliography

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