Chapter 7

Symbolic execution

Course “Model checking”

Martin Steffen

Autumn 2021

Chapter 7

Learning Targets of Chapter "Symbolic execution".

The chapter gives an not too deep introduction to *symbolic* execution and *concolic* execution.

Chapter 7

Outline of Chapter “Symbolic execution”.

Introduction

Symbolic execution

Concolic testing

Section

Introduction

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Introduction

- symbolic execution: “old” technique [3] (from 1976)
- natural also in the context of testing
- concolic execution: extension
- used also in compilers:
  - code generation
  - optimization
  - ...

Code example

```
1  f(int x, int y){
2    if (x*x*x* > 0) {
3      if (x > 0 && y == 10) {
4        fail();
5      }
6    } else {
7      if (x > 0 && y == 20) {
8        fail ();
9      }
10   }
11
12   complete();
13 }
```

How to analyse a program like that?

- testing
- "verification" (whatever that means)
  - could include code review
- model-checking? Hm?
- symbolic and concolic execution (see later)

Testing

- maybe the most used method for ensuring software (and system) "quality"
- broad field
  - many different testing goals, techniques
  - also used in combination, in different phases of software engineering cycle
- here: focus on

"white-box" testing

- AKA structural testing
- program code available (resp. CFG)

Goals

- detect errors
- check corner cases
- provide high ("code") coverage

(Code) coverage

- note: typically a non-concurrent setting (unit testing)
- different coverage criteria
  - nodes
  - edges, conditions
  - combinations thereof
  - path coverage
- defined to answer the question

When have I tested "enough"?

path coverage

- ambitious to impossible (loops)
- note: still not *all reachable states*, i.e., not verified yet

Path coverage

```
1  f(int x, int y) {
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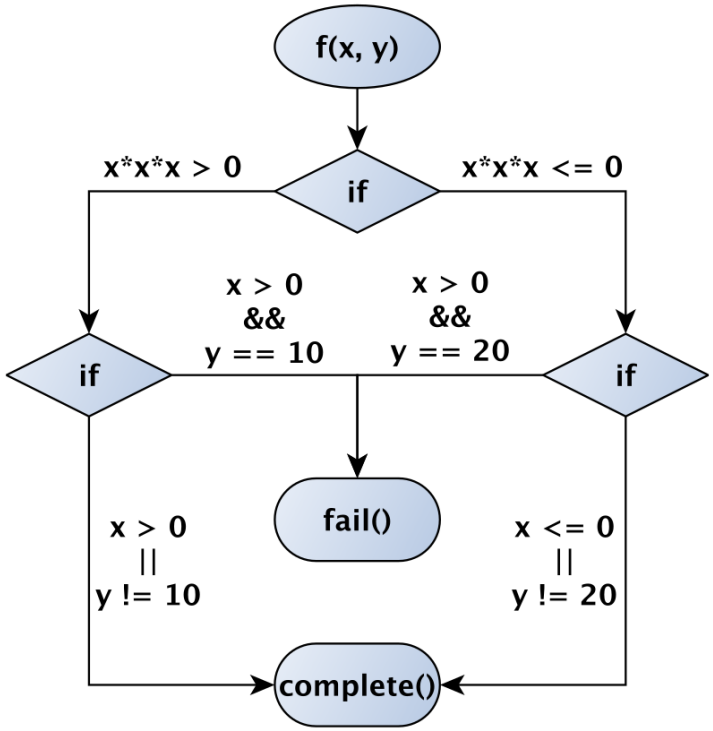

Path coverage

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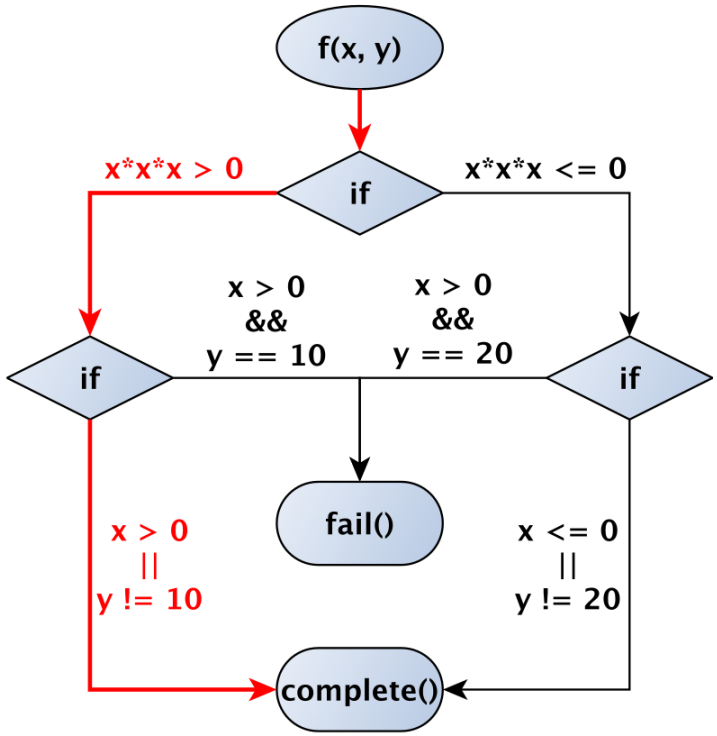

Path coverage

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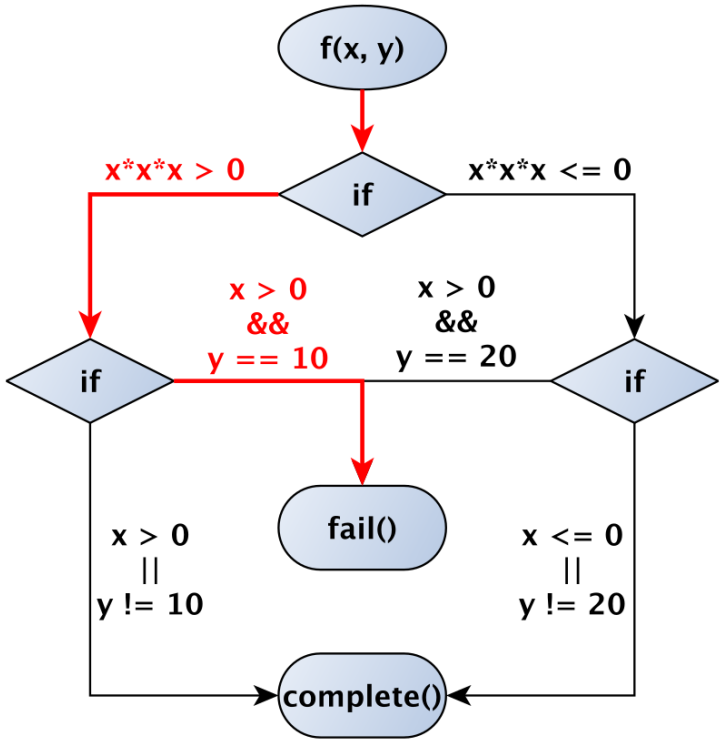

Path coverage

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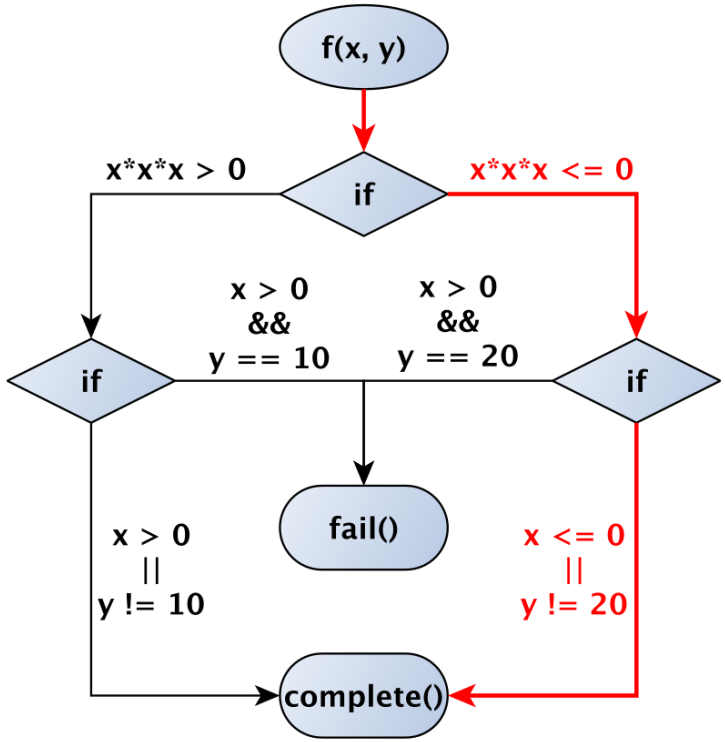

Path coverage

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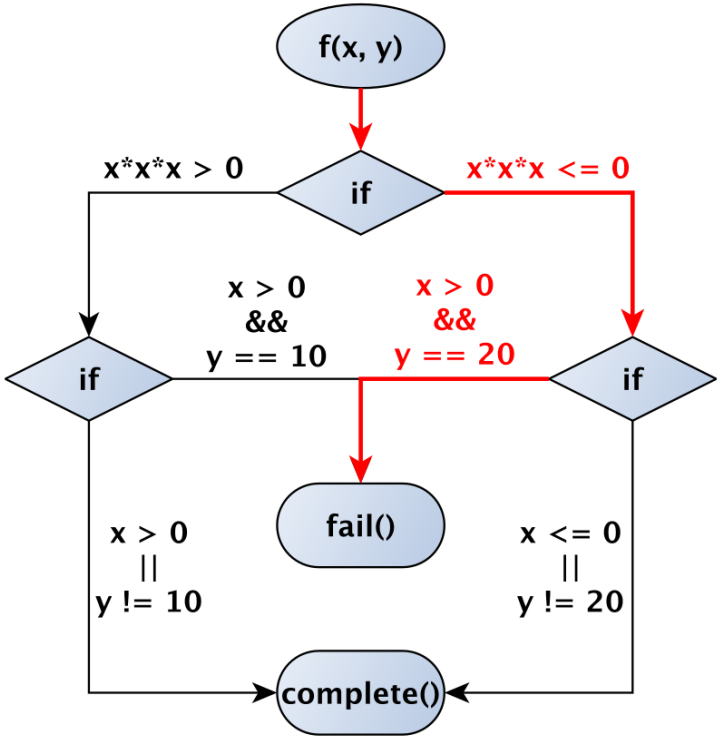
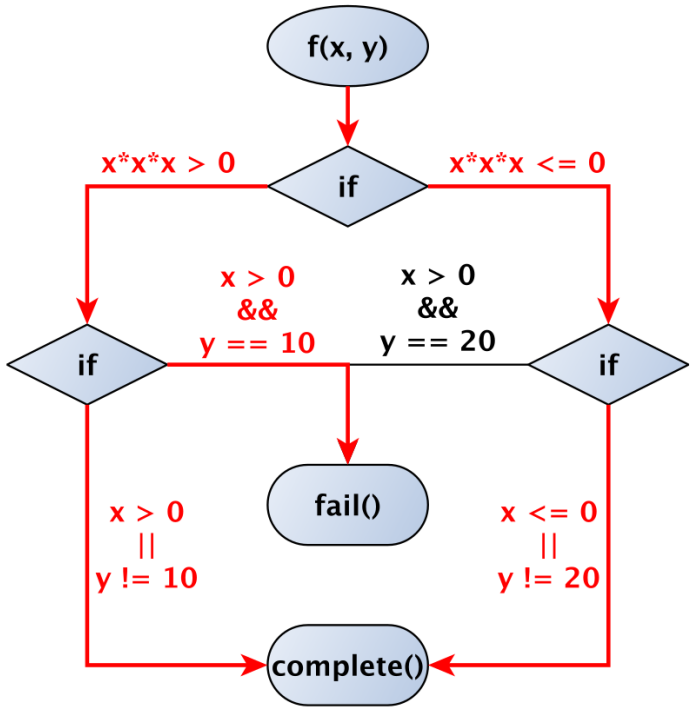


Path coverage



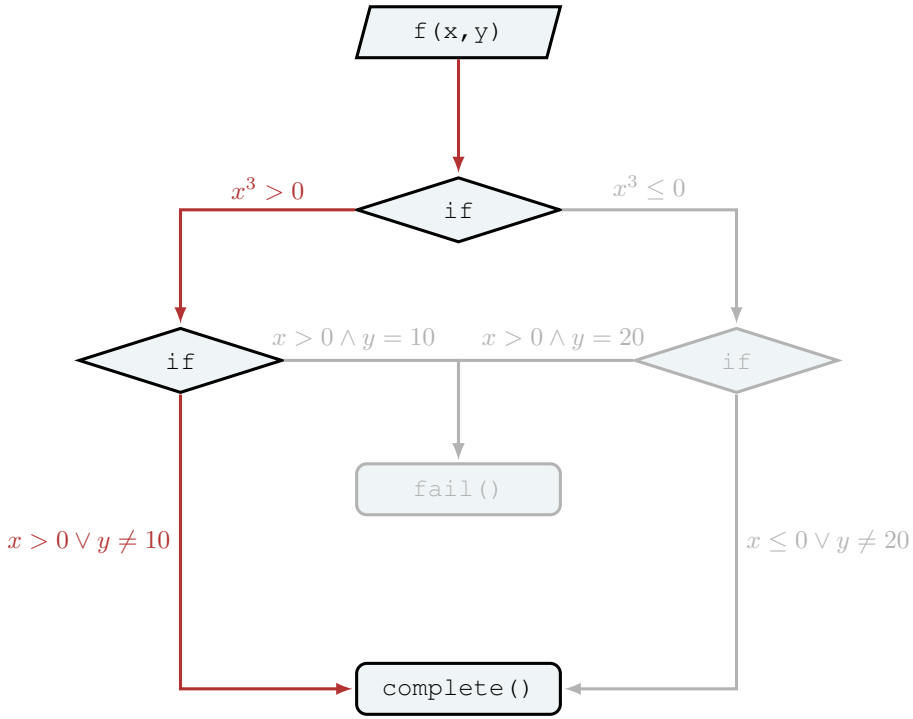
- 3 possible exec. path
- corresponding path conditions
- "optimal": cover all path
- find input set to run program covering all those paths

Random testing

- perhaps most naive way of testing
- generating random inputs
- concrete input values
- dynamic executions of programs
- *observe actual* behavior and
- compare it against *expected behavior*

Random testing

- different inputs, different paths
- maybe
  - $(x, y) = (700, 500)$
  - $(x, y) = (-700, 500)$
  - ...



Random testing

- different inputs, different paths
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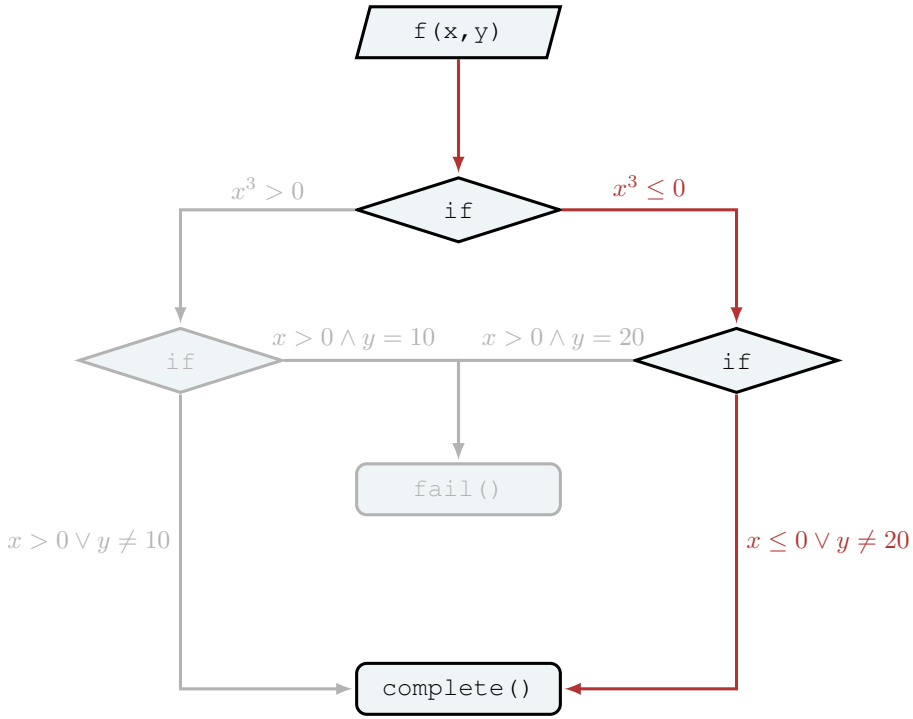
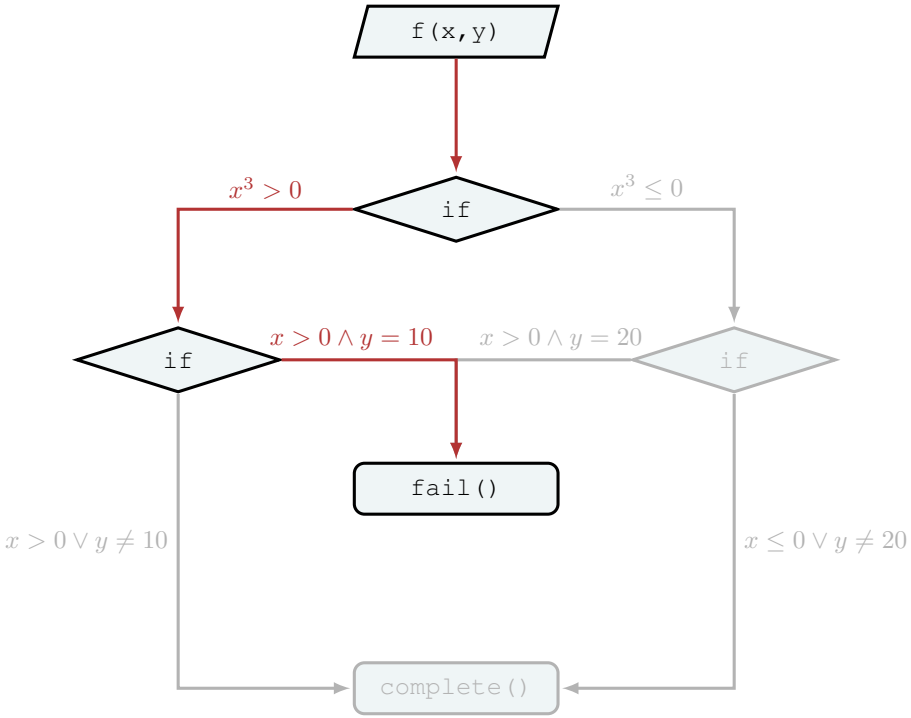
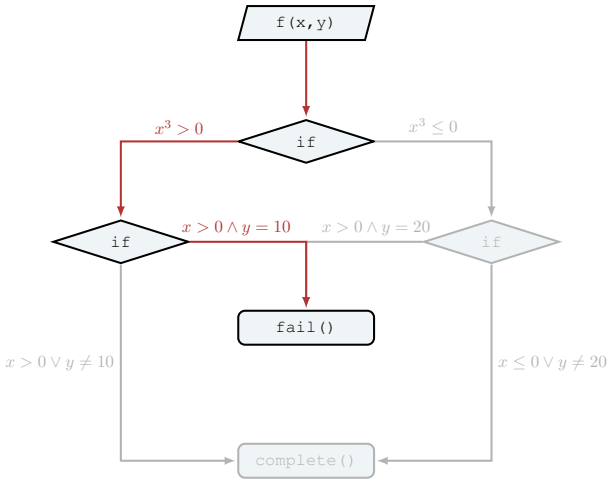
  - $(x, y) = (700, 500)$
  - $(x, y) = (-700, 500)$
  - ...



One path so far missed



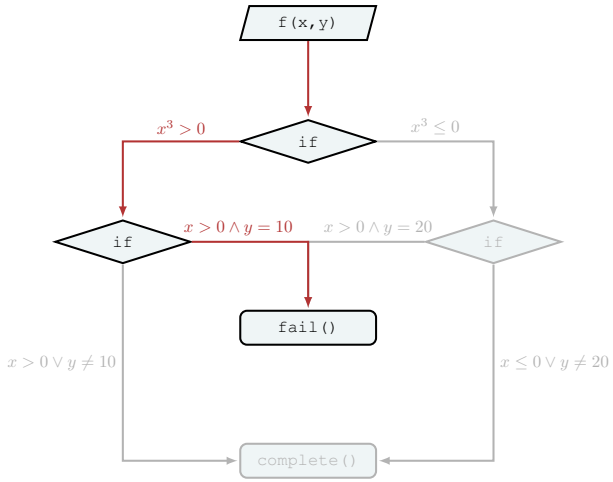
How to get that path (or others)?



- maybe: $(x, y) = (145, 10)$

- path condition

How to get that path (or others)?



- maybe: $(x, y) = (145, 10)$
- by chance: very low probability to randomly get $y = 10$

Symbolic representation

$$x > 0 \wedge y = 10$$

- path condition

Section

Symbolic execution

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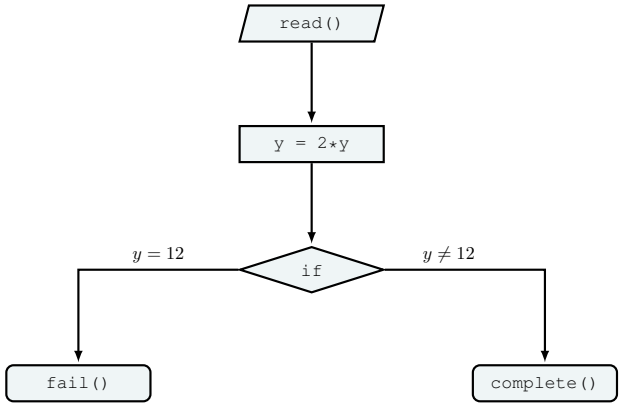
Symbolic execution

- symbols instead of concrete values
- use of path conditions, aka path constraints
- cf. connection to SAT and SMT
- constraint solver computes real values

Simple example

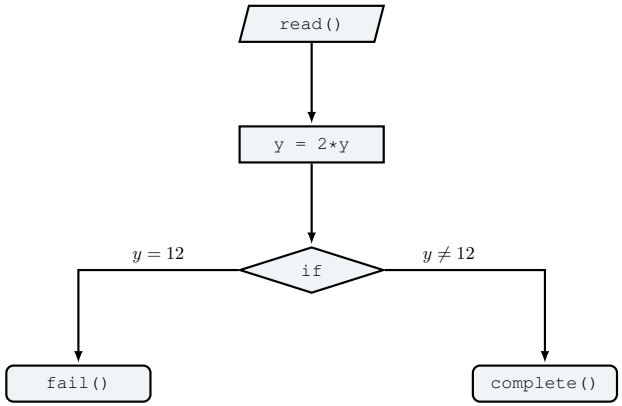
```
1  y = read();
2  y = 2 * y;
3
4  if (y==12) {
5     fail();
6   }
7
8  complete();
```



- in the code: *assignments* not equations (`y := read()`)
- introduce variable `s` for `read()`
- assignments
  - `y := read()` $\Rightarrow y = s$
  - `y := 2*y` $\Rightarrow y = 2s$
- branching point in line 4
  - right: $2s = 12$
  - left: $2s \neq 12$

Which input leads to the error?

```
1  y = read();
2  y = 2 * y;
3
4  if (y==12) {
5    fail();
6   }
7
8  complete();
```



Constraint solver

Solve the path constraint $2s = 12$

- child's play: the solution is $s = 6$
- requires solver that can do "arithmetic", including multiplication

In summary

Symbolic execution for dummies

- take the code (resp. the CFG of the code)
- collect all paths into path conditions
  - big conjunctions of all conditions along each the path
  - each condition b will have
    - one positive mention b in one continuation of the path
    - one negated mention $\neg b$ in the other continuation
- solve the constraints for paths leading to errors with an approriate SMT solver

- works best for loop-free programs
- cf. also SSA
- but there is another problem as well (see next)

How about the program we started with?

```
1  f(int x, int y){
2    if (x*x*x* > 0) {
3      if (x > 0 && y == 10) {
4        fail();
5      }
6    } else {
7      if (x > 0 && y == 20) {
8        fail ();
9      }
10   }
11
12   complete();
13 }
```

Complex condition x^3

```
1  f(int x, int y) {
2    if (x*x*x > 0) {
3      if (x > 0 && y == 10) {
4        fail();
5      }
6    } else {
7      if (x > 0 && y == 20) {
8        fail();
9      }
10   }
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12   complete();
13 }
```

- non-linear constraint
- in general undecidable
- most constraint solvers throw the towel
- for instance: execution stops, no path covered

What can one do?

What can one do (beyond throwing the towel and accept that SE won't cover all paths)?

- "static analysis": abstracting
  - cover both path approximately
- theorem proving? one cannot sell that to testers

Concolic testing

Concrete & Symbolic = "concolic"

Section

Concolic testing

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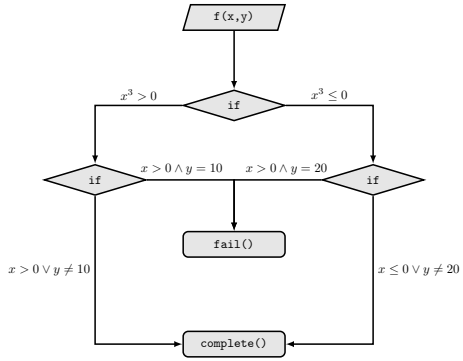
Concolic testing

- here following *DART*
- combination of two techniques

Random testing	Symbolic execution
• concrete values	• symbols, variables
• dynamic execution	• static analysis

- other name: Dynamic symbolic execution (DSE)

Dart (1)

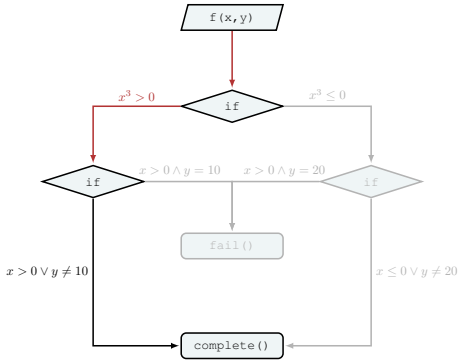


Dynamic execution

- random input: as in random testing
- concrete $(x, y) = 700, 500)$

Symbolic execution

Dart (1)

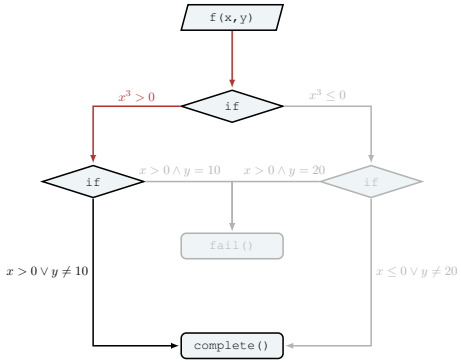


Dynamic execution

- random input: as in random testing
- concrete $(x, y) = 700, 500)$
- `x * x * x > 0`

Symbolic execution

Dart (1)



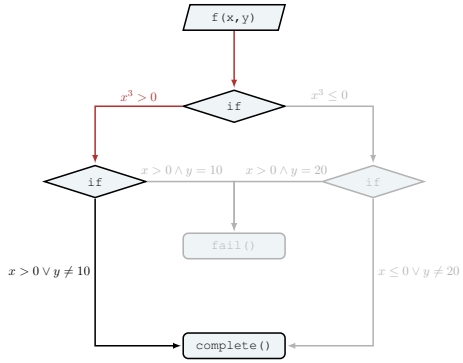
Dynamic execution

- random input: as in random testing
- concrete $(x, y) = 700, 500)$
- `x * x * x > 0`

Symbolic execution

- introduce symbols $x_1 = x, y_1 = y$
- constrain $x^3 \leq 0$

Dart (1)



Dynamic execution

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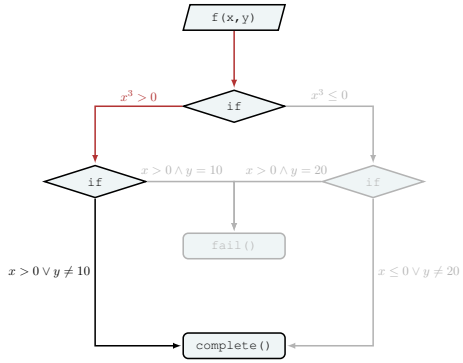
Symbolic execution

- introduce symbols $x_1 = x, y_1 = y$
- constrain $x^3 \leq 0$
- non-linear: fail

Dart (1)



Dynamic execution

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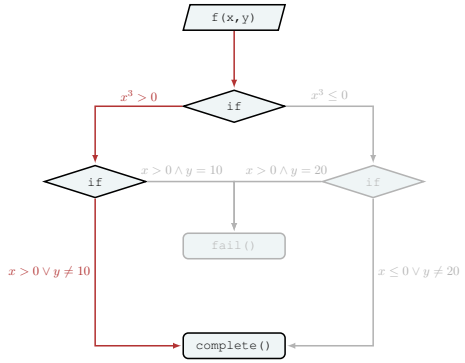
Symbolic execution

- introduce symbols $x_1 = x, y_1 = y$
- constrain $x^3 \leq 0$
- non-linear: fail
- concrete fall-back: $x_1 = 700$

Dart (1)



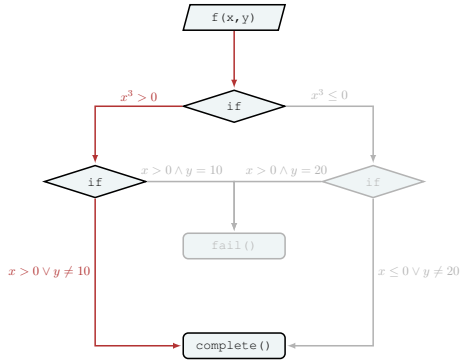
Dynamic execution

- random input: as in random testing
- concrete $(x, y) = 700, 500)$
- `x * x * x > 0`
- `y !=10`

Symbolic execution

- introduce symbols $x_1 = x, y_1 = y$
- constrain $x^3 \leq 0$
- non-linear: fail
- concrete fall-back: $x_1 = 700$

Dart (1)



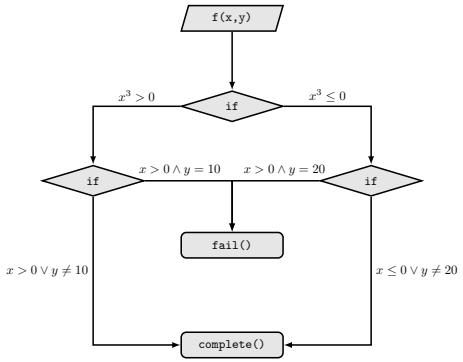
Dynamic execution

- random input: as in random testing
- concrete $(x, y) = 700, 500)$
- `x * x * x > 0`
- `y !=10`

Symbolic execution

- introduce symbols $x_1 = x, y_1 = y$
- constrain $x^3 \leq 0$
- non-linear: fail
- concrete fall-back: $x_1 = 700$
- constrain $y_1 = 10$
- solve the constraint: $(x_1, x_2) = (700, 10)$

Dart (2)

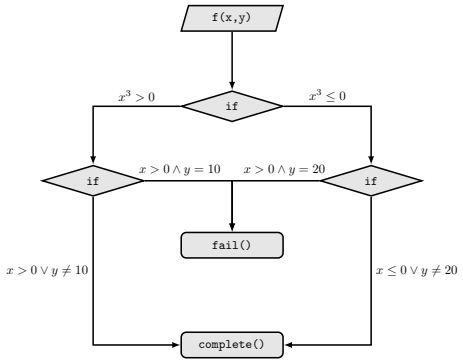


Dynamic execution

- given input $700, 10$

Symbolic execution

Dart (2)



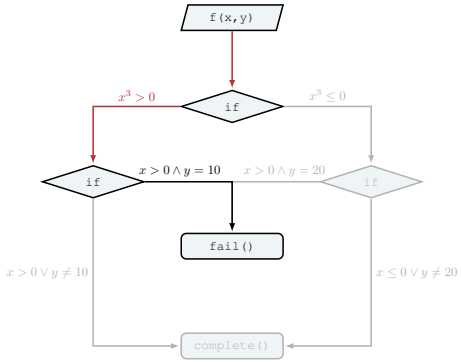
Dynamic execution

- given input $700, 10$

Symbolic execution

- introduce symbols $x_2 = x, y_2 = y$

Dart (2)



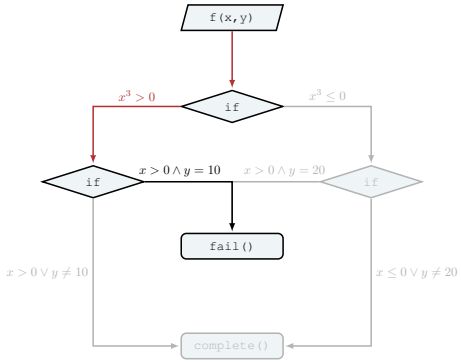
Dynamic execution

- given input $700, 10$
- `x * x * x > 0`

Symbolic execution

- introduce symbols $x_2 = x, y_2 = y$

Dart (2)



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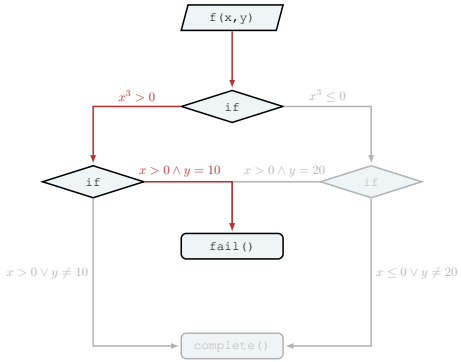
Dart (2)



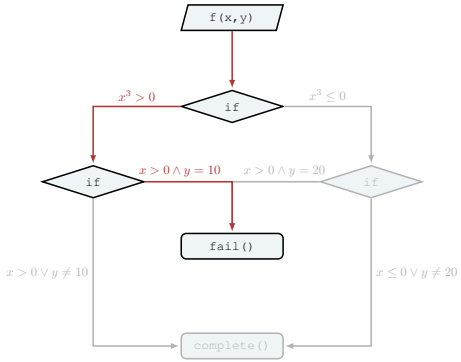
Dynamic execution

- given input $700, 10$
- `x * x * x > 0`
- `x > 0 && y==10`

Symbolic execution

- introduce symbols $x_2 = x, y_2 = y$
- constrain $x^3 \leq 0$
- non-linear: fail
- concrete fall-back: $x_2 = 700$

Dart (2)



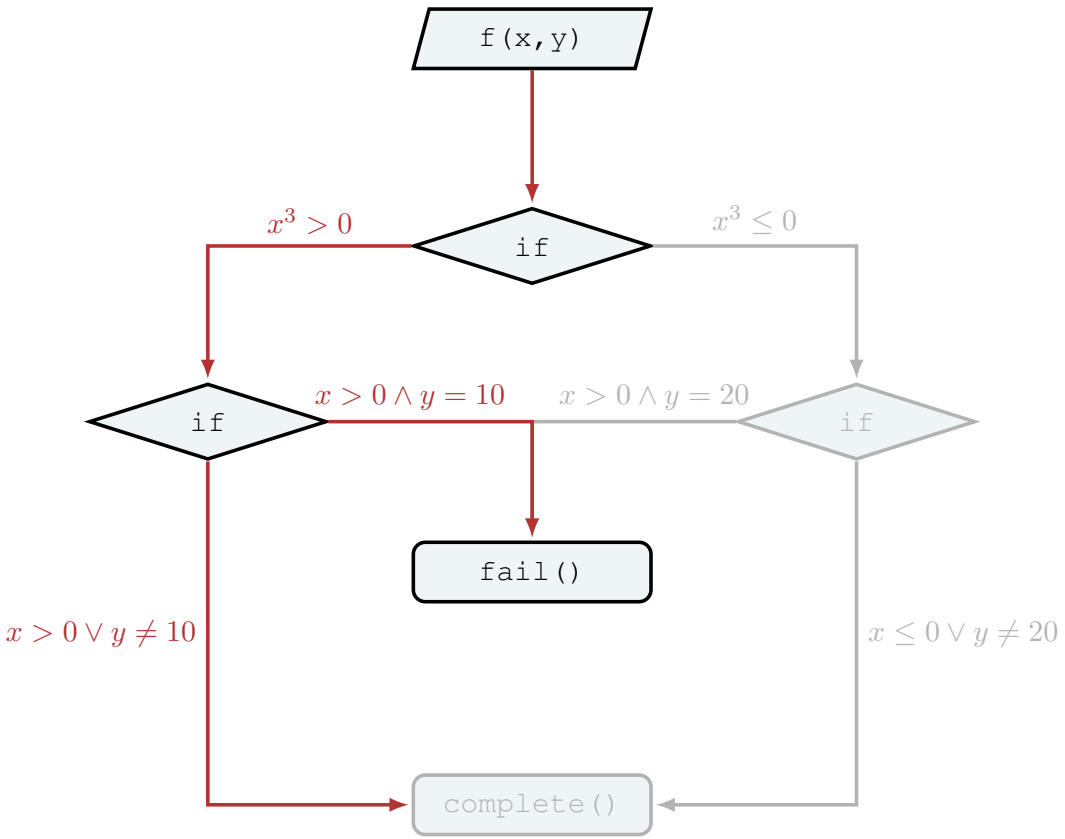
Dynamic execution

- given input $700, 10$
- `x * x * x > 0`
- `x > 0 && y==10`

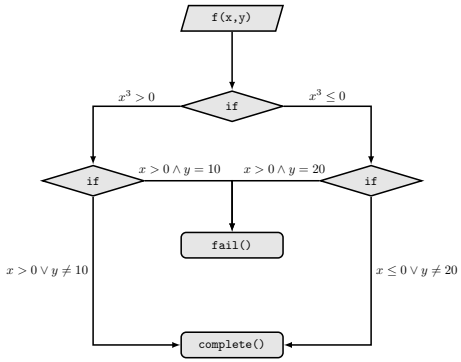
Symbolic execution

- introduce symbols $x_2 = x, y_2 = y$
- constrain $x^3 \leq 0$
- non-linear: fail
- concrete fall-back: $x_2 = 700$
- branch explored, no new input

Dart (3...)


f(x,y)
$x^3 > 0$
if
$x^3 \leq 0$
if
$x > 0 \wedge y = 10$
$x > 0 \wedge y = 20$
if
fail()
$x > 0 \vee y \neq 10$
$x \leq 0 \vee y \neq 20$
complete()

Dart (n)

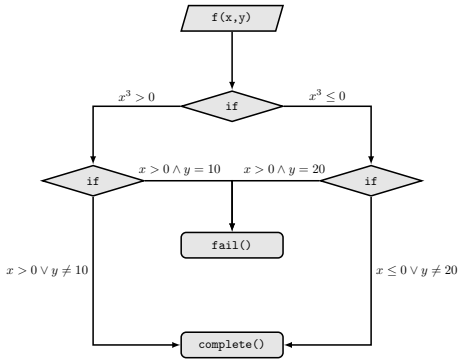


Dynamic execution

- random testing
- random input $-700, 500$

Symbolic execution

Dart (n)



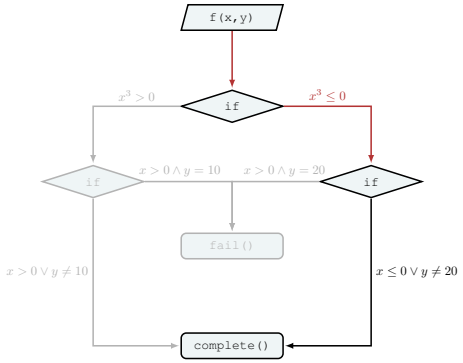
Dynamic execution

- random testing
- random input $-700, 500$

Symbolic execution

- introduce symbols $x_n = x, y_n = y$

Dart (n)



Dynamic execution

- random testing
- random input $-700, 500$
- `x * x * x <= 0`

Symbolic execution

- introduce symbols $x_n = x, y_n = y$

IN5110 – Verification and specification of parallel systems

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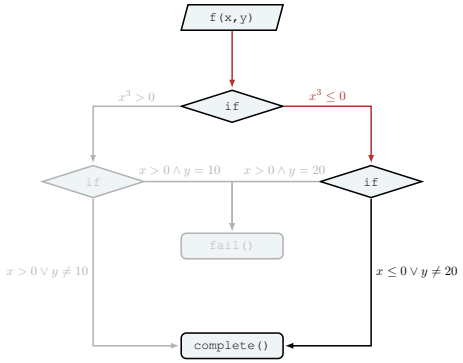
Dart (n)



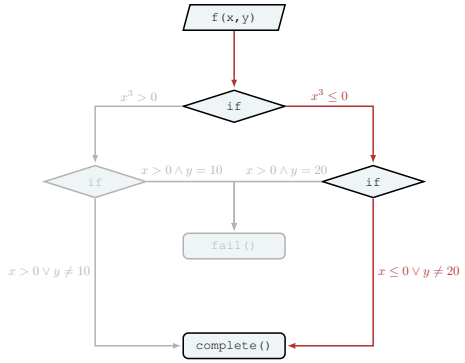
Dynamic execution

- random testing
- random input $-700, 500$
- `x * x * x <= 0`

Symbolic execution

- introduce symbols $x_n = x, y_n = y$
- branch explored, nothing to do

Dart (n)



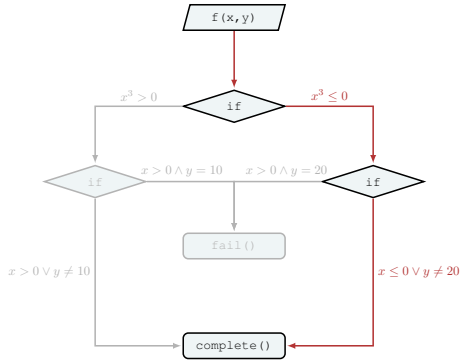
Dynamic execution

- random testing
- random input $-700, 500$
- `x * x * x <= 0`
- `x <= 0 || y!=20`

Symbolic execution

- introduce symbols $x_n = x, y_n = y$
- branch explored, nothing to do

Dart (n)



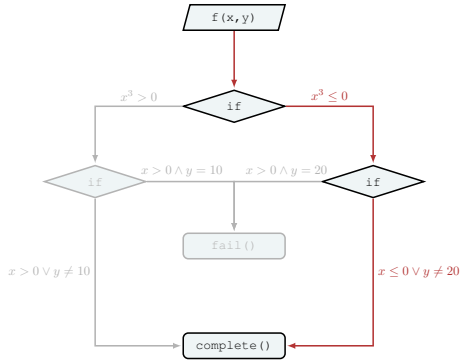
Dynamic execution

- random testing
- random input $-700, 500$
- `x * x * x <= 0`
- `x <= 0 || y!=20`

Symbolic execution

- introduce symbols $x_n = x, y_n = y$
- branch explored, nothing to do
- constrain x and y: $x > 0 \wedge y = 20$

Dart (n)



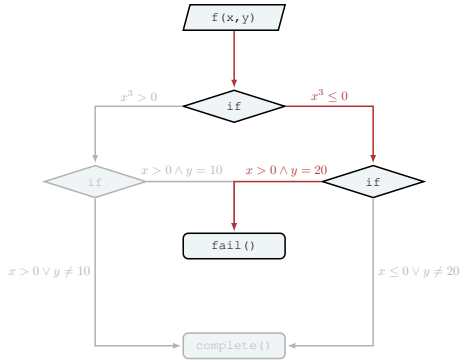
Dynamic execution

- random testing
- random input $-700, 500$
- `x * x * x <= 0`
- `x <= 0 || y!=20`

Symbolic execution

- introduce symbols $x_n = x, y_n = y$
- branch explored, nothing to do
- constrain x and y: $x > 0 \wedge y = 20$
- solution: $x_n = 700, y_n = 20$

Dart (n)



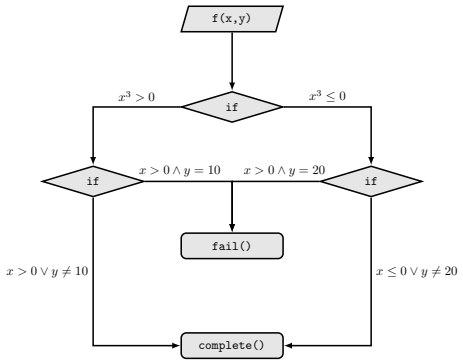
Dynamic execution

- random testing
- random input $-700, 500$
- `x * x * x <= 0`
- `x <= 0 || y!=20`

Symbolic execution

- introduce symbols $x_n = x, y_n = y$
- branch explored, nothing to do
- constrain x and y: $x > 0 \wedge y = 20$
- solution: $x_n = 700, y_n = 20$
- assumed path

Dart $(n+1)$

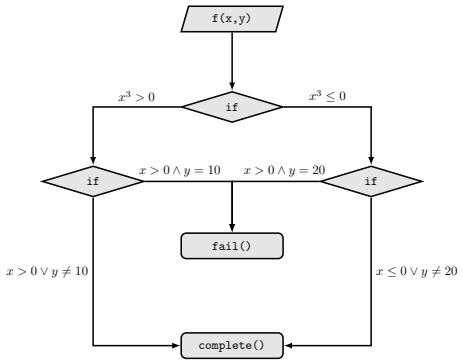


Dynamic execution

- *given* input $700, 20$

Symbolic execution

Dart $(n+1)$



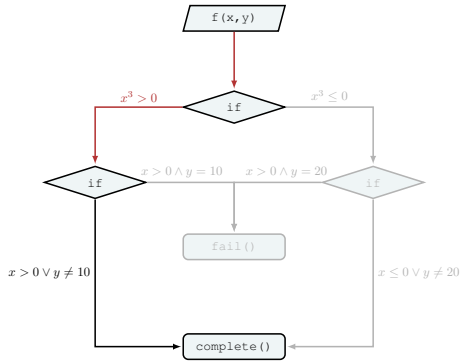
Dynamic execution

- *given* input $700, 20$

Symbolic execution

- introduce symbols $x_{n+1} = x, y_{n+1} = y$

Dart $(n+1)$



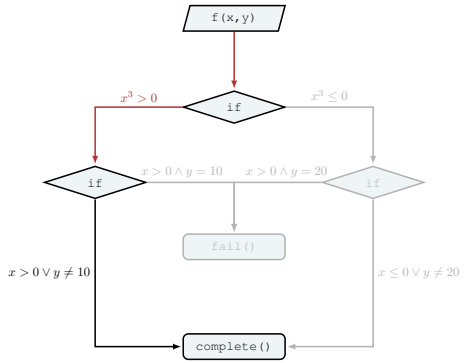
Dynamic execution

- *given* input $700, 20$
- `x * x * x > 0`

Symbolic execution

- introduce symbols $x_{n+1} = x, y_{n+1} = y$

Dart $(n+1)$



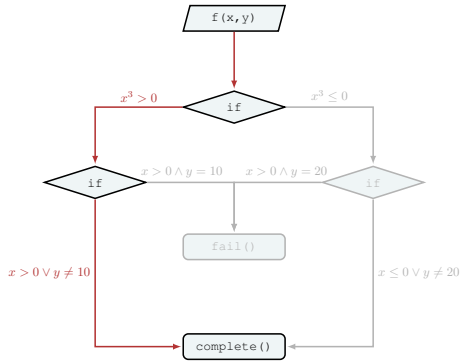
Dynamic execution

- *given* input $700, 20$
- `x * x * x > 0`

Symbolic execution

- introduce symbols $x_{n+1} = x, y_{n+1} = y$
- branch explored, nothing to do

Dart $(n+1)$



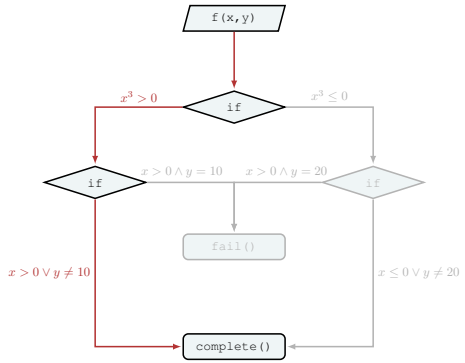
Dynamic execution

- *given* input $700, 20$
- `x * x * x > 0`
- `x > 0 || y!=10`

Symbolic execution

- introduce symbols $x_{n+1} = x, y_{n+1} = y$
- branch explored, nothing to do

Dart $(n+1)$



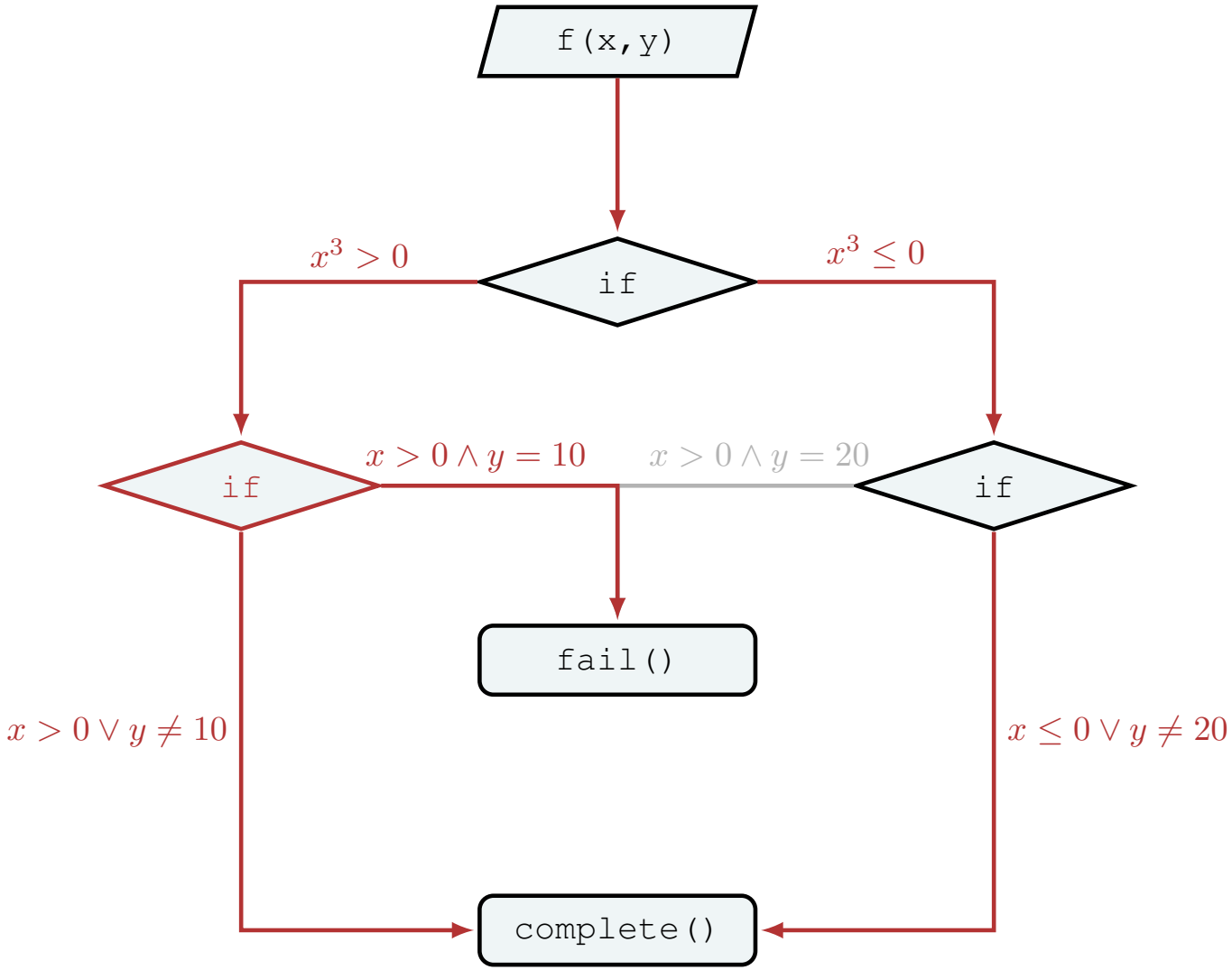
Dynamic execution

- *given* input $700, 20$
- `x * x * x > 0`
- `x > 0 || y!=10`

Symbolic execution

- introduce symbols $x_{n+1} = x, y_{n+1} = y$
- branch explored, nothing to do
- branch explored, nothing to do

Dart completed



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[2] Godefroid, P., Klarlund, N., and Sen, K. (2005). Dart: Directed automated runtime testing. In *ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI)*, pages 213–223. ACM.

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