

# Refinement III

## The general case

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# Topics

- Sequential composition
- Negative behaviour
- Refinement in the general case

# Sequential composition

# Basic rules

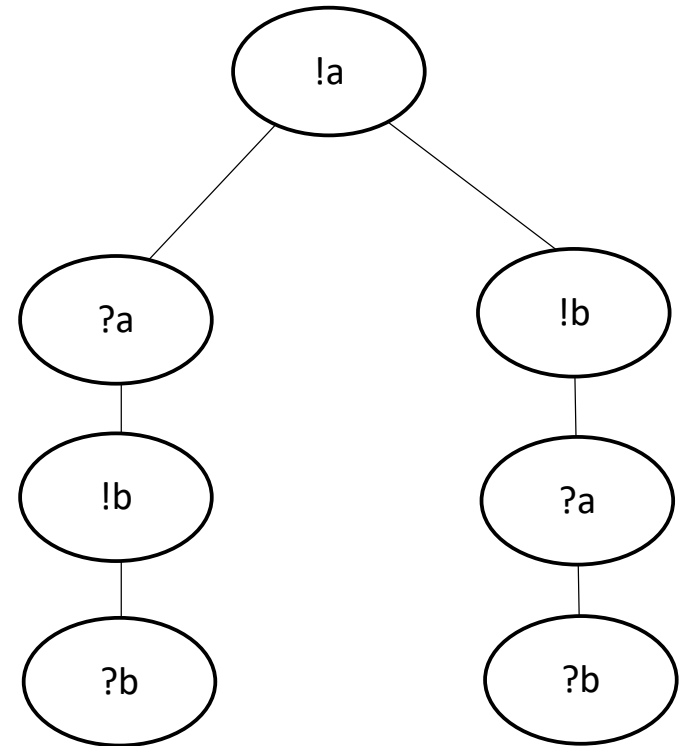
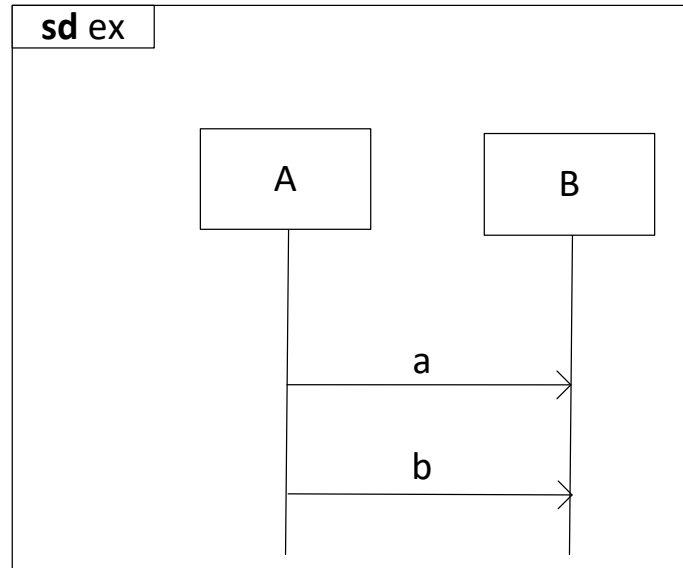
## Causality

- a message can never be received before it has been transmitted
- the transmission event for a message is therefore always ordered before the reception event for the same message

## Weak sequencing

- events from the same lifeline are ordered in the trace in the same order as on the lifeline (from top to bottom)

# Example



Mathematically  $!a$  and  $?a$  (etc.) are shorthands for  $!(a,A,B)$  and  $?(a,A,B)$   
Hence, each event contains the names of its sending and receiving lifelines

# Sequential composition of trace sets $s_1$ and $s_2$

$$s_1 \succcurlyeq s_2$$
$$=$$

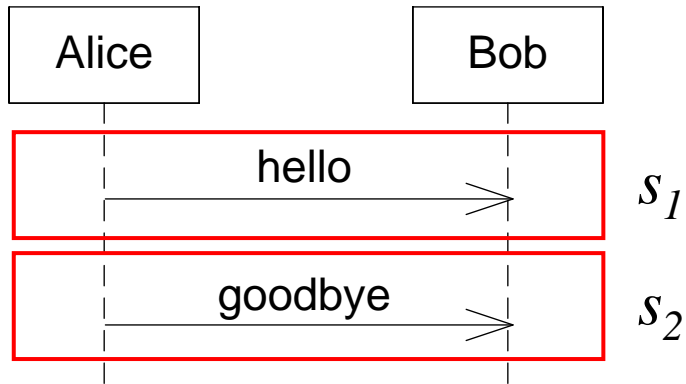
the set of all traces obtained by merging traces

$t_1$  from  $s_1$  and  $t_2$  from  $s_2$

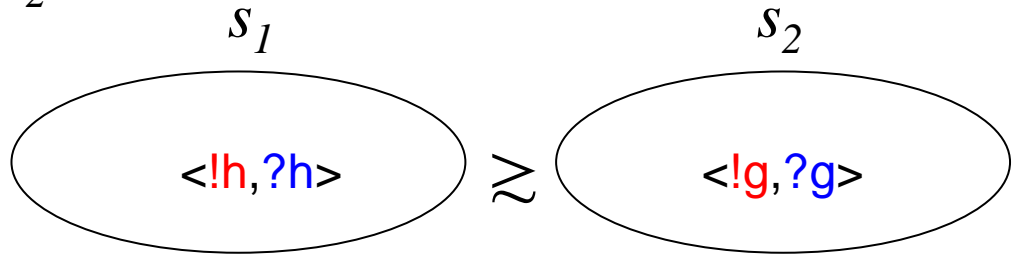
in such a way that for each lifeline,

the events from  $t_1$  comes before the events from  $t_2$

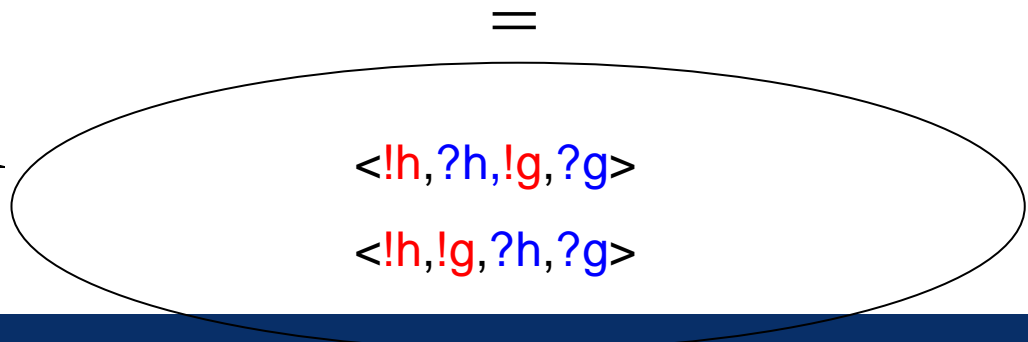
# Sequential composition of trace sets



Red events occur on Alice, blue events on Bob



$S_1 \approx S_2$  is the set of positive traces for the diagram



## Note

- if  $s_1$  or  $s_2$  is empty then  $s_1 \succcurlyeq s_2$  is also empty



# Sequential composition of interaction obligations

- $(p_1, n_1) \succeq (p_2, n_2) \stackrel{\text{def}}{=} (p_1 \succeq p_2, (n_1 \succeq p_2) \cup (n_1 \succeq n_2) \cup (p_1 \succeq n_2))$
- Traces composed exclusively by positive traces become positive
- Traces composed with at least one negative trace become negative

## Formal semantics of **seq**

- $[[d_1 \text{ seq } d_2]] \stackrel{\text{def}}{=} \{o_1 \succsim o_2 \mid o_1 \in [[d_1]] \wedge o_2 \in [[d_2]]\}$
- $o_i$  is shorthand for  $(p_i, n_i)$

## Remember: By sequential composition

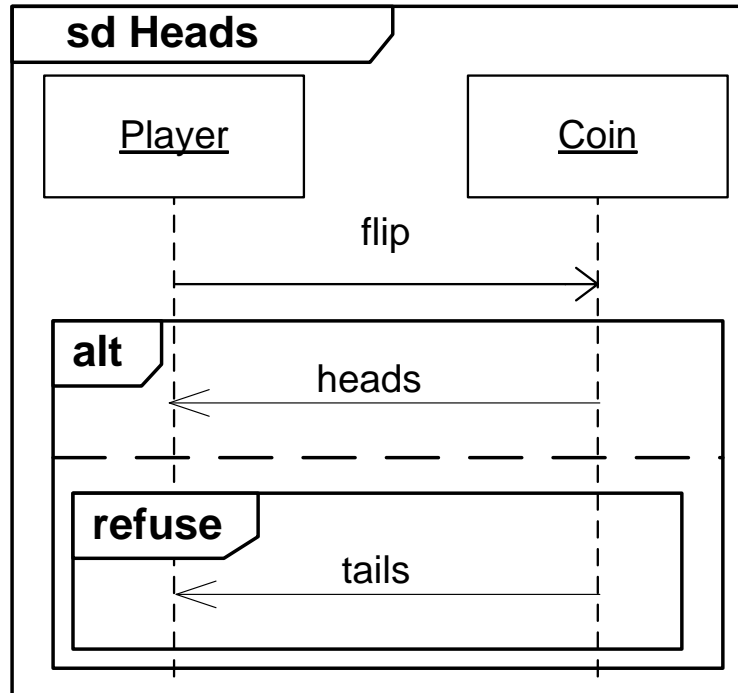
- positive followed by positive is positive
- positive followed by negative is negative
- negative followed by negative is positive
- negative followed by positive is negative

# Negative behaviour

## Specifying negative behaviour with **refuse**

- $[[\text{refuse } d]] \stackrel{\text{def}}{=} \{(\{ \}, p \cup n) \mid (p, n) \in [[d]]\}$
- All interaction obligations in  $[[\text{refuse } d]]$  have empty positive sets
- Hence, all interaction obligations in  $[[d_1 \text{ seq } (\text{refuse } d_2)]]$  have empty positive sets
- The same applies to  $[[\text{(refuse } d_1) \text{ seq } d_2]]$

## Example use of **refuse**



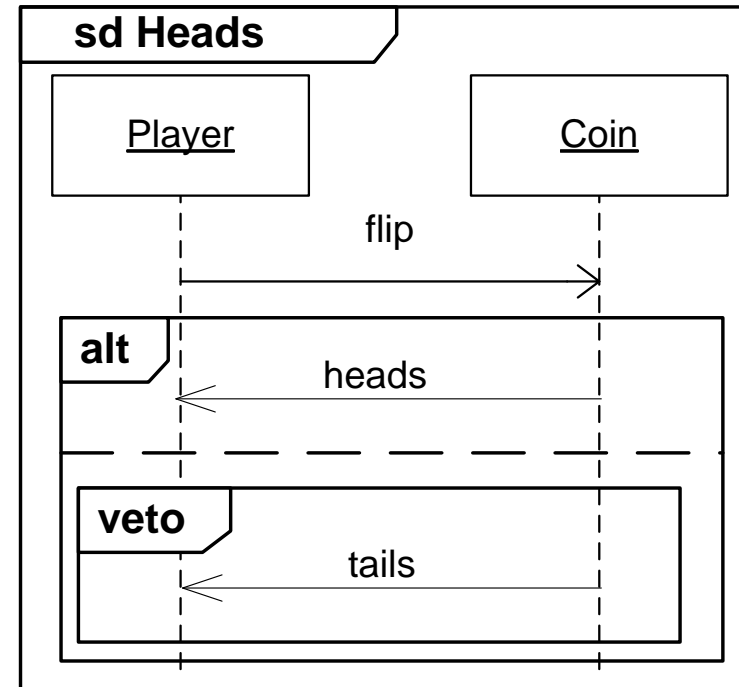
- $[[\text{Heads}]] = \{(\{<!f, ?f, !h, ?h>\}, \{<!f, ?f, !t, ?t>\})\}$

# Specifying negative behaviour with **veto**

$[[\text{veto } d]] \stackrel{\text{def}}{=} [[\text{skip alt (refuse } d)]]$

This means:

$[[\text{veto } d]] = \{(\{\langle \rangle\}, p \cup n) \mid (p, n) \in [[d]]\}$



$[[\text{Heads}]] = \{(\{\langle !f, ?f, !h, ?h \rangle, \langle !f, ?f \rangle\}, \{\langle !f, ?f, !t, ?t \rangle\})\}$

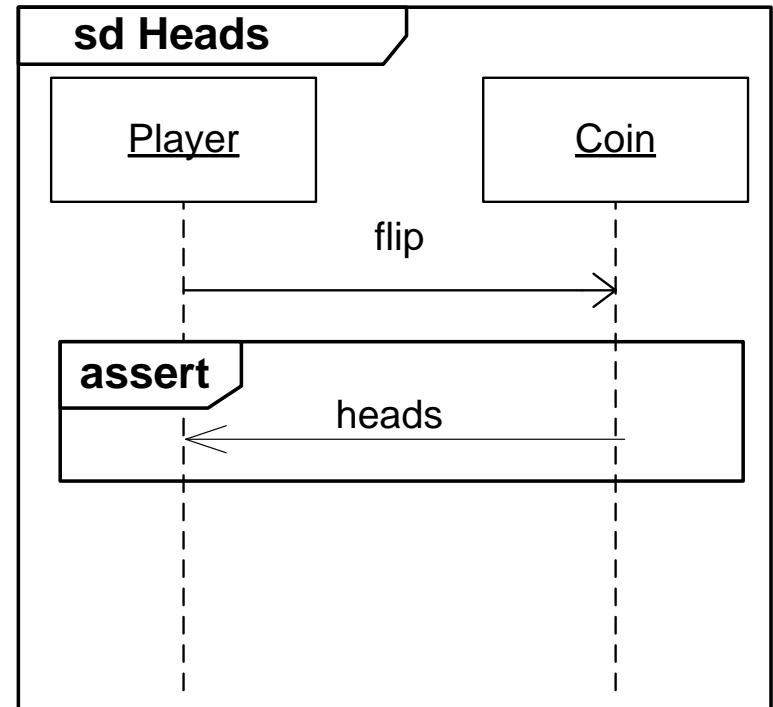
## Specifying negative behaviour with **assert**

- By using **assert**, all inconclusive traces are redefined as negative
- This ensures that for each interaction obligation, at least one of its positive traces will be implemented in the final implementation
- $[[\text{assert } d]] \stackrel{\text{def}}{=} \{(p, n \cup (\mathcal{H} \setminus p)) \mid (p, n) \in [[d]]\}$
- $\mathcal{H}$  = all possible traces
- $\mathcal{H} \setminus p$  = all possible traces minus those in  $p$



## Example use of **assert**

- $[[\text{Heads}]] = \{(\{<!f, ?f, !h, ?h>\}, n)\}$



- $n =$  all traces where the first event on the lifeline of Player is  $!f$  and the first event on the lifeline of Coin is  $?f$  except the trace  $<!f, ?f, !h, ?h>$

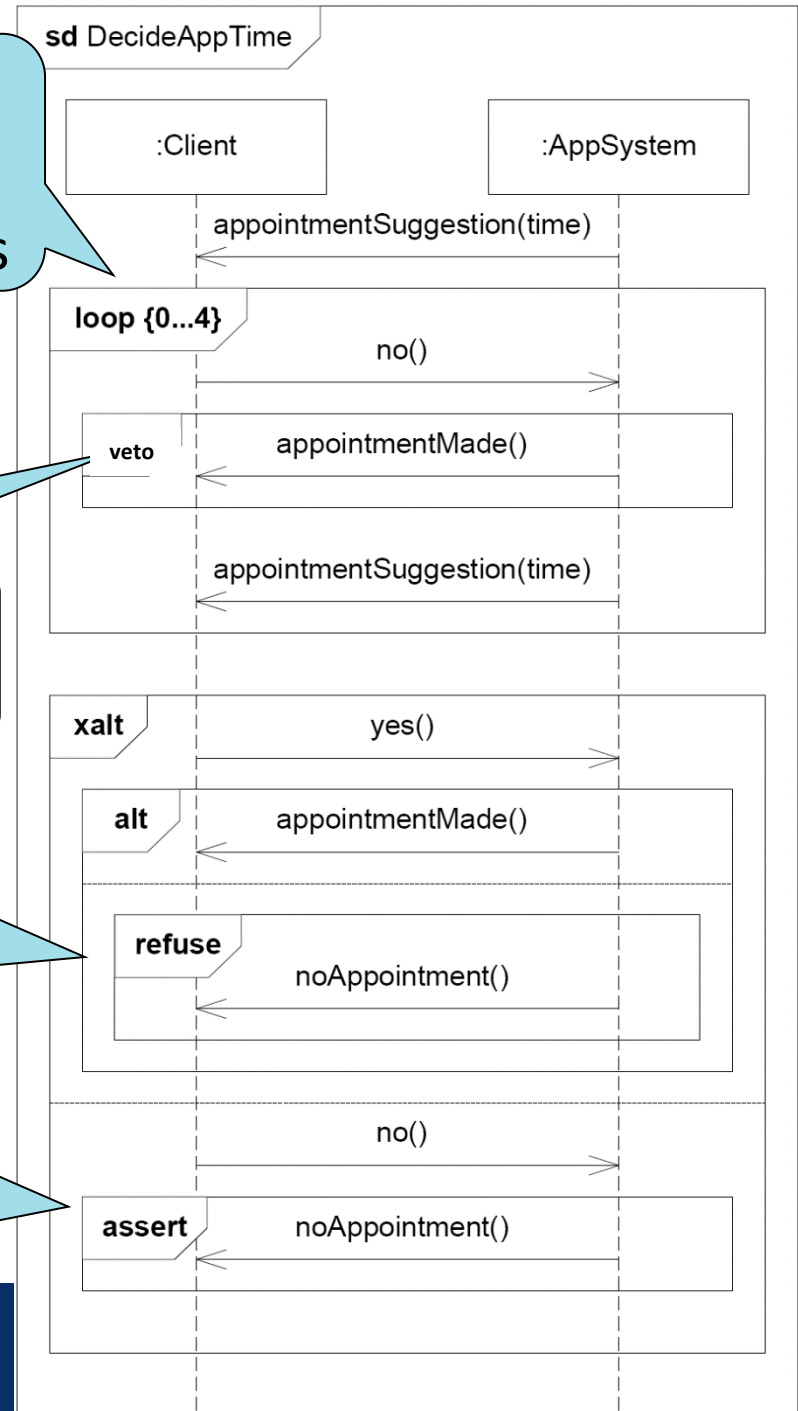
# Negative behaviour

From 0 to 4 iterations

appointmentMade() may not occur here

noAppointment() may not occur instead of appointmentMade() here

noAppointment () is the only message that may occur here



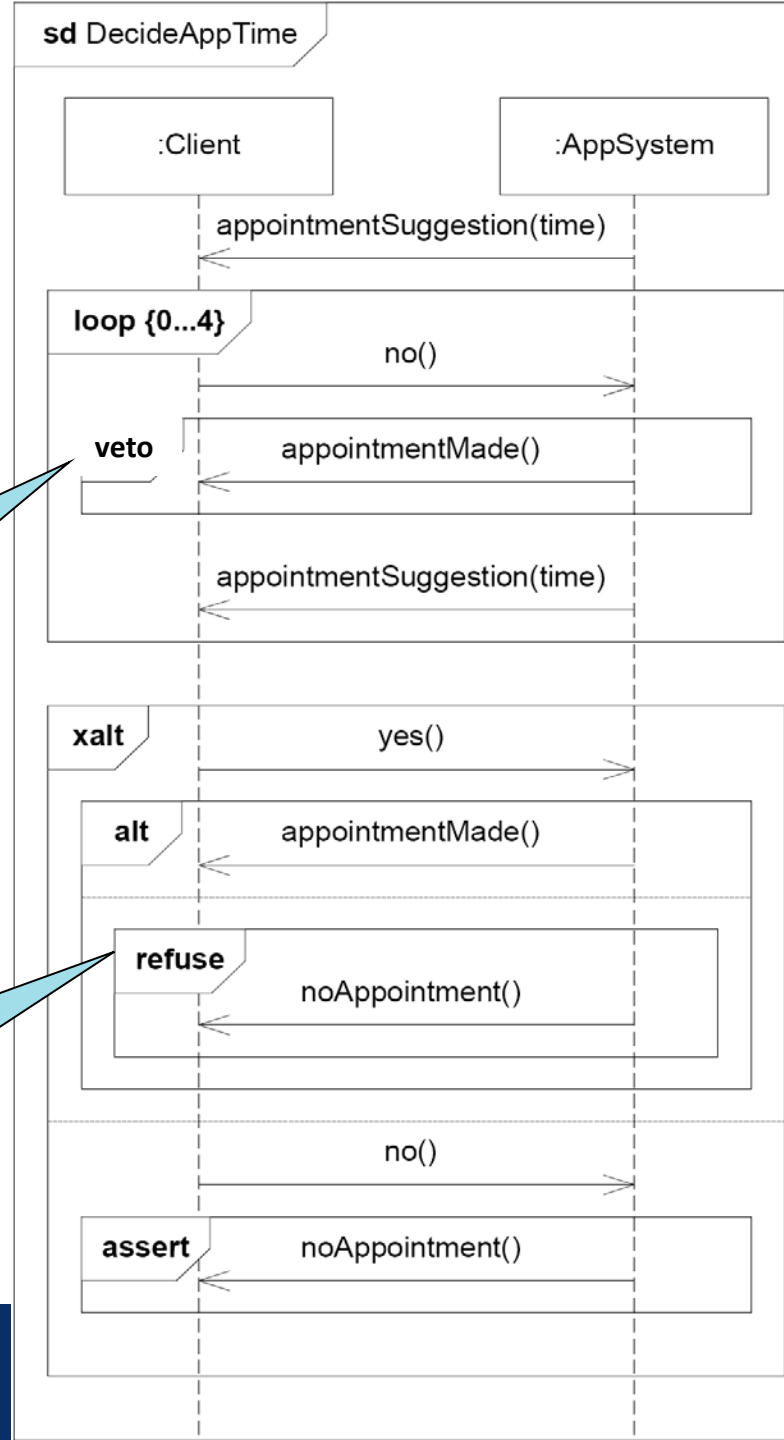
## veto or refuse?

Should doing nothing be possible in the otherwise negative situation?

- If yes, use veto
- If no, use refuse

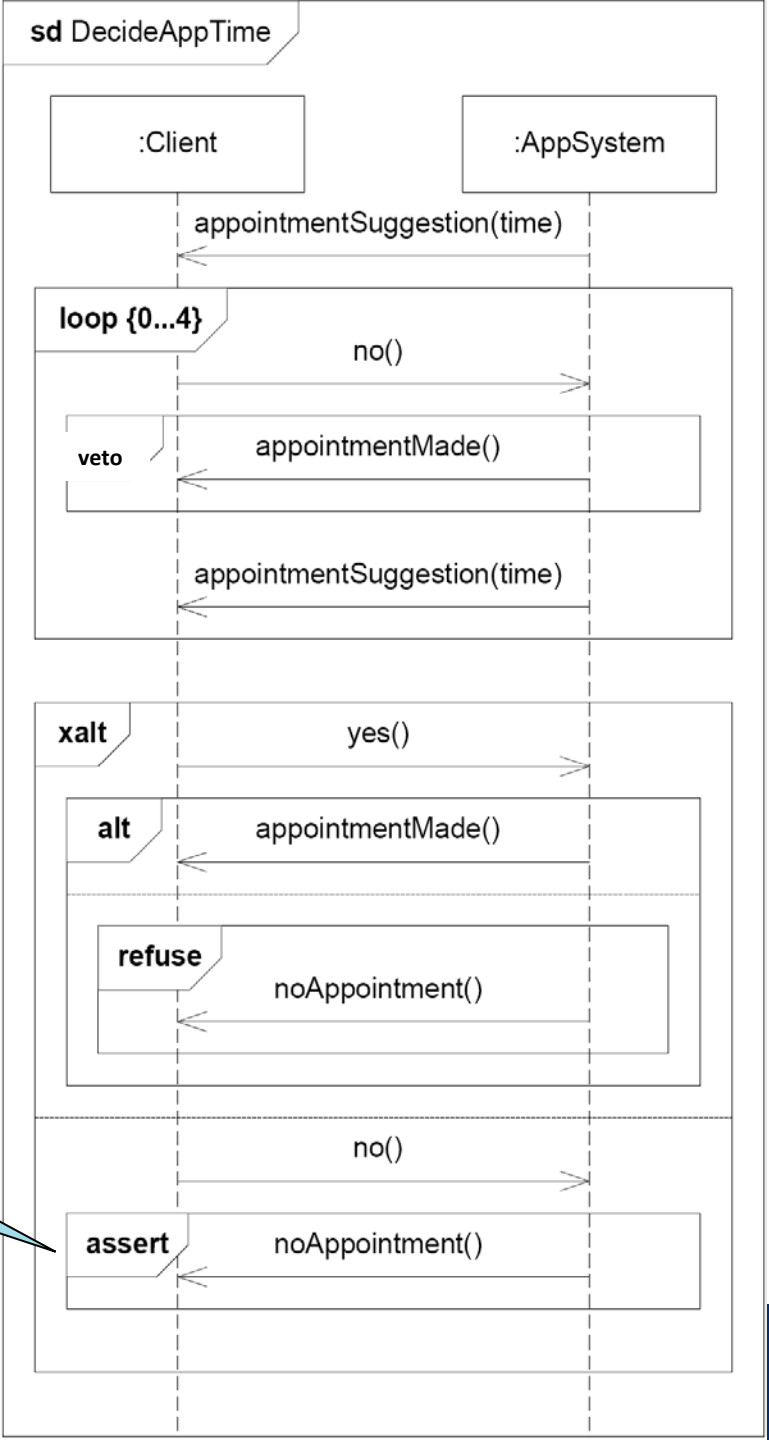
ok to do nothing between no() and appointment-Suggestion(time)

not ok to do nothing after yes()



# When to use assert?

Sending noAppointment() is the only acceptable response to the no() message at this point



# The pragmatics of negative behaviour

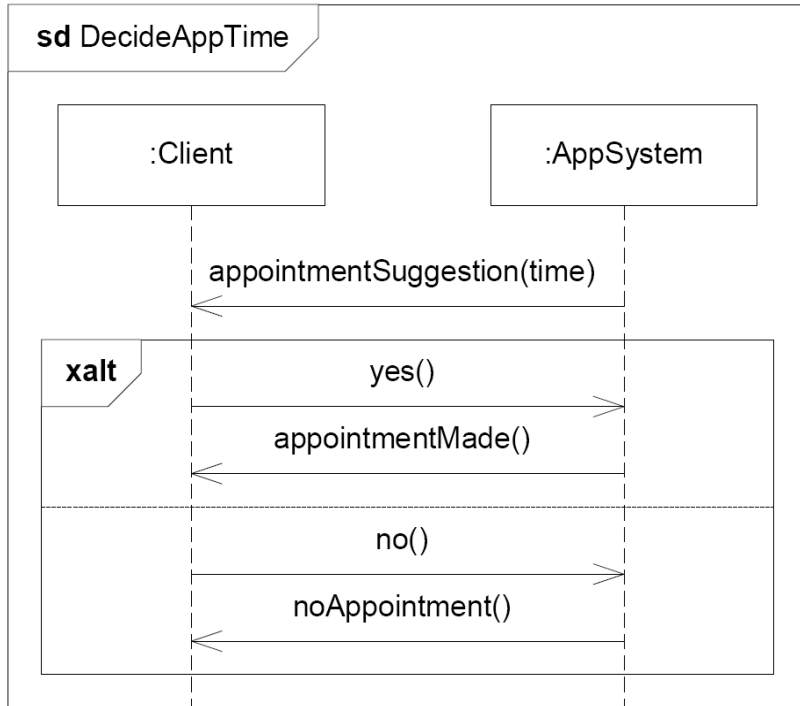
- To effectively constrain the implementation, the specification should include a reasonable set of negative traces
- Use **refuse** when specifying that one of the alternatives in an **alt** represents negative traces
- Use **veto** when the empty trace (i.e. doing nothing) should be positive, as when specifying a negative message in an otherwise positive scenario
- Use **assert** on an interaction fragment when all positive traces for that fragment have been described



# Refinement in the general case

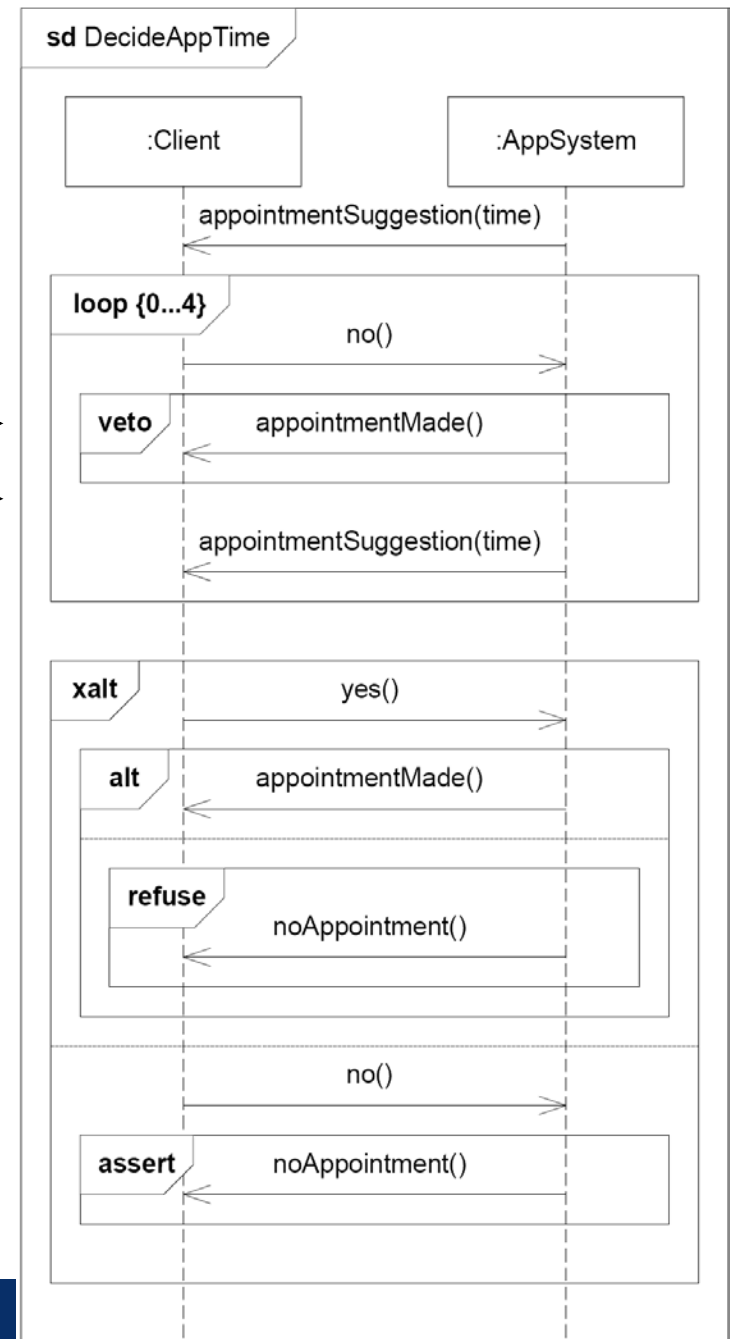
# Supplementing

- Inconclusive trace are recategorized as either positive or negative (for an interaction obligation)
- New situations are considered
  - adding fault tolerance
  - new user requirements
  - ...
- Typically used in early phases

# Example of supplementing



Positive   
 Negative 





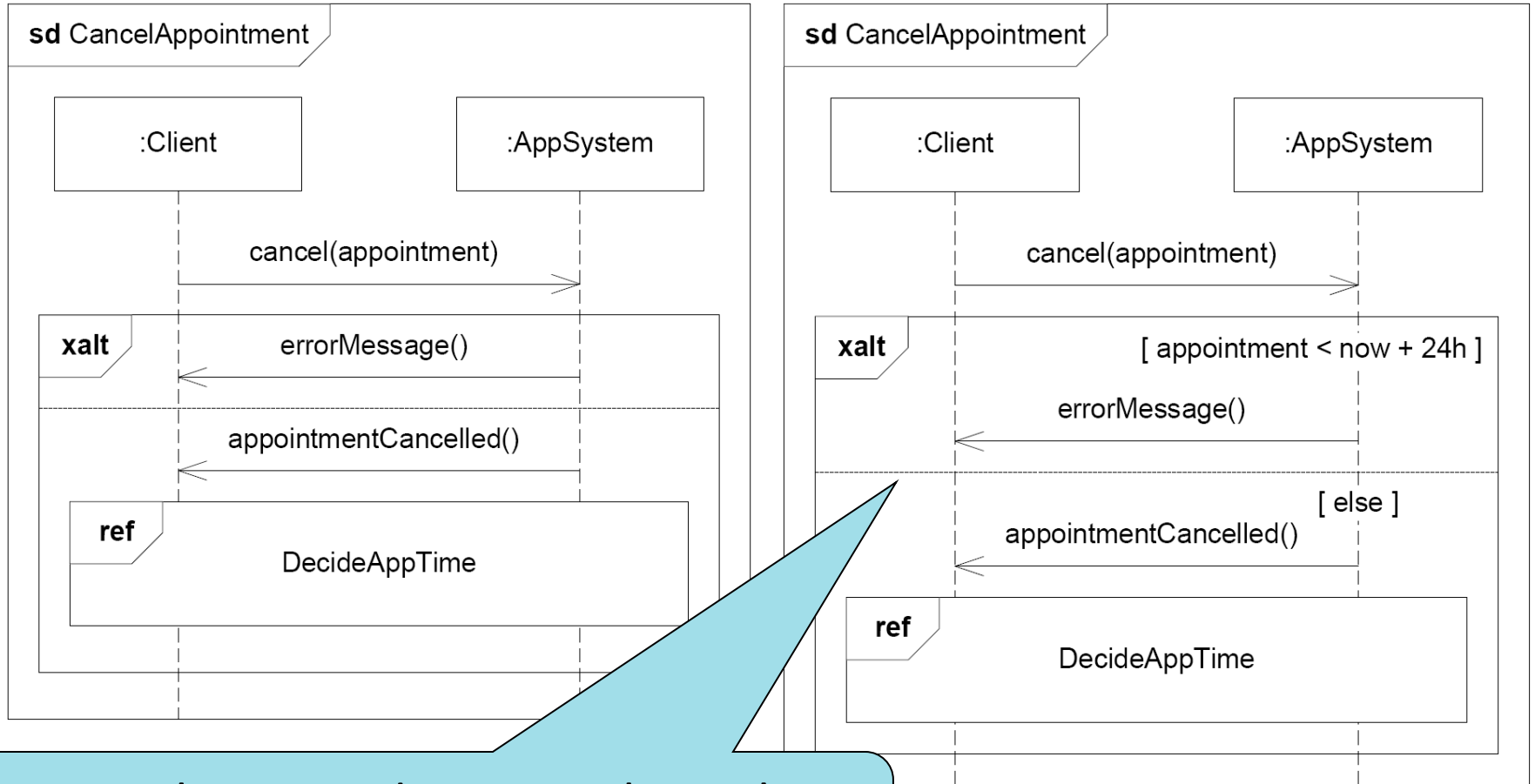
# The pragmatics of supplementing

- Use supplementing to add positive or negative traces to the specification
- When supplementing, all of the original positive traces must remain positive, and all of the original negative traces must remain negative
- Do not use supplementing on the operand of an assert
  - no traces are inconclusive in the operand

# Narrowing

- Reduce underspecification by redefining positive traces as negative
- For example adding guards, or replacing a guard with a stronger one
  - traces where the guard is false become negative

# Example of narrowing



For each operand, traces where the guard is false become negative

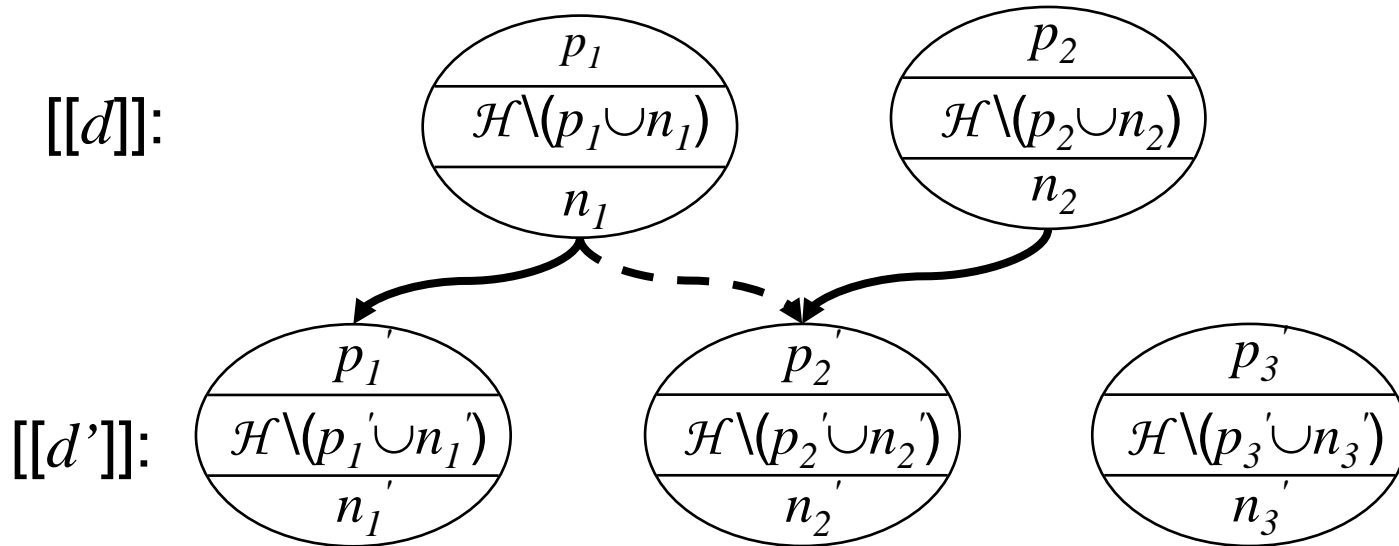
# The pragmatics of narrowing

- Use narrowing to remove underspecification by redefining positive traces as negative
- In cases of narrowing, all of the original negative traces must remain negative
- Guards may be added to an **alt** as a legal narrowing step
- Guards may be added to an **xalt** as a legal narrowing step
- Guards may be narrowed, i.e. the refined condition must imply the original one

# General refinement

- $d'$  is a general refinement of  $d$  if
  - for every interaction obligation  $o$  in  $[[d]]$  there is at least one interaction obligation  $o'$  in  $[[d']]$  such that  $o'$  is a refinement of  $o$
- Interaction obligations that do not refine any obligation at the abstract level may be added

# General refinement illustrated



# The pragmatics of general refinement

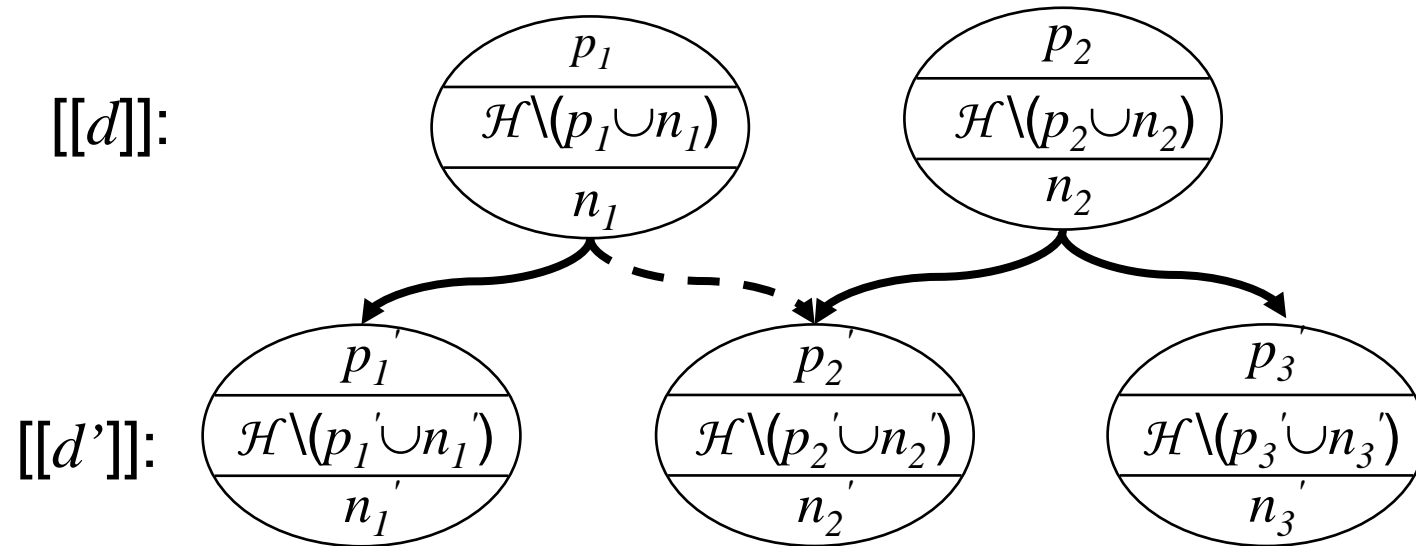
- General refinement is required for specifications with **xalt**
- It corresponds to the pointwise application of refinement for each single interaction obligation
- General refinement supports the introduction of additional inherent nondeterminism

# Limited refinement

- $d'$  is a limited refinement of  $d$  if
  - $d'$  is a general refinement of  $d$ , and
  - every interaction obligation in  $[[d']]$  is a refinement of at least one interaction obligation in  $[[d]]$
- Limits the possibility of adding new interaction obligations
- Typically used at a later stage



# Limited refinement illustrated



# The pragmatics of limited refinement

- Limited refinement is a special case of general refinement
- Limited refinement disallows the introduction of additional inherent nondeterminism
- Limited refinement is normally used in the later stages of a system development

# Compositionality

A refinement operator  $\rightsquigarrow$  is compositional if it is

reflexive:  $d \rightsquigarrow d$

transitive:  $d \rightsquigarrow d' \wedge d' \rightsquigarrow d'' \Rightarrow d \rightsquigarrow d''$

the operators refuse, veto, alt, xalt and seq are monotonic w.r.t.  $\rightsquigarrow$  :

$d \rightsquigarrow d' \Rightarrow \text{refuse } d \rightsquigarrow \text{refuse } d'$

$d \rightsquigarrow d' \Rightarrow \text{veto } d \rightsquigarrow \text{veto } d'$

$d_1 \rightsquigarrow d_1' \wedge d_2 \rightsquigarrow d_2' \Rightarrow d_1 \text{ alt } d_2 \rightsquigarrow d_1' \text{ alt } d_2'$

$d_1 \rightsquigarrow d_1' \wedge d_2 \rightsquigarrow d_2' \Rightarrow d_1 \text{ xalt } d_2 \rightsquigarrow d_1' \text{ xalt } d_2'$

$d_1 \rightsquigarrow d_1' \wedge d_2 \rightsquigarrow d_2' \Rightarrow d_1 \text{ seq } d_2 \rightsquigarrow d_1' \text{ seq } d_2'$

- Transitivity allows stepwise development
- Monotonicity allow different parts of the specification to be refined separately

Supplementing, narrowing, general refinement and limited refinement are all compositional 😊

# The mathematical foundation

- Haugen, Husa, Runde, Stølen: *STAIRS towards formal design with sequence diagrams*, 2005. SoSyM, Springer.
  - <http://heim.ifi.uio.no/~ketils/kst/Articles/2005.SoSyM-onlinefirst.pdf>
- Runde, Haugen, Stølen: *The Pragmatics of STAIRS*, 2006. Springer-Verlag. LNCS 4111.
  - <http://heim.ifi.uio.no/~ketils/kst/Articles/2006.FMCO-LNCS4111.pdf>