

Obligatory Exercise II

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Question I

a)

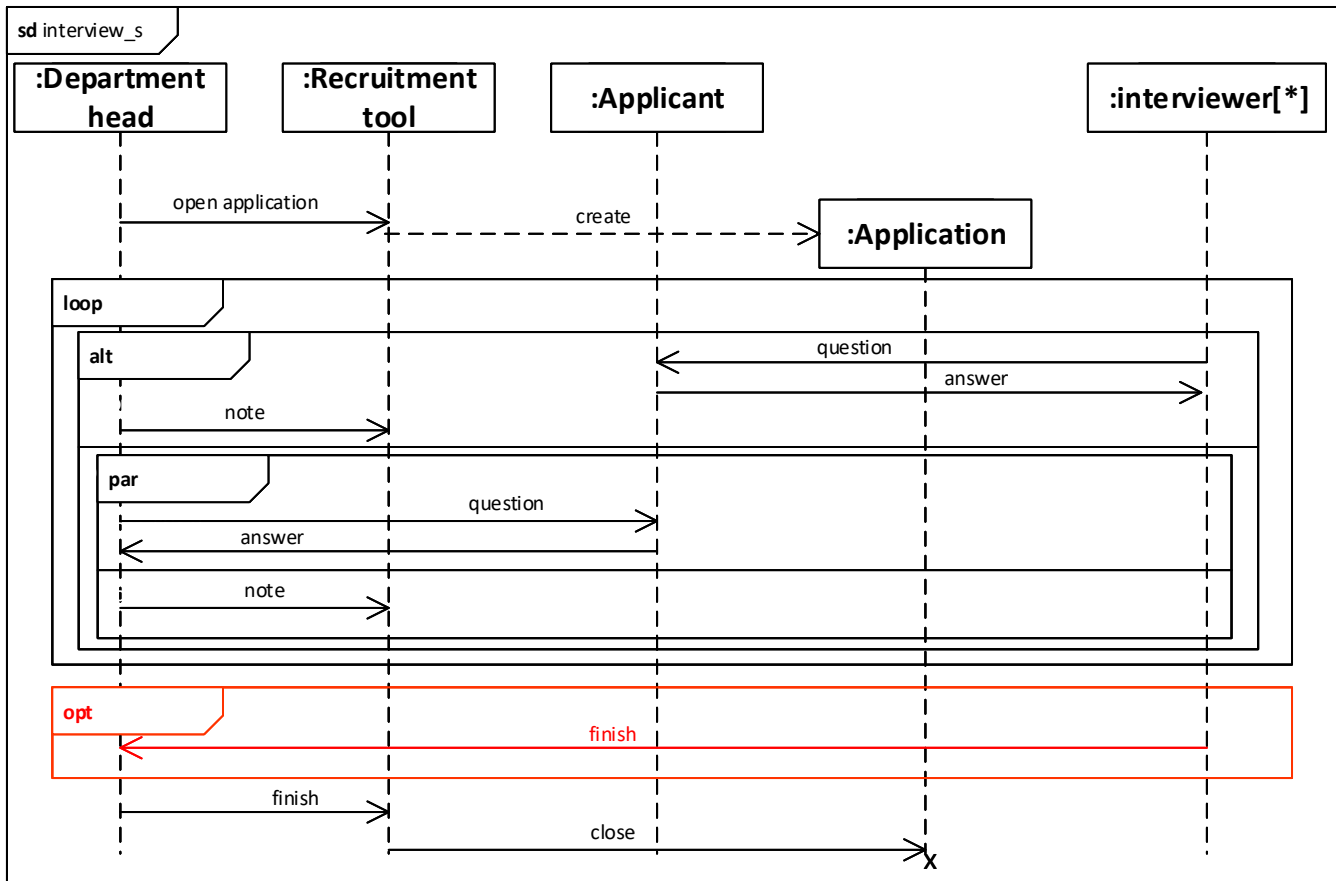
The first events in the diagram could be either **Department head sends open application** or **Interviewer sends question**. The reason is that these are sending events (causality) and are also the first events of their corresponding lifeline (weak sequencing).

b)

The last events in the diagram could be either **Application receives close message** or **Interviewer receives answer**. These two events are the last events of their corresponding lifeline and are reception events.

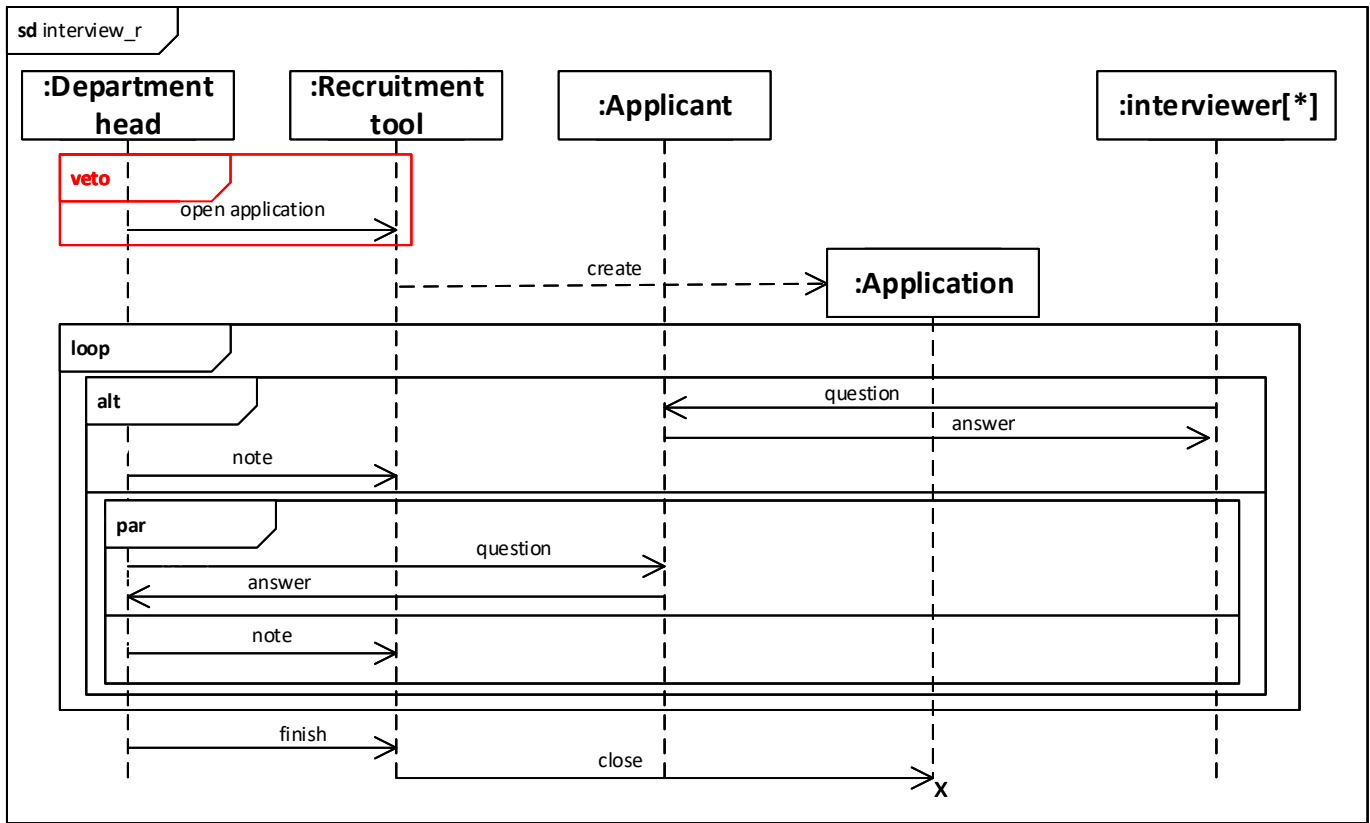
c)

One possibility of supplementing the *interview* diagram is introducing an optional fragment as the diagram *interview_s* below. Accordingly, the positive traces of *interview_s* are **all the positive traces of the interview diagram** (without optional fragment) as well as **those positive traces with sending and receiving finish message events** (with optional fragment). The negative traces set of this diagram remains unchanged (i.e. empty set). Since we are only moving traces which are inconclusive to positive set, this is pure supplementing.



d)

By moving the *open application* message into a *veto fragment* as showed in the *interview_r* diagram below, the negative traces of the diagram *interview_r* are **all of the positive traces** of the *interview* diagram (narrowing). In addition, the positive traces of the diagram *interview_r* are the positive traces of the the *interview* diagram **without sending and receiving open application message events** (supplementing).



Question II

a)

Since sending messages question/answer and note occur parallel, there are 15 possible traces (t1 to t15) for the second operand of the alt construct as showed below. Also, it has no negative traces. Hence,

$$p2 = \bigcup_{i=1}^{15} \{t_i\}$$

$$n2 = \emptyset$$

$$[[Op2]] = \{(p2, n2)\} = \left\{ \left(\bigcup_{i=1}^{15} \{t_i\}, \emptyset \right) \right\}$$

forwarded note after sending answer. Thus, for each positive trace t_i (in p_2), there are several possible negative traces $T_{i,j}$ created by adding these two events respectively. Accordingly, there are 55 negative traces (from N_1 to N_{55}) as showed below:

$N_1 = T_{1_1} = \langle !q, !n, ?n, !f, ?q, !a, ?f, ?a \rangle$
 $N_2 = T_{1_2} = \langle !q, !n, ?n, !f, ?q, !a, ?a, ?f \rangle$
 $N_3 = T_{1_3} = \langle !q, !n, ?n, ?q, !f, !a, ?f, ?a \rangle$
 $N_4 = T_{1_4} = \langle !q, !n, ?n, ?q, !f, !a, ?a, ?f \rangle$
 $N_5 = T_{1_5} = \langle !q, !n, ?n, ?q, !a, !f, ?f, ?a \rangle$
 $N_6 = T_{1_6} = \langle !q, !n, ?n, ?q, !a, !f, ?a, ?f \rangle$
 $N_7 = T_{1_7} = \langle !q, !n, ?n, ?q, !a, ?a, !f, ?f \rangle$
 $N_8 = T_{2_8} = \langle !q, !n, ?q, ?n, !f, !a, ?f, ?a \rangle$
 $N_9 = T_{2_9} = \langle !q, !n, ?q, ?n, !f, !a, ?a, ?f \rangle$
 $N_{10} = T_{2_{10}} = \langle !q, !n, ?q, ?n, !a, !f, ?f, ?a \rangle$
 $N_{11} = T_{2_{11}} = \langle !q, !n, ?q, ?n, !a, !f, ?a, ?f \rangle$
 $N_{12} = T_{2_{12}} = \langle !q, !n, ?q, ?n, !a, ?a, !f, ?f \rangle$
 $N_{13} = T_{3_{13}} = \langle !q, !n, ?q, !a, ?n, !f, ?f, ?a \rangle$
 $N_{14} = T_{3_{14}} = \langle !q, !n, ?q, !a, ?n, !f, ?a, ?f \rangle$
 $N_{15} = T_{3_{15}} = \langle !q, !n, ?q, !a, ?n, ?a, !f, ?f \rangle$
 $N_{16} = T_{4_{16}} = \langle !q, !n, ?q, !a, ?a, ?n, !f, ?f \rangle$
 $N_{17} = T_{5_{17}} = \langle !q, ?q, !n, ?n, !f, !a, ?f, ?a \rangle$
 $N_{18} = T_{5_{18}} = \langle !q, ?q, !n, ?n, !f, !a, ?a, ?f \rangle$
 $N_{19} = T_{5_{19}} = \langle !q, ?q, !n, ?n, !a, !f, ?f, ?a \rangle$
 $N_{20} = T_{5_{20}} = \langle !q, ?q, !n, ?n, !a, !f, ?a, ?f \rangle$
 $N_{21} = T_{5_{21}} = \langle !q, ?q, !n, ?n, !a, ?a, !f, ?f \rangle$
 $N_{22} = T_{6_{22}} = \langle !q, ?q, !n, !a, ?n, !f, ?f, ?a \rangle$
 $N_{23} = T_{6_{23}} = \langle !q, ?q, !n, !a, ?n, !f, ?a, ?f \rangle$
 $N_{24} = T_{6_{24}} = \langle !q, ?q, !n, !a, ?n, ?a, !f, ?f \rangle$
 $N_{25} = T_{7_{25}} = \langle !q, ?q, !n, !a, ?a, ?n, !f, ?f \rangle$
 $N_{26} = T_{8_{26}} = \langle !q, ?q, !a, !n, ?n, !f, ?f, ?a \rangle$
 $N_{27} = T_{8_{27}} = \langle !q, ?q, !a, !n, ?n, !f, ?a, ?f \rangle$
 $N_{28} = T_{8_{28}} = \langle !q, ?q, !a, !n, ?n, ?a, !f, ?f \rangle$
 $N_{29} = T_{9_{29}} = \langle !q, ?q, !a, !n, ?a, ?n, !f, ?f \rangle$
 $N_{30} = T_{10_{30}} = \langle !q, ?q, !a, ?a, !n, ?n, !f, ?f \rangle$
 $N_{31} = T_{11_{31}} = \langle !n, ?n, !f, !q, ?q, !a, ?f, ?a \rangle$
 $N_{32} = T_{11_{32}} = \langle !n, ?n, !f, !q, ?q, !a, ?a, ?f \rangle$
 $N_{33} = T_{11_{33}} = \langle !n, ?n, !q, !f, ?q, !a, ?f, ?a \rangle$
 $N_{34} = T_{11_{34}} = \langle !n, ?n, !q, !f, ?q, !a, ?a, ?f \rangle$
 $N_{35} = T_{11_{35}} = \langle !n, ?n, !q, ?q, !f, !a, ?f, ?a \rangle$
 $N_{36} = T_{11_{36}} = \langle !n, ?n, !q, ?q, !f, !a, ?a, ?f \rangle$
 $N_{37} = T_{11_{37}} = \langle !n, ?n, !q, ?q, !a, !f, ?f, ?a \rangle$
 $N_{38} = T_{11_{38}} = \langle !n, ?n, !q, ?q, !a, !f, ?a, ?f \rangle$
 $N_{39} = T_{11_{39}} = \langle !n, ?n, !q, ?q, !a, ?a, !f, ?f \rangle$
 $N_{40} = T_{12_{40}} = \langle !n, !q, ?n, !f, ?q, !a, ?f, ?a \rangle$
 $N_{41} = T_{12_{41}} = \langle !n, !q, ?n, !f, ?q, !a, ?a, ?f \rangle$
 $N_{42} = T_{12_{42}} = \langle !n, !q, ?n, ?q, !f, !a, ?f, ?a \rangle$
 $N_{43} = T_{12_{43}} = \langle !n, !q, ?n, ?q, !f, !a, ?a, ?f \rangle$

N44 = T12_44 = <!n, !q, ?n, ?q, !a, !f, ?f, ?a>
 N45 = T12_45 = <!n, !q, ?n, ?q, !a, !f, ?a, ?f>
 N46 = T12_46 = <!n, !q, ?n, ?q, !a, ?a, !f, ?f>
 N47 = T13_47 = <!n, !q, ?q, ?n, !f, !a, ?f, ?a>
 N48 = T13_48 = <!n, !q, ?q, ?n, !f, !a, ?a, ?f>
 N49 = T13_49 = <!n, !q, ?q, ?n, !a, !f, ?f, ?a>
 N50 = T13_50 = <!n, !q, ?q, ?n, !a, !f, ?a, ?f>
 N51 = T13_51 = <!n, !q, ?q, ?n, !a, ?a, !f, ?f>
 N52 = T14_52 = <!n, !q, ?q, !a, ?n, !f, ?f, ?a>
 N53 = T14_53 = <!n, !q, ?q, !a, ?n, !f, ?a, ?f>
 N54 = T14_54 = <!n, !q, ?q, !a, ?n, ?a, !f, ?f>
 N55 = T15_55 = <!n, !q, ?q, !a, ?a, ?n, !f, ?f>

Hence,

$$\begin{aligned}
 p1 &= \bigcup_{i=16}^{30} \{t_i\} \\
 n1 &= \bigcup_{i=1}^{55} \{N_i\} \\
 [[Op1]] &= \{(p1, n1)\} = \left\{ \left(\bigcup_{i=16}^{30} \{t_i\}, \bigcup_{i=1}^{55} \{N_i\} \right) \right\}
 \end{aligned}$$

c)

$$[[Op1 \text{ alt } Op2]] = \{(p1 + 2, n1 + 2)\} = \{(p1 \cup p2, n1 \cup n2)\} = \left\{ \left(\bigcup_{i=1}^{30} \{t_i\}, \bigcup_{i=1}^{55} \{N_i\} \right) \right\}$$

That means the positive traces are the total 30 traces (t1 to t30) and the negative traces are the traces in n1.

d)

$$[[Op1 \text{ xalt } Op2]] = [[Op1]] \cup [[Op2]] = \{(p1, n1), (p2, n2)\} = \left\{ \left(\bigcup_{i=1}^{15} \{t_i\}, \emptyset \right), \left(\bigcup_{i=16}^{30} \{t_i\}, \bigcup_{i=1}^{55} \{N_i\} \right) \right\}$$

e)

Here we consider only semantic of the loop fragment since any other messages outside the loop would only add traces to the positive traces set and not result in new interaction obligation.

The loop fragment could occur from 0 to n times.

When $n = 0$ there would be only 1 interaction obligation since there is no xalt construct.

When $n = 1$ there would be 2 interaction obligation as showed in previous question.

When $n = 2$, the resulting sequence diagram could be considered as an alt construct with either 0, 1 and 2 xalt constructs in each operand. Hence, resulting in 8 interaction obligation (the operand #0 is not included since it has no trace):

$$\begin{aligned}
 & xalt1 \text{ alt } [[xalt2 \text{ seq } xalt3]] = \\
 & \underbrace{\{(p1, n1), (p2, n2)\}}_{\text{operand \#1}} \uplus \underbrace{\left\{ \{(p1, n1), (p2, n2)\} \succ \{(p1, n1), (p2, n2)\} \right\}}_{\text{operand \#2}} = \\
 & \underbrace{\{(p1, n1), (p2, n2)\}}_{2^1} \underbrace{\uplus}_{\times} \underbrace{\left\{ \left\{ ((p1, n1) \succ (p1, n1)), ((p1, n1) \succ (p2, n2)), ((p2, n2) \succ (p1, n1)), ((p2, n2) \succ (p2, n2)) \right\} \right\}}_{2^2=2^3 \text{ interaction obligations}}
 \end{aligned}$$

Similarly, $n = 3$ would result in $2^1 \times 2^2 \times 2^3 = 2^6$ interaction obligations. Generally, there would be $2^{\frac{n(n+1)}{2}}$ ($n \geq 0$) interaction obligations. (in order words, there would be infinitely many interaction obligations).