UiO : Department of Informatics
University of Oslo

IN5230
Electronic noise – Estimates and countermeasures

Lecture 10 (Mot 7)

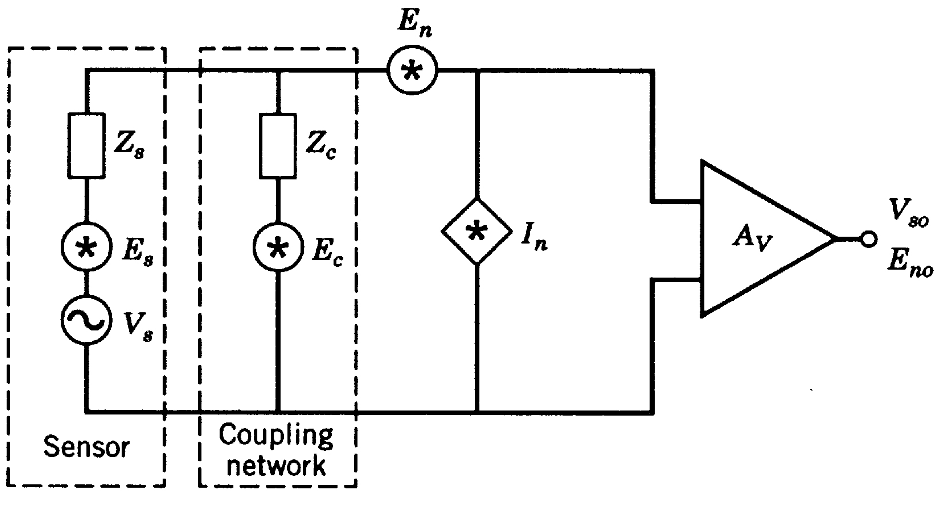
Modelling system noise

- Modelling of noise must include:
  - Sensors
  - Bias and coupling network
  - Amplifiers

- We use our standard method:
  1. Determine the total noise at output: *Eno*
  2. Determine the system gain: *Kt*
  3. Divide *Eno* with *Kt*: $Eni^2 = Eno^2/Kt^2$

A general voltage noise model

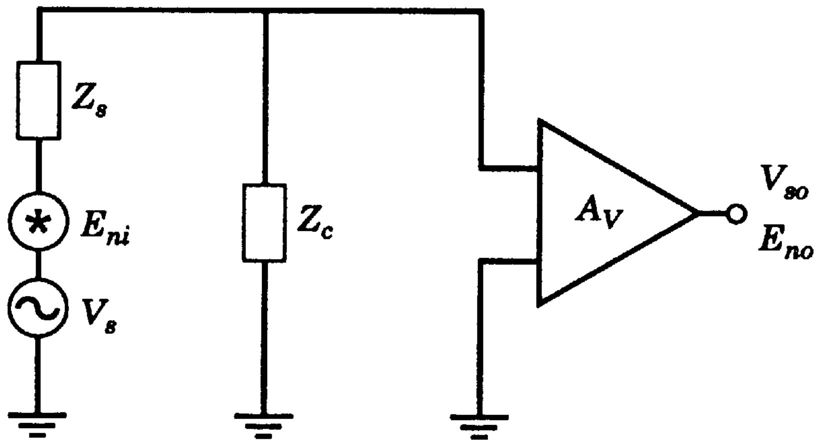
- Equivalent noise voltage at the input – general expression:



(a)

$$E_{ni}^2 = A^2 E_S^2 + B^2 E_n^2 + C^2 I_n^2 Z_S^2 + D^2 E_C^2$$

  - A, B, C and D are functions of resistors, capacitors, coils etc and not functions of currents or voltages.



(b)

A general current noise model

- Equivalent noise current at the input – general expression:

$$I_{ni}^2 = J^2 I_{ns}^2 + \frac{K^2 E_n^2}{Z_S^2} + L^2 I_n^2 + \frac{M^2 E_C^2}{Z_C^2}$$

- Neither J, K, L nor M are functions of voltage or current.
- It is irrelevant whether one calculates the equivalent input noise voltage or current.

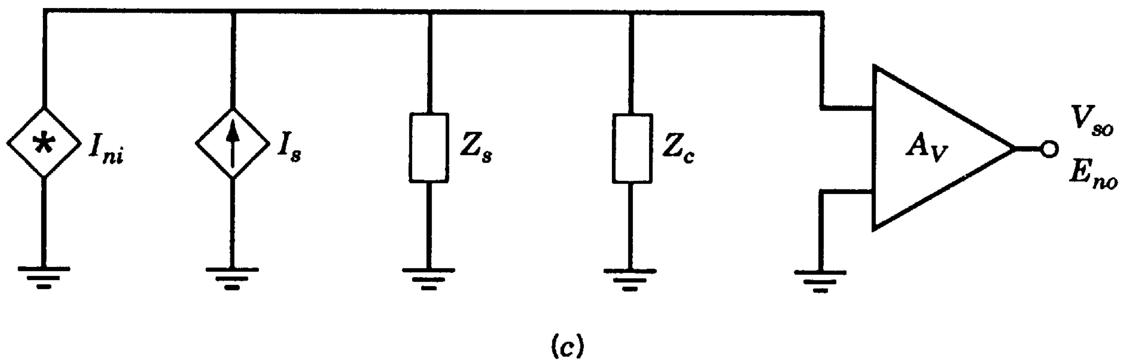
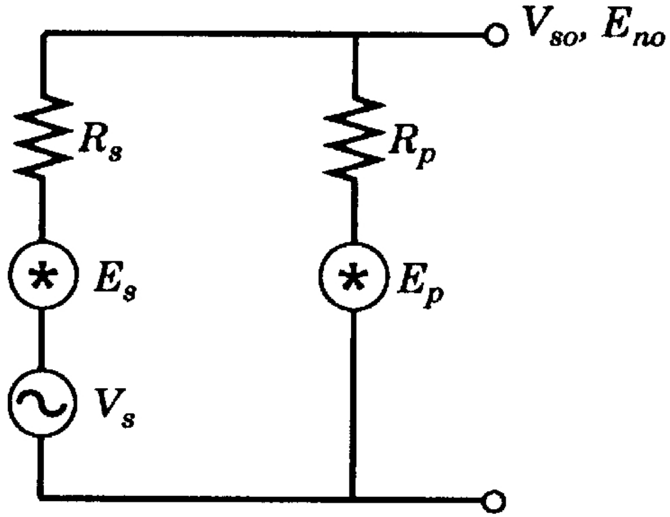


Figure 7-1 System noise model.

Effect of parallel load resistance 1/2

- Input (i.e. without Rp):

$$\frac{S}{N} = \frac{V_{so}^2}{E_{no}^2} = \frac{V_S^2}{E_S^2}$$



- Output (i.e. with Rp).

$$V_{so} = \frac{R_p}{R_S + R_P} V_S$$

$$E_{no}^2 = \left(\frac{R_P}{R_S + R_P} E_S \right)^2 + \left(\frac{R_S}{R_P + R_S} E_P \right)^2$$

$$\frac{S_{ut}}{N_{ut}} = \frac{V_{so}^2}{E_{no}^2} = \frac{\left(\frac{R_p}{R_s + R_p} \right)^2 V_s^2}{\left(\frac{R_p}{R_s + R_p} \right)^2 E_s^2 + \left(\frac{R_s}{R_p + R_s} \right)^2 E_p^2} = \frac{V_s^2}{E_s^2 + \left(\frac{R_s}{R_p} \right)^2 E_p^2}$$

Effect of parallel load resistance 2/2

$$\frac{S_{ut}}{N_{ut}} = \frac{V_{so}^2}{E_{no}^2} = \frac{\left(\frac{R_p}{R_s + R_p}\right)^2 V_s^2}{\left(\frac{R_p}{R_s + R_p}\right)^2 E_s^2 + \left(\frac{R_s}{R_p + R_s}\right)^2 E_p^2} = \frac{V_s^2}{E_s^2 + \left(\frac{R_s}{R_p}\right)^2 E_p^2}$$

When $R_s = R_p$ then $E_s = E_p$ and we get that $(S/N)_{ut} = \frac{1}{2}(V_s^2/E_s^2) = \frac{1}{2}(S/N)_{inn}$. R_p equally reduces V_s and E_s but contribute in addition with its own noise. When $R_s \gg R_p$ decreases $(S/N)_{ut}$ towards zero, while when $R_s \ll R_p$ will $(S/N)_{ut}$ increase towards $(S/N)_{inn}$ which is the best that can be achieved.

Calculation with amplifier noise

1. Noise at output

$$E_{no}^2 = E_S^2\left(\frac{R_p}{R_S + R_p}\right)^2 + E_n^2 + I_n^2\left(R_p \parallel R_S\right)^2 + I_{np}^2\left(R_p \parallel R_S\right)^2$$

2. System gain

$$K_t = \frac{R_p}{R_S + R_p}$$

3. Equivalent input noise

Yellow: Equation

$$E_{ni}^2 = \frac{E_{no}^2}{K_t^2} = E_S^2 + \left(\frac{R_S + R_p}{R_p}\right)^2 E_n^2 + \left(I_n^2 + I_{np}^2\right)R_S^2$$

We compare with our well-known equation:

$$E_{ni}^2 = \frac{E_{no}^2}{K_t^2} = E_s^2 + E_n^2 + I_n^2 R_S^2$$

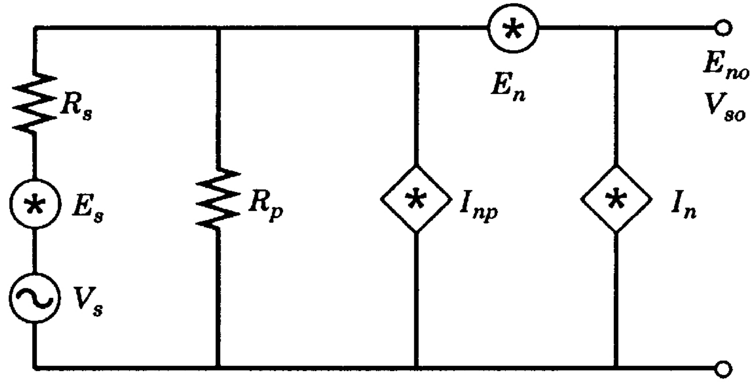


Figure 7-3 Amplifier and sensor models with shunt resistance.

- Terms in front of E_n: If $R_p \ll R_s$ we have that E_n will contribute a lot. If $R_s = R_p$ the contribution from E_n will be equal to $4E_n^2$. If $R_p \gg R_s$ contributes E_n with only E_n^2
- $I_{np}^2 R_S^2$ is a new term. This is the thermal noise in R_p.

Increased Rp & Vb

1)

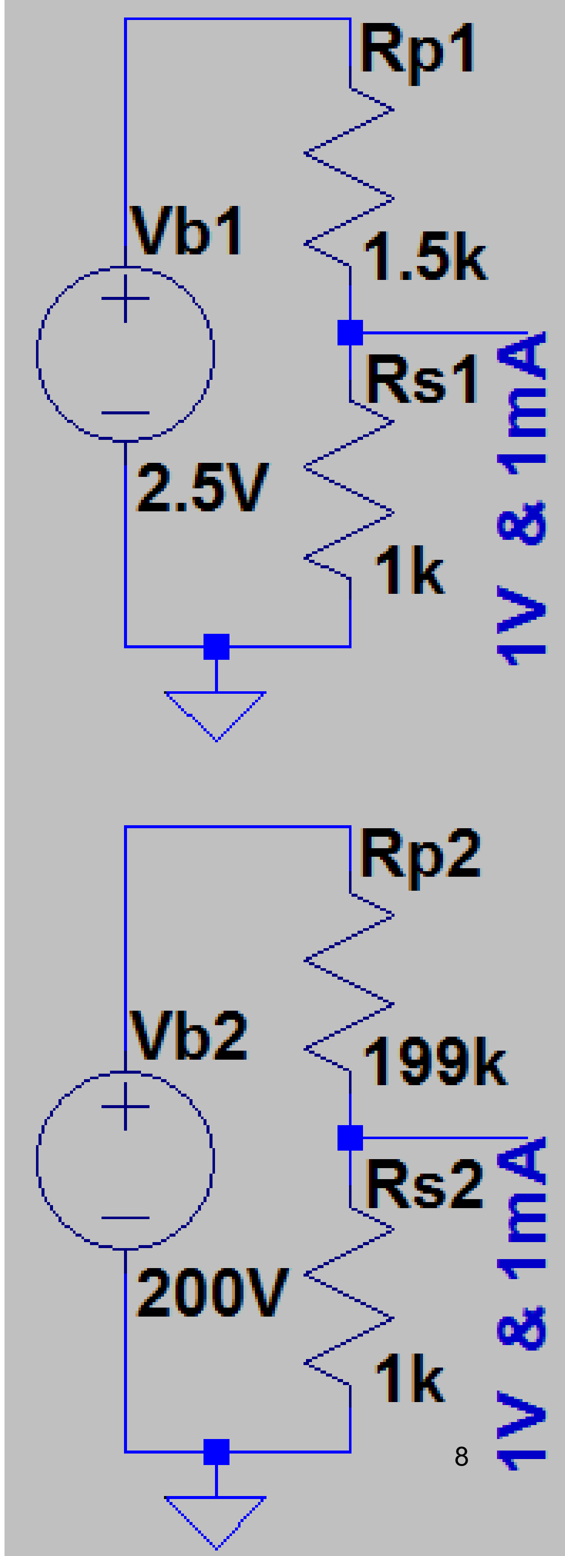
Rp must be made as large as possible. If a certain voltage is required over *Rs* or a certain current through *Rs* we may change the bias voltage accordingly.

Example: R_S=1kΩ, R_p=1.5kΩ and V_B=2.5V. If we change to new values R_p=199kΩ and V_B=200V the voltage over the sensor and the current through the sensor remains the same but the noise is significantly reduced.

2)

Alternatively, maybe it is possible to use a coil instead of R_p?

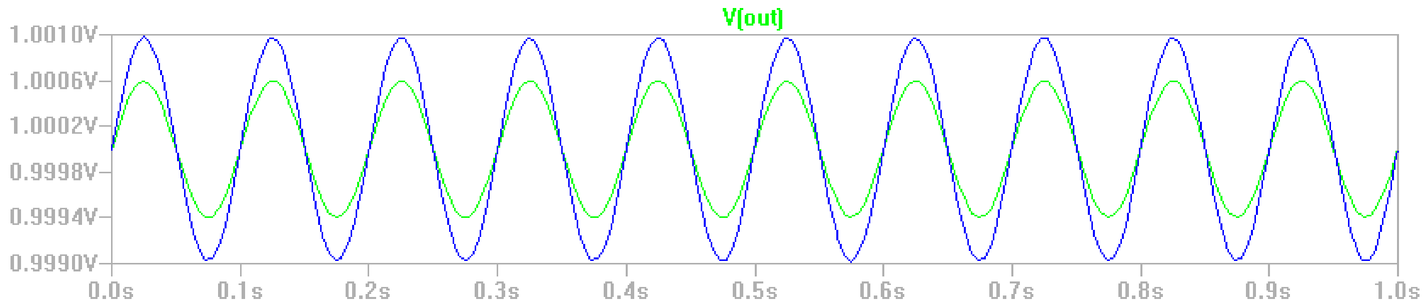
$$E_{ni}^2 = E_n^2\left[\frac{R_S}{j\omega L}+1\right]^2 + I_n^2 R_S^2 + E_S^2$$

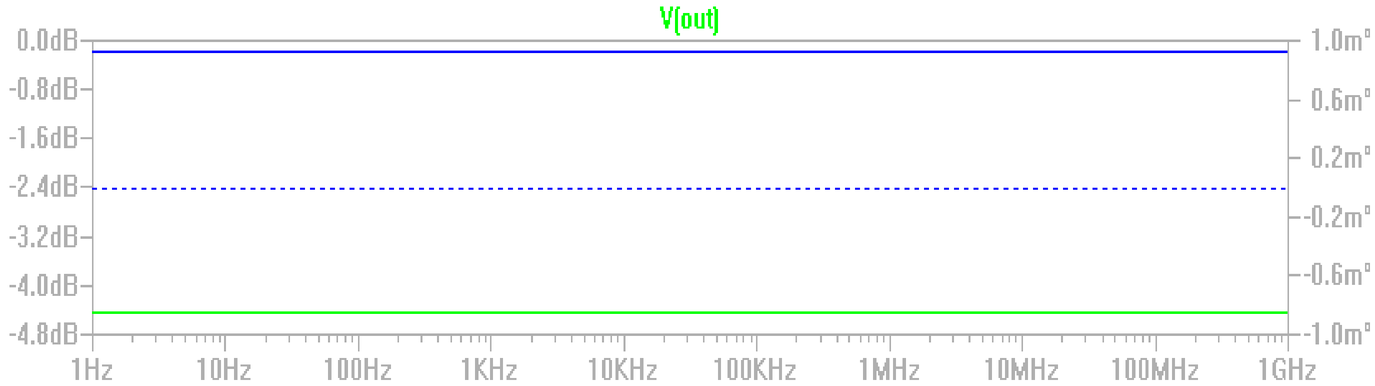


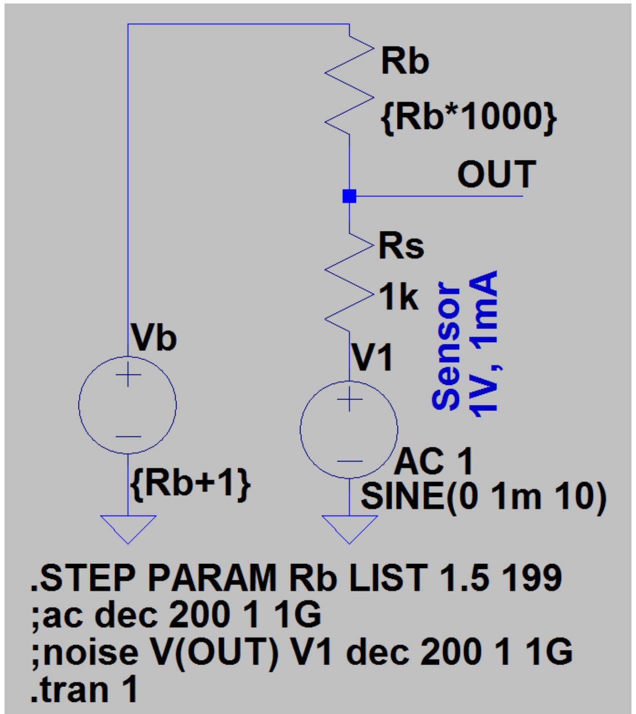
1kΩ sensor requiring 1V and 1mA

Green: 1.5k, 2.5V

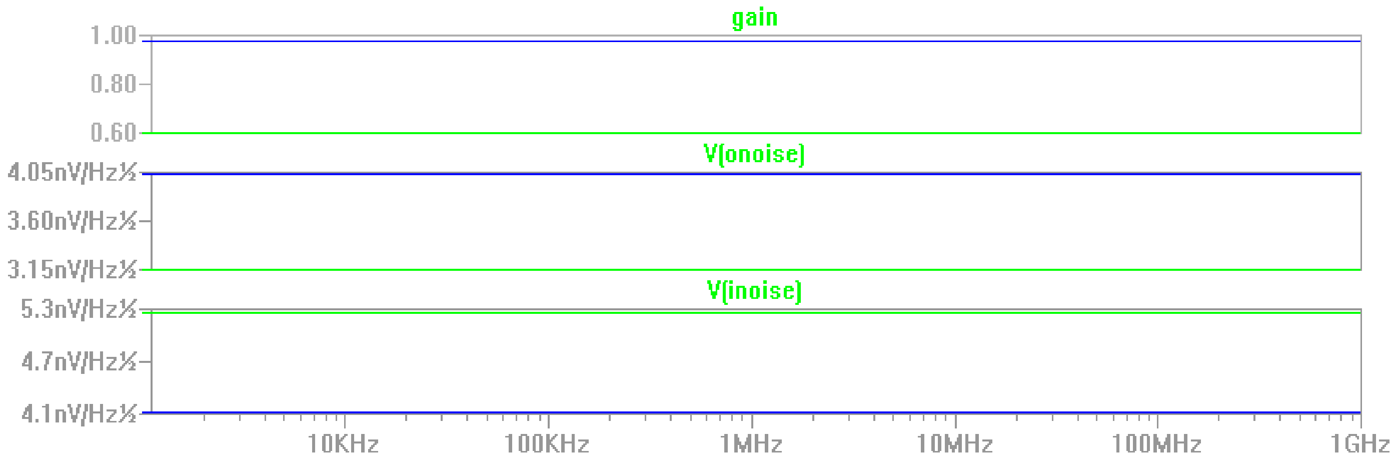
Blue: 199k, 200V







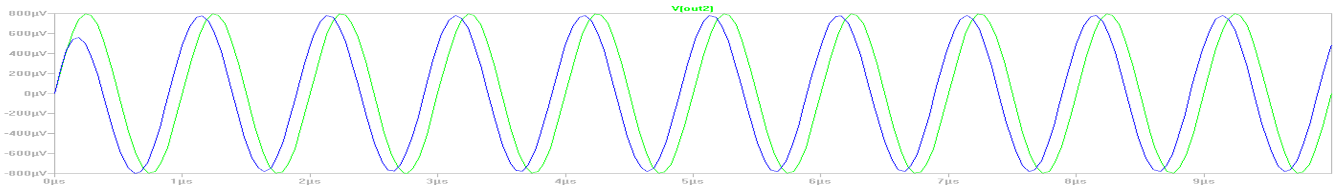
	1.5kΩ, 2.5V	49kΩ, 50V
AC gain	-4.4dB	-0.2dB
Noise gain	0.60	0.98
Noise out	3.2nV/√Hz	4.0nV/√Hz
Noise eq. input	5.3nV/√Hz	4.1nV/√Hz

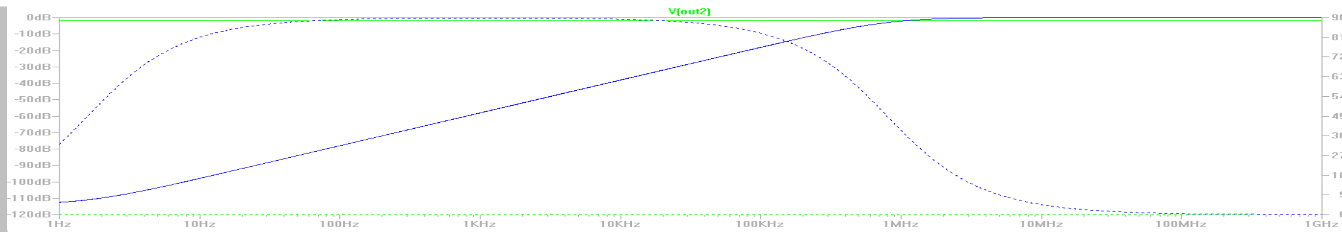


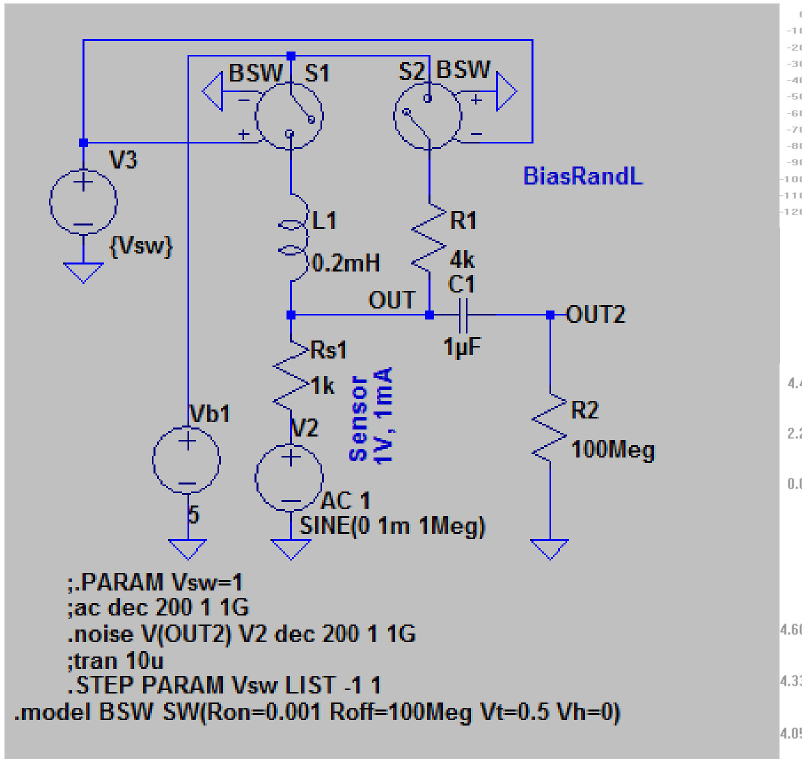
Bias impedance: R and L

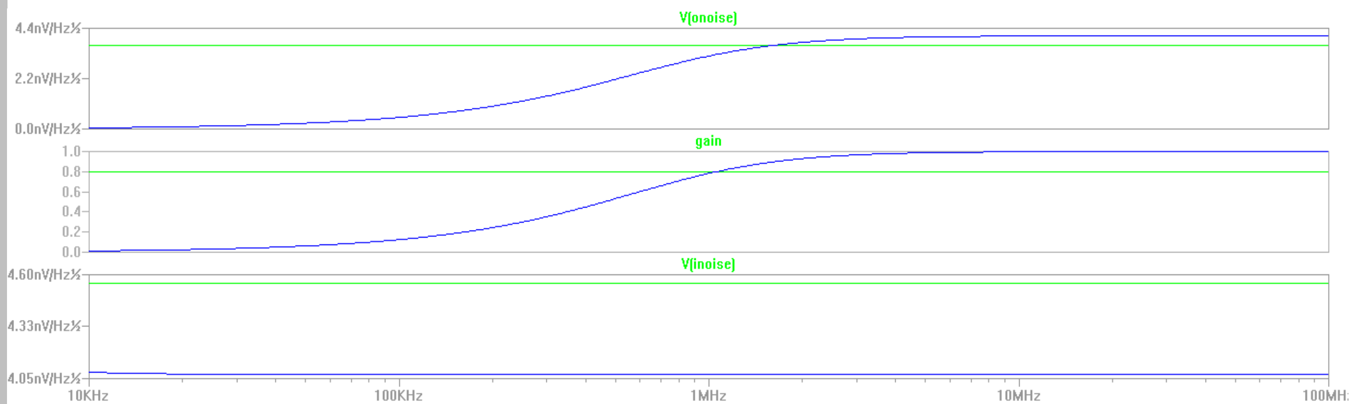
- 1MHz signal frequency
- Load R or L
- Simulations:
  - Transient
  - AC
  - Noise
- (HP-filter on output to get same DC)

At 1MHz	R 4kΩ	L 200µH
Gain	0.80	0.78
V(onoise)	3.6nV/√Hz	3.2nV/√Hz
V(inoise)	4.6nV/√Hz	4.1nV/√Hz



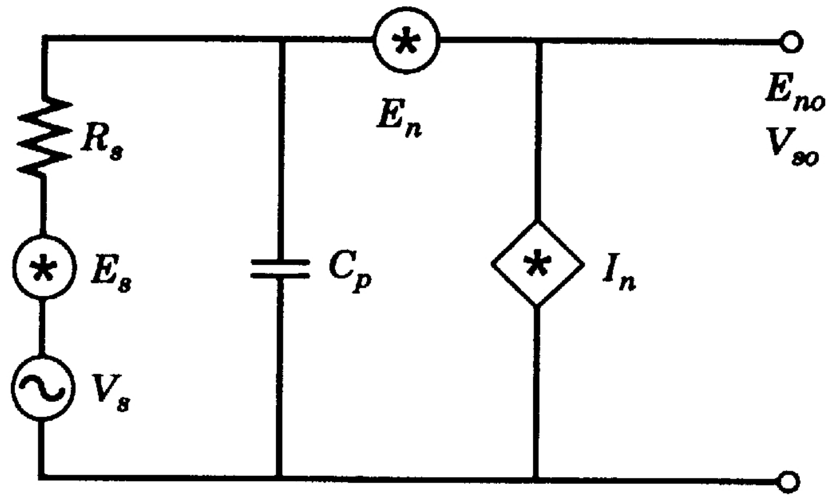






Effect of shunt capacitances

Here we have replaced the resistance Rp with a capacitor Cp



1)

$$E_{no}^2 = E_S^2\left(\frac{\frac{1}{j\omega C_p}}{R_S + \frac{1}{j\omega C_p}}\right)^2 + E_n^2 + I_n^2\left(\frac{R_S\frac{1}{j\omega C_p}}{R_S + \frac{1}{j\omega C_p}}\right)^2 =$$

$$E_S^2\left(\frac{1}{R_s^2 C_p^2 \omega^2 + 1}\right) + E_n^2 + I_n^2\left(\frac{R_S^2}{R_S^2 C_p^2 \omega^2 + 1}\right)$$

2)

$$K_t^2 = \left(\frac{\frac{1}{j\omega C_p}}{R_S + \frac{1}{j\omega C_p}}\right) = \frac{1}{R_S^2 C_p^2 \omega^2 + 1}$$

3)

$$E_{ni}^2 = \frac{E_{no}^2}{K_t^2} = E_S^2 + E_n^2\left(R_S^2 C_p^2 \omega^2 + 1\right) + I_n^2 R_S^2$$

Yellow: Equation

We compare with our well-known expression:

$$E_{ni}^2 = \frac{E_{no}^2}{K_t^2} = E_s^2 + E_n^2 + I_n^2 R_S^2$$

$\Rightarrow$ E_n^2 is multiplied by $(R_s^2 C_p^2 \omega^2 + 1)$. *Note !* $R_s^2 C_p^2 \omega^2$ will often be substantially less than 1.

Only the *En²* contribution increases.
NB! *Cp* is no noise source!
Cp is not the input capacitance of the amplifier. This is included in *En, In* and *Kt*.

Noise in a resonant circuit

1) Noise at output

$$E_{no}^2 = E_S^2\left(\frac{X_{L_P} \parallel X_{C_p}}{R_S + X_{L_P} \parallel X_{C_p}}\right)^2 + E_n^2 + I_n^2\left(R_S \parallel X_{L_p} \parallel X_{C_p}\right)^2$$

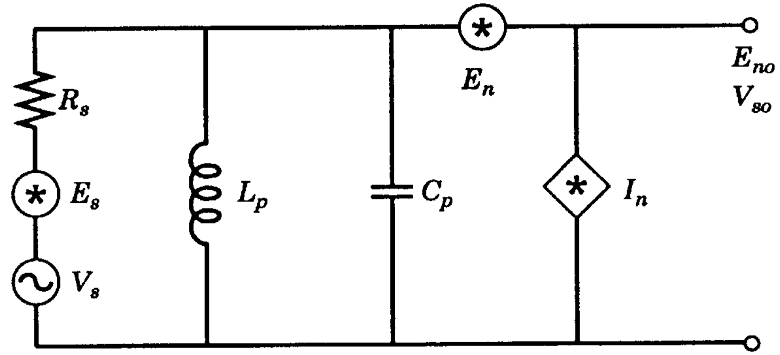


Figure 7-5 Resonant sensor equivalent circuit.

Calculation of parts:

$$\frac{X_{L_p} \parallel X_{C_p}}{R_S + X_{L_p} \parallel X_{C_p}} = \frac{1}{R_S\left(j\omega C_p + \frac{1}{j\omega L_p}\right) + 1} = \frac{1}{R_S\left(\frac{1-\omega^2 C_p L_p}{j\omega L_p}\right) + 1} = \frac{j\omega L_p}{j\omega L_p + R_s - \omega^2 L_p C_p R_S}$$

$$R_S \parallel X_{L_p} \parallel X_{C_p} = \frac{1}{\frac{1}{R_S} + \frac{1}{j\omega L_p} + j\omega C_p} = \frac{j\omega R_S L_p}{j\omega L_p + R_S - \omega^2 R_S C_p L_p}$$

$$E_{no}^2 = E_S^2\left(\frac{1}{R_S\left(j\omega C_p + \frac{1}{j\omega L_p}\right) + 1}\right)^2 + E_n^2 + I_n^2\left(\frac{1}{\frac{1}{R_S} + \frac{1}{j\omega L_p} + j\omega C_p}\right)^2$$

2) Signal gain

$$K_t^2 = \left(\frac{X_{L_p} \parallel X_{C_p}}{R_S + X_{L_p} \parallel X_{C_p}}\right)^2 = \left(\frac{1}{R_S\left(jC_p\omega + \frac{1}{jL_p\omega}\right)+1}\right)^2$$

3) Equivalent input noise

$$E_{ni}^2 = \frac{E_{no}^2}{K_t^2} = E_S^2 + E_n^2\left|1 + \frac{R_S\left(1-\omega^2 C_p L_p\right)}{j\omega L_p}\right|^2 + I_n^2\left(\frac{R_S\left(j\omega C_p + \frac{1}{j\omega L_p}\right)+1}{\frac{1}{R_S} + \frac{1}{j\omega L_p} + j\omega C_p}\right)^2 =$$

Yellow:
Equation

$$E_S^2 + E_n^2\left|1 + \frac{R_S\left(1-\omega^2 C_p L_p\right)}{j\omega L_p}\right|^2 + I_n^2 R_S^2$$

⇒The *In*-coefficient is independent of frequency

⇒The *En*-coefficient will have a weight larger than 1 except at resonance. At the resonance *($\omega^2CpLp=1$)* is the reactance element 0 and the coefficient equal to 1. We will then end up with our well-known expression (without *Cp* and *Lp*).

⇒*Lp* and *Cp* does not contribute itself but affects others.

•

The gain (top figure) is largest at resonance frequency. That is also the case for the noise output (in the middle). However, the equivalent noise at the input (bottom) is lowest at the resonance. This means that the signal is amplified more than the increase in noise at resonance.

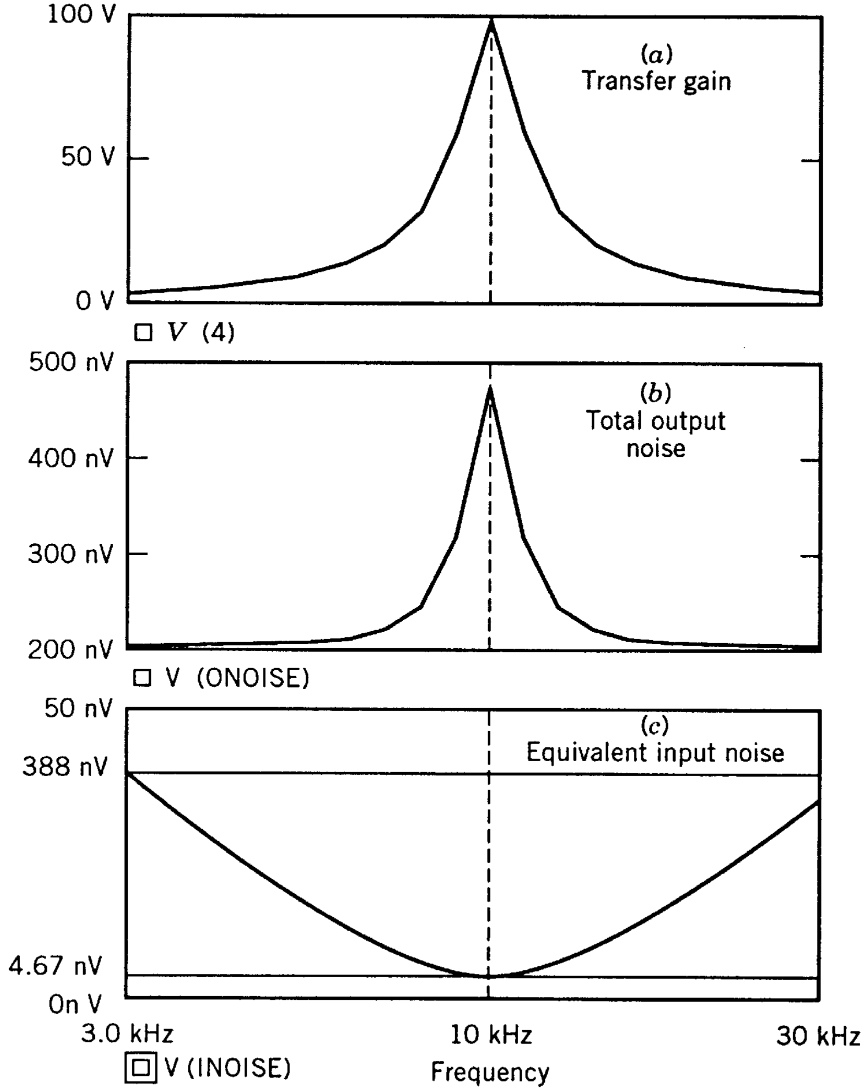


Figure 7-7 Plot of noise for *RLC* model.

Summary

- Resistance in parallel: $$E_{ni}^2 = E_S^2 + \left(\frac{R_S}{R_P} + 1\right)^2 E_n^2 + \left(I_n^2 + I_{np}^2\right) R_S^2$$
- Capacitor in parallel: $$E_{ni}^2 = E_S^2 + \left(R_S^2 C_p^2 \omega^2 + 1\right) E_n^2 + I_n^2 R_S^2$$
- Coil in parallel: $$E_{ni}^2 = E_S^2 + \left(\frac{R_S^2}{\omega^2 L^2} + 1\right) E_n^2 + I_n^2 R_S^2$$
- *Xs* in signal path and *Xp* in parallel: $$E_{ni}^2 = E_S^2 + \left(\frac{R_S + X_S}{X_p} + 1\right)^2 E_n^2 + \left(R_S + X_S\right)^2 I_n^2$$

Yellow: Equations

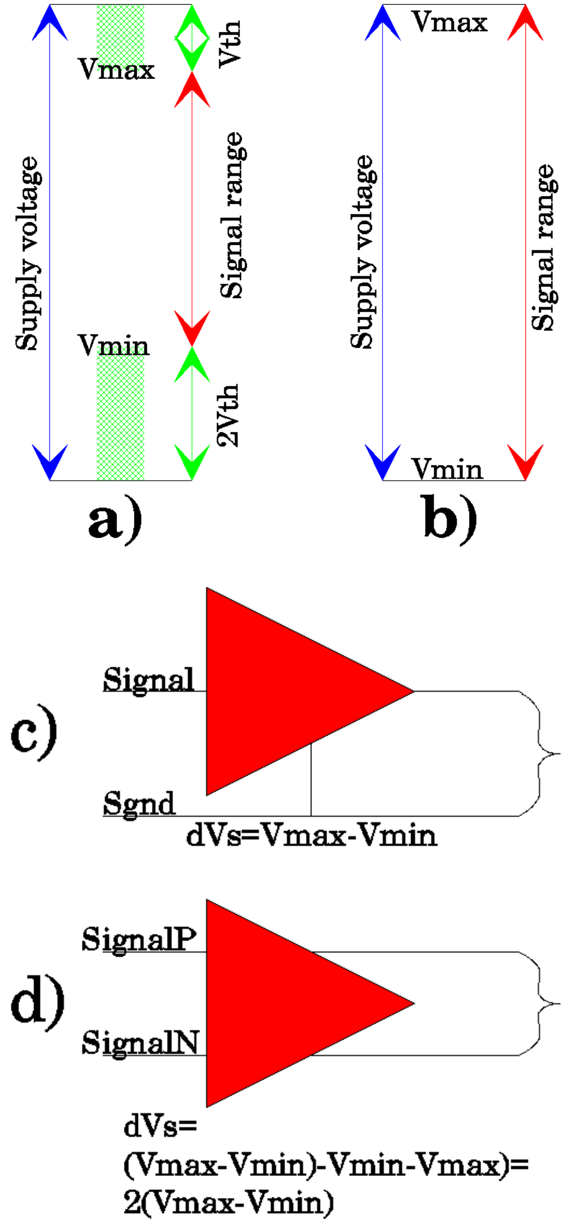
Xs and *Xp* are combinations of capacitances and coils in series and/or parallel (No resistances!)

- *XLC* is a coil and a capacitor in parallel: $$X_{LC} = \frac{j\omega L \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1 - \omega^2 LC}$$

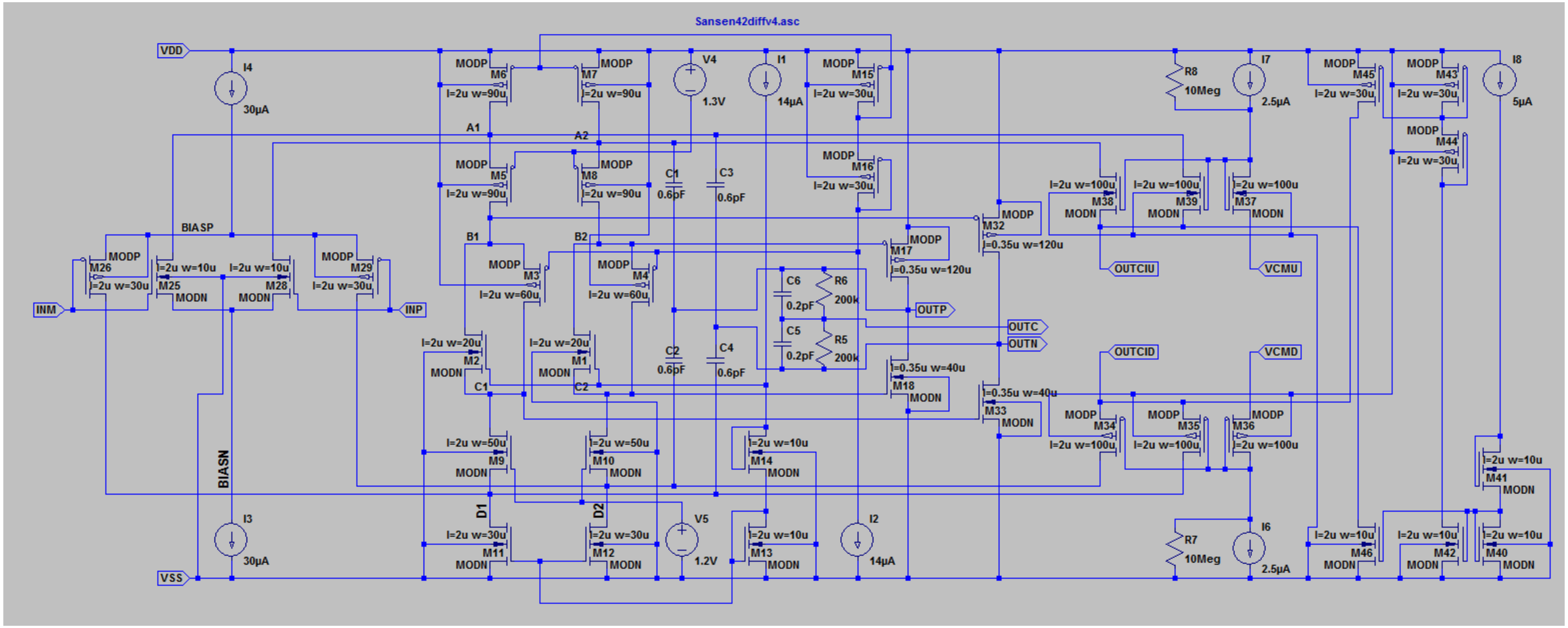
Red: All figures

When we can not reduce the noise…

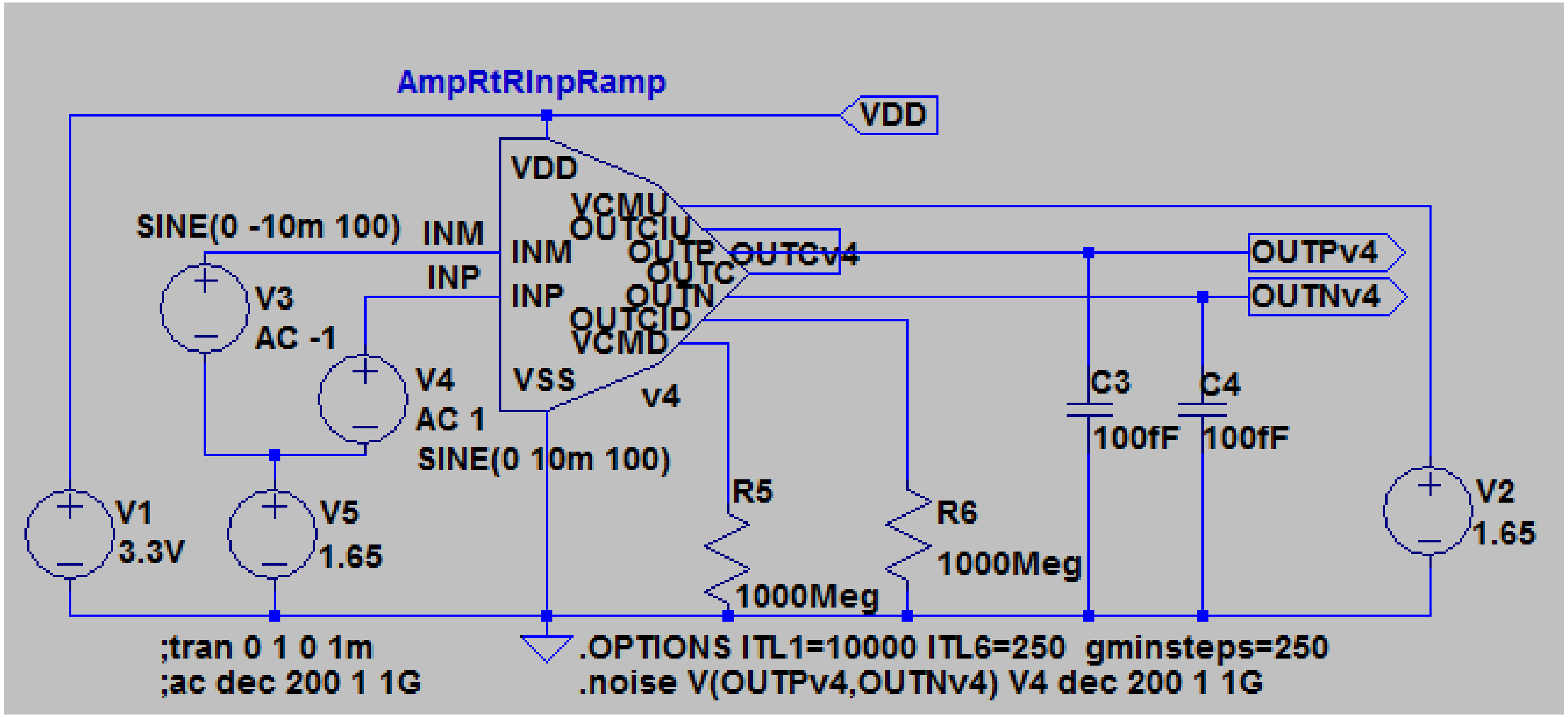
- Large SNR and DR is achieved either by reducing the noise or increasing the signal. Is it any potential for increasing the maximum signal range?
- Make the signal range approach the supply voltage range
  - The signal range in traditional amplifiers is only a limited region of the voltage supply region a)
  - Rail-to-rail amplifiers approach the full supply voltage range. However these amplifiers are much more complex. b)
- Use differential amplifiers
  - Differential amplifiers has twice the signal range (and better CMRR) but the component noise increases (from √2 and upwards).



Rail-to-rail diff in/out amplifier



Rail-to-rail test bench



Rail-to-rail amplifier

- Vout_cm=1.65, Vin_cm=0-3.3V
- Vin_cm=1.65, Vout_cm=0-3.3V
- AC (Vout_cm=Vin_cm=1.65V)
- Noise (Vout_cm=Vin_cm=1.65V)