UiO : Department of Informatics
University of Oslo

IN5230
Electronic noise – estimates and countermeasures

Lecture no 5

Component noise intro (Mot1)

Definition of noise:

- **Unwanted:**

  Most definitions focus on that noise is unwanted like in Motchenbacher: ”*Any unwanted disturbance that obscures or interfaces with a desired signal*”. In general this is a describing characteristic with some small exception that noise (due to it randomness) is wanted in some random number generators, cryptosystems etc. The presence of noise can also be used to confirm connections/assembly and quality of a system.

- **Unpredictability:**

  Another important feature that one should include when defining noise is that it is unpredictable. In advance it is not possible to say what strength the unwanted signal will have at a specific time in the future. The exception is the low-frequency components that are present also immediately beforehand. But even if one can not predict exactly the undesirable components, one can describe the statistical probability and distribution with respect to amplitude and frequency.

Definition of noise:

Before Motchenbacher discusses component noise, the book briefly discusses coupling noise entitled as “external noise”.

External sources:

- Electrostatic (capacitive coupling)
- Electromagnetic (inductive coupling)
- AC-power/DC-power
- Signal wires
- Radio transmitters
- Electrical storms
- Galactic radiation
- Mechanical vibrations

May be “eliminated” through adequate:

- Shielding
- Filtering
- Altering layout
- Distance
- Make parallel
- Make serial
- Change external components like adding a separate power supply for the front-end amplifiers.

Definition of noise:

Internal sources:

"Noise" in this book means component noise. It is also entitled as "internal noise", "true noise" and "fundamental noise".

It comes from spontaneous fluctuations due to the physics of the devices and materials that make up the electrical system.

Example:

- Thermal noise in resistors (and parasitic resistance in transistors, conductors, coils and capacitors etc.)

More difficult (impossible) to eliminate than coupling noise. However, it is helpful and necessary to estimate the size of this noise.

The resolution of the sensor signal is typically decided by the sensor noise. A common goal is to reduce the electrical noise down to the same level as the sensor noise.

Example: "Snow storm" on TV-screens

- Main cause: Thermal noise in the input amplifier.

Not necessarily the preamplifier that generates most noise but this is (typically) the place where the signal is at its weakest and where the noise has the greatest effect on the signal to noise ratio.

Characteristics of noise

While noise from the 50Hz mains can be very predictable, thermal noise are unpredictable when it comes to amplitude and phase. However we can in both cases find statistical characteristics like the RMS (root-mean-square).

Gaussian noise

Thermal noise and some other kind of noise have a normal (or Gaussian) distribution. The figure shows the normal distribution and what it may look like on an oscilloscope.

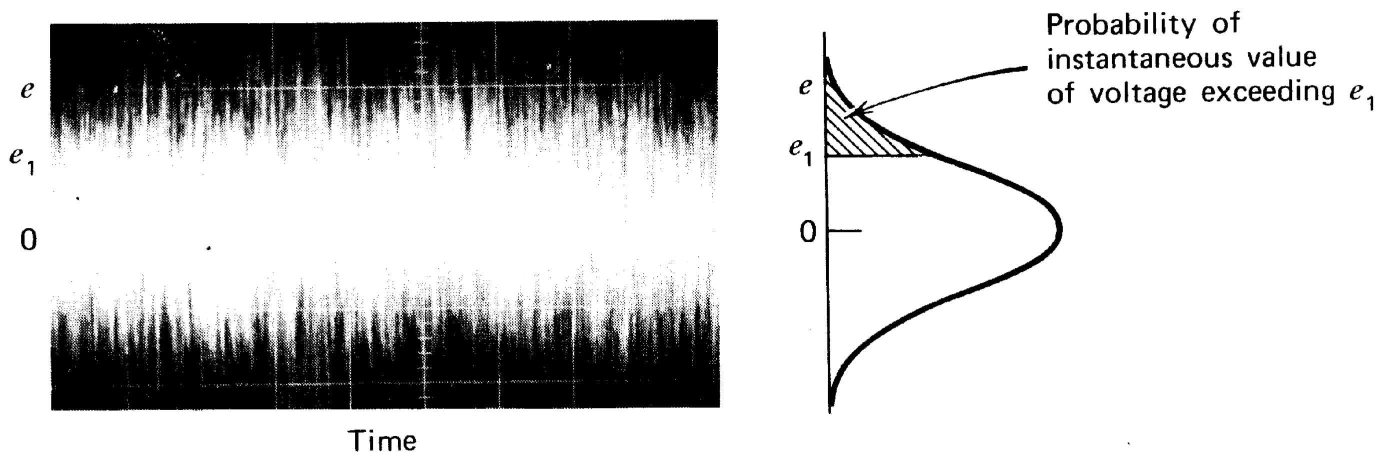


Figure 1-1 Noise waveform and Gaussian distribution of amplitudes.

The normal distribution

The normal distribution describes the probability that the noise has a certain value at a given time. The mathematical expression for the normal distribution is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{(x-\mu)^2}{2\sigma^2}\right]}$$

Here μ is the average value measured. Of all values this is the most probable value. The function *f(x)* is a *probability density function* or for short: *pdf*.

For typical considerations we say that the noise is within +/- 3σ of μ.

	Inside	Outside
[-σ,σ]	68%	32%
[-2σ,2σ]	95%	5%
[-3σ,3σ]	99.7%	0.3%
[-4σ,4σ]	99.994%	0.006%

1) σ is no upper limit and 2) noise: μ=0

σ is no upper limit!

σ tells about the amplitude of the noise but it is not an upper limit! If we want 95% of the noise to be below a limit, we must reduce the noise so that 2σ is within this limit. If we want more, the noise must be reduced further but we will never ensure that all noise will be below a certain limit. If noise may trigger a latch, reset a sensor etc. we should have an idea of how often this may occur. Typical time values used are from a few hours to the life-time of the universe.

Noise: μ=0

In the case of noise, μ is equal to 0. If we find an average value different from zero over some time, this will be something that can be characterised and compensated in hardware or software. Thus, this average value is not unpredictable and hence not a part of what we consider as noise.

RMS (Root-Mean-Square)

RMS is a general term that applies not only to the Gaussian distribution. **The RMS-value for a Gaussian-distribution is equal to σ.**

RMS is defined as:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

V(t): The signal (or noise) voltage as a function of time.

T: the time we are integrating over.

- The integral will grow with T to infinity (as long as V(t) ≠ 0). When we divide by T, we find that the average slope of the integral is equal to the square of Vrms.
- If v(t) has a cyclic behaviour we have to integrate over a whole number of periods for the answer to be completely accurate. If we do not integrate a whole number of periods, the last unfinished period contributes with an error. However this error will be relative to the contribution from all the whole periods. This is useful knowledge if one does not know the exact frequencies to integrate a signal over: Hence instead of integrating over a complete number of periods we integrate for a time that is many times longer than the longest expected period. Then a possible part of a period will give only a small contribution.

RMS: "Effective heating"

The RMS value can be seen as an expression of the effective heating effect of a signal. The RMS value of a random signal is the DC value that will give as rapid heating through a heating element in water as the signal itself.

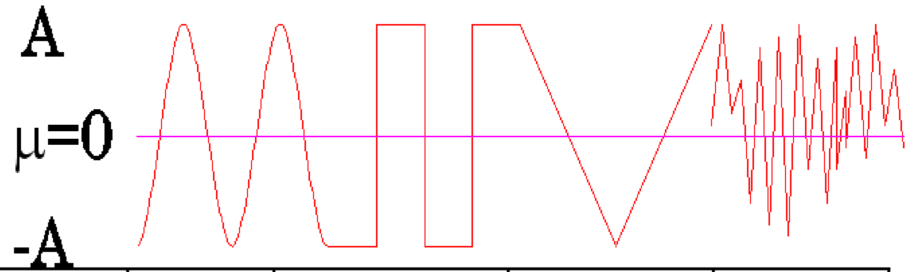
Example: The mains power network
Our mains power network carry sinus voltages with a frequency of 50Hz. The average value is 0V, the peak value is 310V and the RMS value equal to 220V. When we want to find the maximum effect in a 10A-circuit we multiply the 10A with the RMS voltage (220V) and get 2200W. *Vrms* is thus the effective "heating tension" and not eg. peak-voltage.

Volt meters finding wrong RMS

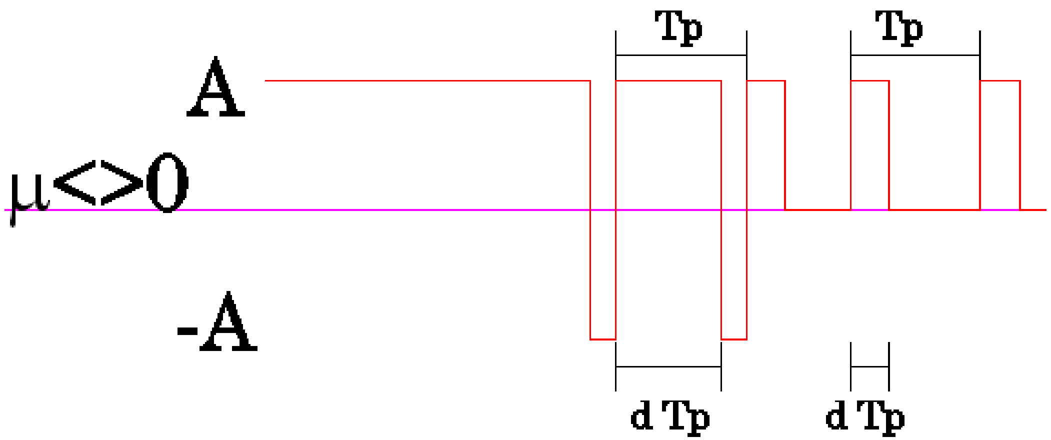
An apropos:

Some voltmeters find the RMS value by dividing the peak value by $\sqrt{2}$. This is only correct when the curve is a sine. In true-RMS voltmeters the RMS value is found according to the expression given ahead.

Examples



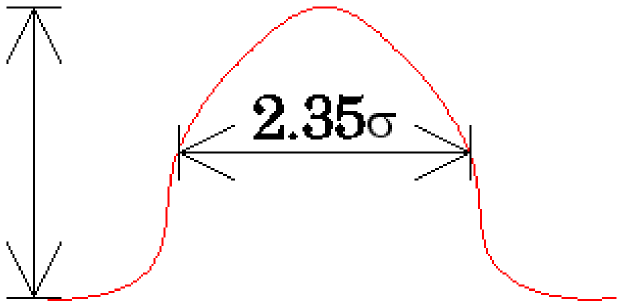
(μ=0)	Sinus	Square (50% duty)	Triangular	Gauss
Vpeak	A	+/-A	+/-A	∞
Vrms	A/√2	A	A/√3	σ



(μ<>0)	DC*	Square High a part *d* of the time	
Vpeak	A	+/-A	A, (0)
Vrms	A	A	A√d
μ	A	A(2d-1)	Ad

FWHM

Within some physic fields the notation Full-Width-Half-Maximum (FWHM) is common.



FWHM means the width of the portion of the signal that has a probability that is more than half of the maximum value. This width is a constant scaling of the standard variation and can be expressed as:

$$fwhm = \sigma\sqrt{8\ln 2} \approx 2{,}35\sigma$$

Thermal noise

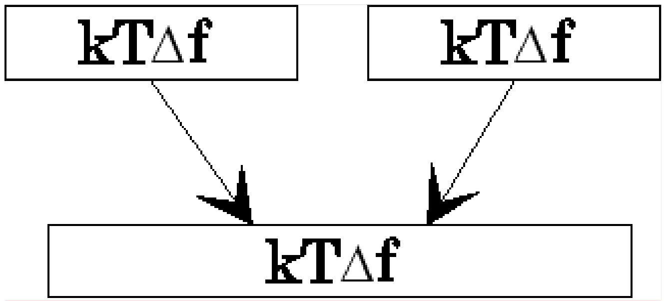
- Due to "Brownian" motion of charges in a conductor.
- First observed by J. B. Johnson in 1927. Theoretically analysed by H. Nyquist in 1928. Also called "Johnson noise" and "Nyquist noise".
- Over time is the average voltage zero but the random motion of charges results in that we at different time points can measure voltage differences over the terminals.

The available noise power in a conductor is: $$N_t = kT\Delta f$$

k: Boltzmans constant: 1.38E-23 Ws/K

T: Temperature in Kelvin

Δf: Bandwidth of the "measurement system"



Thermal noise – Nt examples

1) We assume both:

- Room temperature (17°C or 290K)
- 1Hz bandwidth

$\Rightarrow$ Nt =-204dB relative to 1W

2) In RF communication we give the noise power relative to 1mW and we have:

$$Noise\_power\_in\_dB_m = 10\log_{10}\left(\frac{4\times10^{-21}}{10^{-3}}\right) = -174dB_m$$

-174dBm is generally called the noise floor and is the minimum noise level one can achieve in a system that operates at room temperature.

(1Hz = 1Hz)

Basic equations:

- Power

$$P = UI = \frac{U^2}{R} = RI^2$$

- Energy:

$$W = P \cdot t$$

- Relative signal strength (decibels):

$$RSS_{dB} = 10\log\frac{P}{P_{ref}} = 10\log\frac{U^2/R}{U_{ref}^2/R_{ref}} =$$

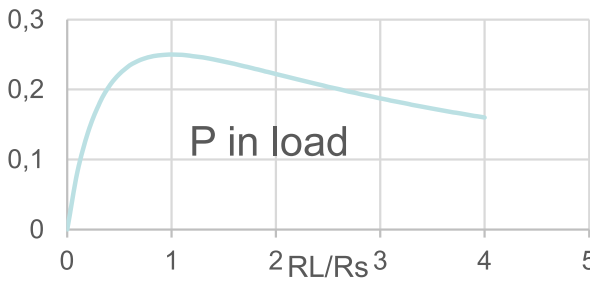
$$10\log\left(\frac{U}{U_{ref}}\right)^2 + 10\log\left(\frac{R_{ref}}{R}\right) =$$

$$20\log\left(\frac{U}{U_{ref}}\right) + 10\log\left(\frac{R_{ref}}{R}\right)$$

Noise voltage

The available noise power is the amount of power a resistive source can supply a noiseless resistive load when both resistances are equal.

I.e. this is a worst case scenario where the resistor is matched for maximum noise power transfer.



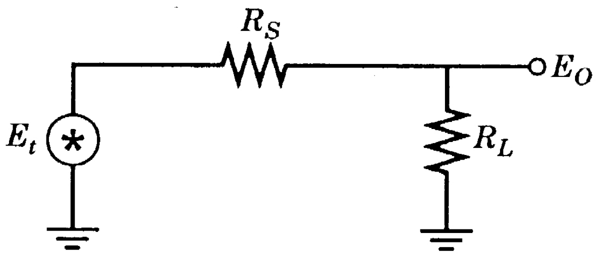


Figure 1-2 Circuit for determination of noise voltage.

Method: Often, the load resistance is not equal to the source resistance. The method used is that we first assume that these are equal, calculates backward to a theoretical voltage at the source resistance and then use this voltage with the actual load resistance.

$$N_t = \frac{E_0^2}{R_L} = \frac{E_t^2}{4R_L} = \frac{E_t^2}{4R_S} = kT\Delta f$$

$$N_t = \frac{E_0^2}{R_L} = \frac{E_t^2}{4R_L} = \frac{E_t^2}{4R_S} = kT\Delta f$$

We make an expression where the deposed power on the theoretical load is expressed only by values in relation to the source:

$$E_t = \sqrt{4kTR\Delta f}$$

$$4kT = 1.61 \times 10^{-20} \_(at \_ 290K)$$

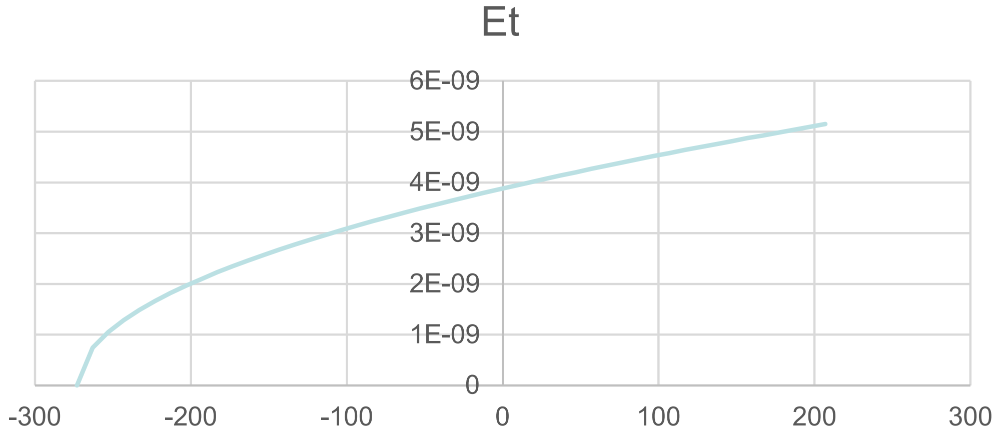
Example:

- **1kΩ, 1Hz and 290K (17°C)**

⇒ 4nV/√Hz

(5kΩ ? ⇒ Multiply by √5,

1000Hz? ⇒ Multiply by √1000)



(For those especially interested)

A more accurate expression is:

$$E_t^2 = 4kTR\Delta f \cdot p(f)$$

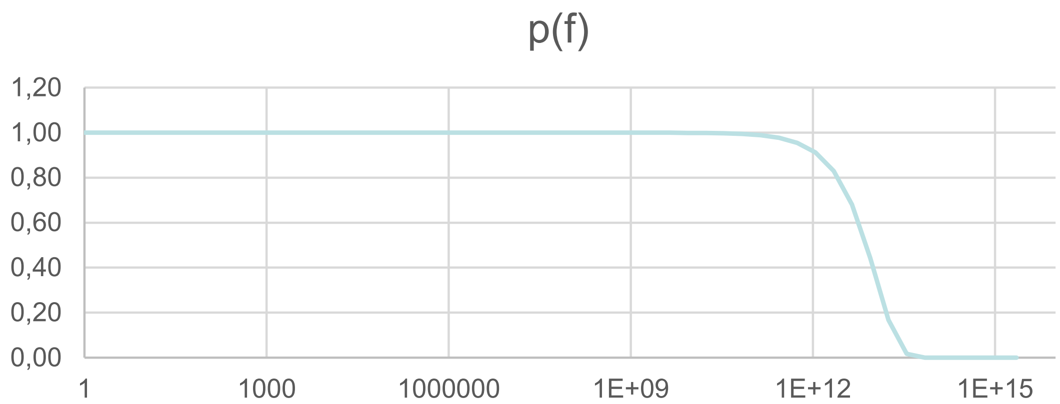
$$p(f) = \left(hf / kT\right) / \left(e^{(hf/kT)} - 1\right)$$

where p(f) ≈1 at room temperature with standard frequencies.

P(f) starts to decrease towards zero when

- The frequency is above 1THz or
- The temperature is below milli Kelvins.

I.e. in all practical cases p(f) can be ignored.



Noise bandwidth

Signal Bandwidth ≠ Noise Bandwidth

Signal Bandwidth: The frequency range with an attenuation of less than -3dB of the centre or maximum signal.

Noise Bandwidth: The noise bandwidth is set so that the product of the noise bandwidth and the maximum noise signal is equal to an area. The area is equal to the integral of the noise integrated over all frequencies. This can be expressed with the formula:

$$\Delta f = \frac{1}{G_0}\int_0^\infty G(f)df$$

Here is:

G(f) noise power as a function of frequency.

G_0: maximum noise effect.

Δf: noise bandwidth..

However since often the voltage is measured instead of the power it can be useful to express the noise bandwidth as a function of the noise voltage:

$$\Delta f = \frac{1}{A_{vo}^2}\int_0^\infty \left|A_v(f)\right|^2 df$$

Examples of "bandwidths"

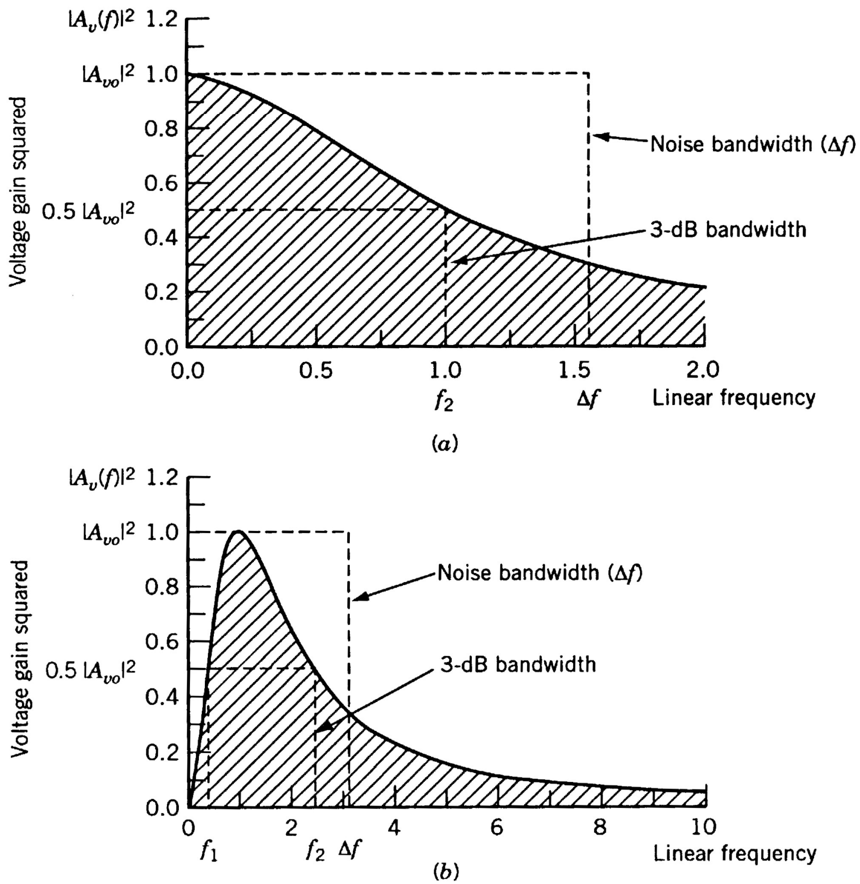


Figure 1-3 Definition of noise bandwidth.

a) Low pass filter

b) Band pass filter

NB ! The scale on the frequency axis is linear.

Calculations for low pass filter

Filter function for first order low pass filter: $$A_v(f) = \frac{1}{1 + jf / f_2}$$

f_2 *is the* -3dB frequency
Normalized: Gain equal to 1 at DC.

Size of gain: $$\left|A_v(f)\right| = \frac{1}{\sqrt{1 + (f / f_2)^2}}$$

Interprets the signal as a noise signal: $$\Delta f = \int_0^\infty \frac{df}{1 + (f / f_2)^2}$$

Substitutes $f = f_2 \tan\theta$ and $df = f_2 \sec^2 \theta d\theta$

and get $$\Delta f = f_2 \int_0^{\pi/2} d\theta = \frac{\pi f_2}{2} = 1.571 f_2$$

I.e. interpreted as noise the frequency width is 57.1% greater than if it were an ordinary signal.

Calculation for two first order LP-filters

The total gain is now:

$$A_v(f) = \left| \frac{1}{1 + jf/f_2} \right|^2$$

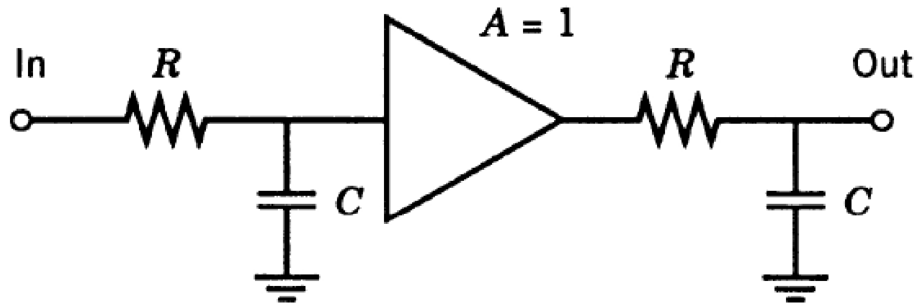


Figure 1-4 Cascaded low-pass filter.

Interpreted as noise the bandwidth is:

$$\Delta f = \int_0^\infty \left| \frac{1}{1 + (f/f_2)^2} \right|^2 df$$

With the same substitutions we get

$$\Delta f = \int_0^{\pi/2} \frac{f_2 d\theta}{1 + \tan^2\theta} = \frac{\pi f_2}{4} = 0.785 f_2$$

However here f2 is the -3dB limit for each step and not for the entire system. We must find the -3dB limit to the system:

$$\frac{1}{\sqrt{2}} = \frac{1}{1 + \left(f_a / f_2\right)^2}$$

This gives us $f_a = 0.6436 f_2$

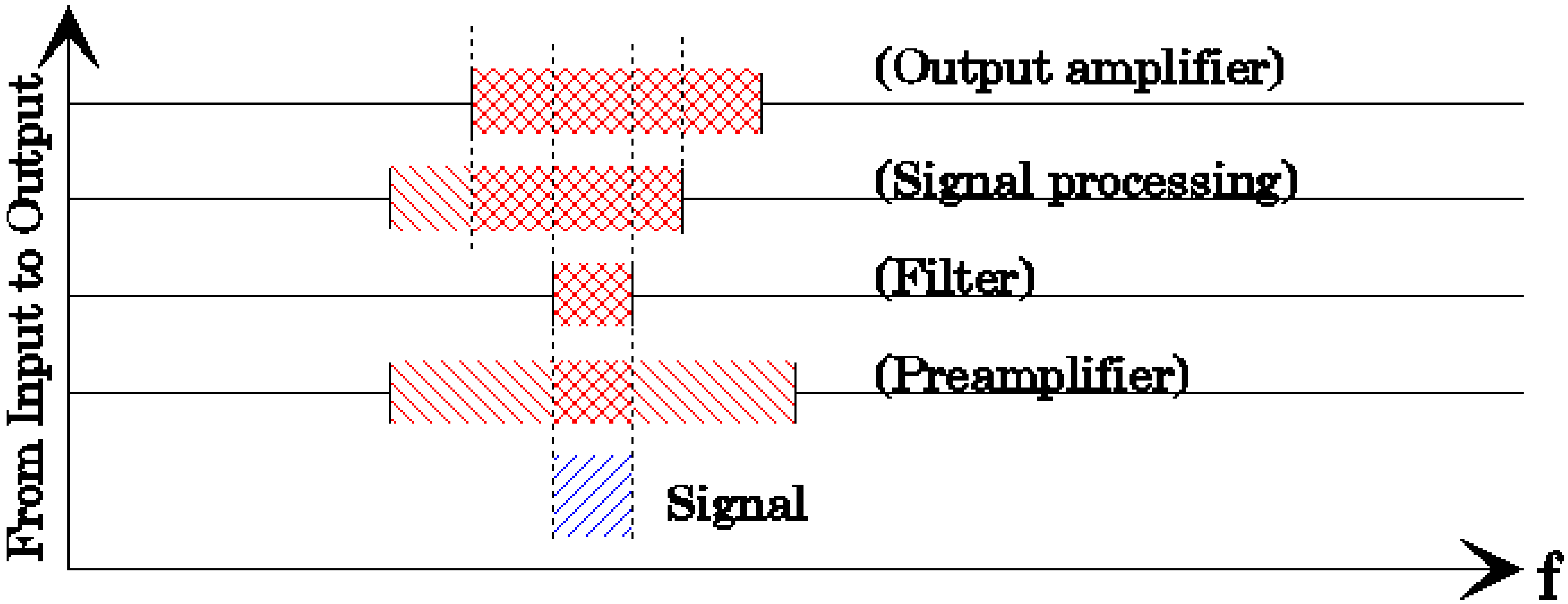
At the end we get: $\Delta f = \frac{\pi f_2}{4} = \frac{\pi f_a}{4 \times 0.6436} = 1.222 f_a$

I.e. Δf (noise) is 1.22 fa (signal).

Conclusion:

With sharper edges (higher order) the "noise bandwidth" will approach "the signal bandwidth“ from above. (The noise bandwidth will always be larger than the signal bandwidth.)

An illustration of how the noise generated at the different steps propagate towards the output.



Correlation

$$V_{Srms}^2 = \frac{1}{T}\int_0^T V_S(t)^2\,dt$$

$$V_S(t) = V_A(t) + V_B(t)$$

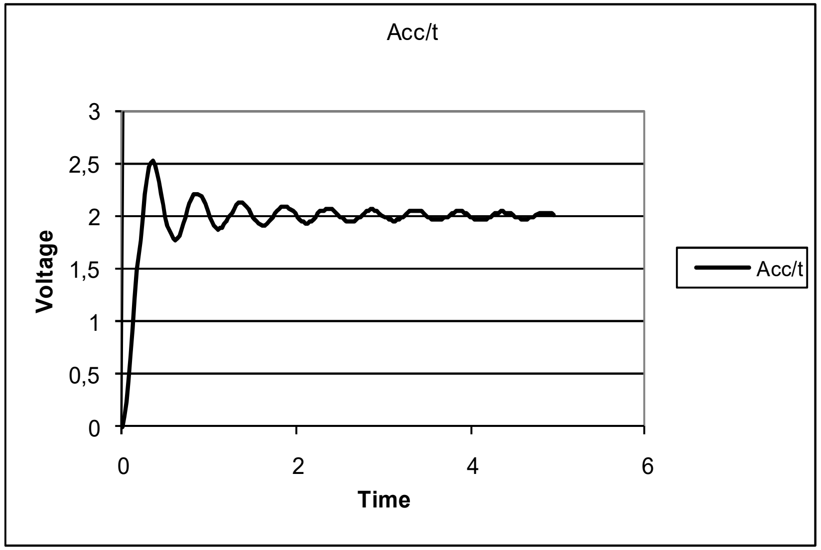
$$(a+b)^2 = a^2 + 2ab + b^2$$

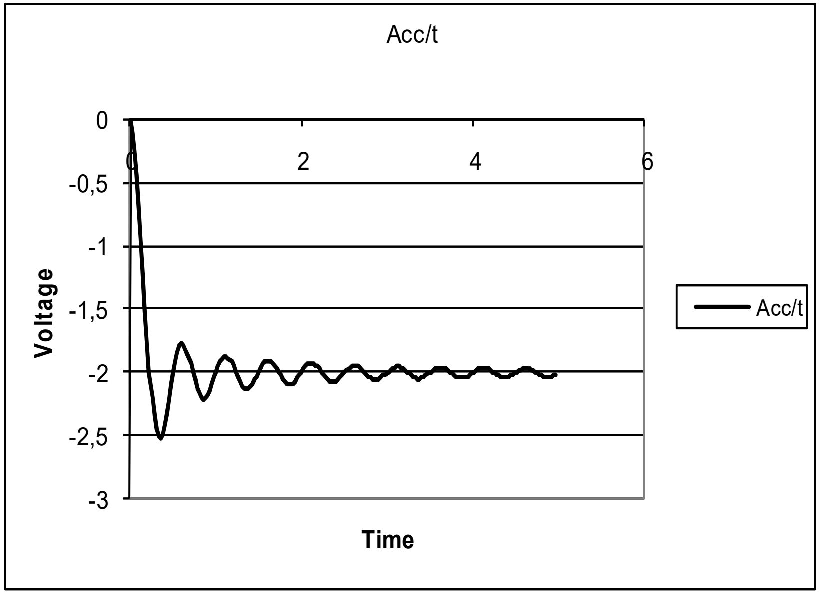
$$V_{Srms}^2 = V_{Arms}^2 + V_{Brms}^2 + \frac{2}{T}\int_0^T V_A(t)V_B(t)\,dt$$

Examples of correlation term (last term):

Top $f_A = f_B$ and in phase:

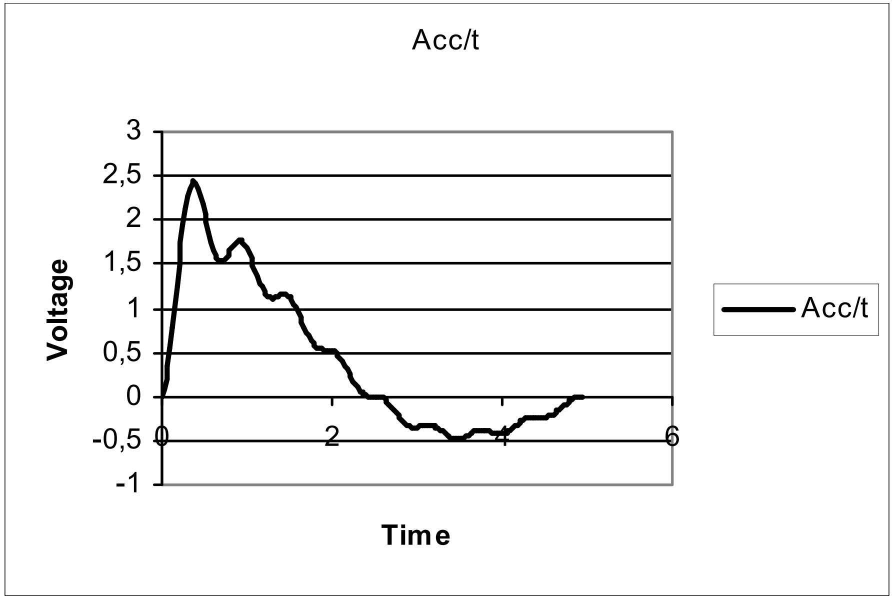
Bottom $f_A = f_B$ but in opposite phase:

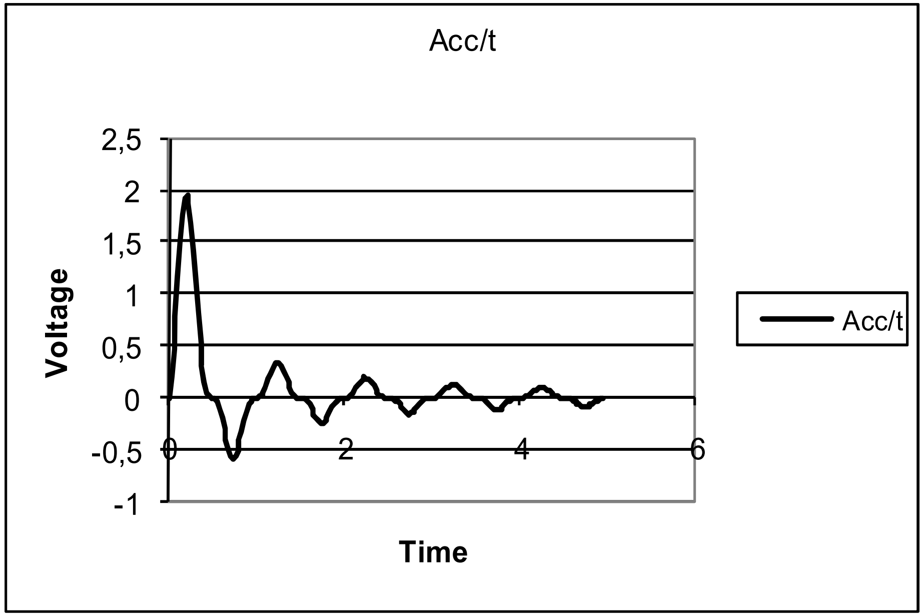


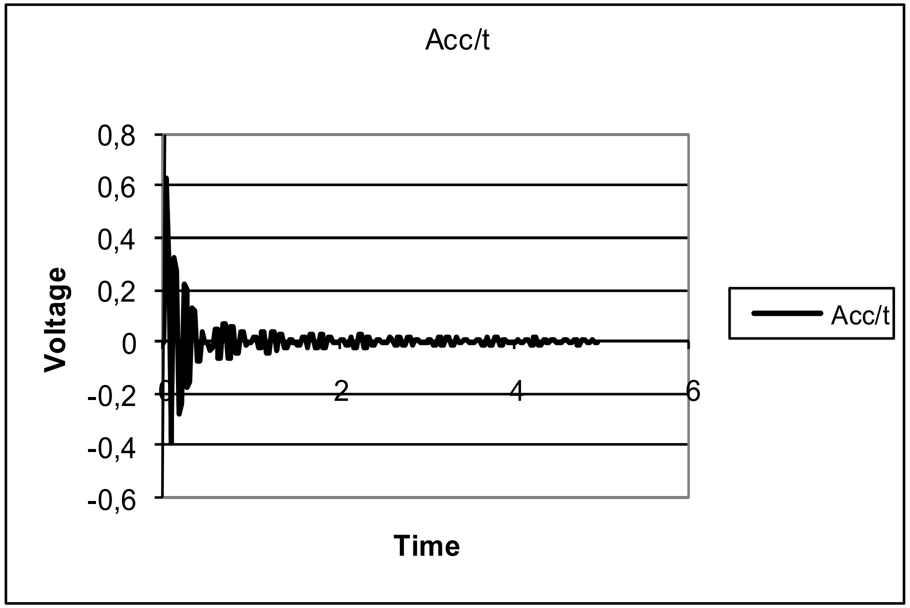


Uncorrelated

- Top right: $f_A=0.8f_B$
- Bottom left: $f_A=2f_B$
- Bottom right: $f_A=10f_B$







Examples with and without correlation

Vs(t)=Va(t)+Vb(t): $V_s(t)^2 = (V_a(t)+V_b(t))^2 = V_a(t)^2 + V_b(t)^2 + 2V_a(t)V_b(t)$

Example: Assume that

- *Va(t)* and *Vb(t)* has the same amplitude
- *Va(t)* and *Vb(t)* has similar shape (both are sine, triangular etc.)

$V_s(t)_{rms}^{\ 2} = \frac{1}{T}\int_0^T V_s(t)^2\,dt =$		$\frac{1}{T}\int_0^T V_a(t)^2\,dt$	$+\frac{1}{T}\int_0^T V_b(t)^2\,dt$	$+\frac{2}{T}\int_0^T V_a(t)V_b(t)dt$		
Uncorrelated: $\omega_a \neq \omega_b$		$V_a(t)_{rms}^{\ 2}$	$+V_a(t)_{rms}^{\ 2}$	$+0$	$=2V_a(t)_{rms}^{\ 2}$	√2
Correlated: $\omega_a = \omega_b$	$\theta_a = \theta_b$	$V_a(t)_{rms}^{\ 2}$	$+V_a(t)_{rms}^{\ 2}$	$+2V_a(t)_{rms}^{\ 2}$	$=4V_a(t)_{rms}^{\ 2}$	2
	$\theta_a = \theta_b + \pi$	$V_a(t)_{rms}^{\ 2}$	$+V_a(t)_{rms}^{\ 2}$	$-2V_a(t)_{rms}^{\ 2}$	$=0$	0

We see that two correlated noise sources will be able to provide between 0% and 141% times the noise as an uncorrelated case.

Addition of uncorrelated noise voltages

Example 1:

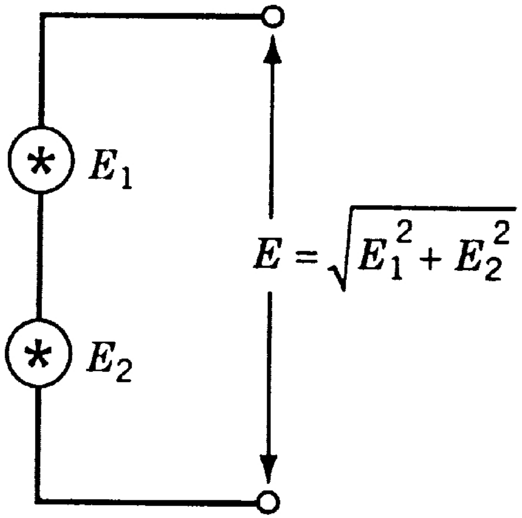


Figure 1-8 Addition of uncorrelated noise voltages.

We must here make an rms addition and not a standard linear addition. We will then have: $E^2 = E_1^2 + E_2^2$

As an approach we can ignore noise contributions that are less than 1/10 of more dominating noise contributions.

Eksempel 2:

In b) is the source resistance in a) split into two equal parts. First, we calculate with standard (and incorrect) linear mathematics.

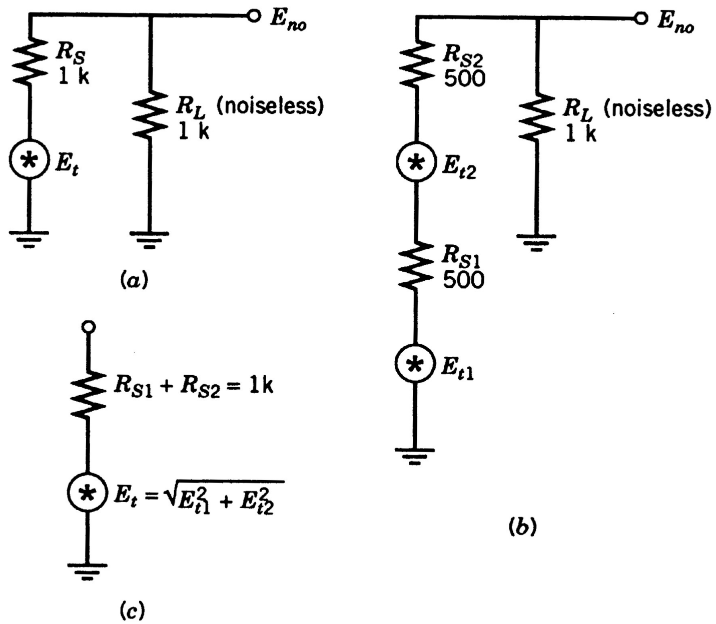


Figure 1-9 Circuits with noise voltages: (*a*) simple circuit, (*b*) equivalent circu (*c*) correct resultant circuit.

a):

$$E_{no} = \frac{R_L}{R_S + R_L} E_t = 0.5\left(4nV / Hz^{1/2}\right) = 2nV / Hz^{1/2}$$

b):

$$E_{no} = \frac{R_L}{R_{S1} + R_{S2} + R_L} E_{t1} + \frac{R_L}{R_{S1} + R_{S2} + R_L} E_{t2} =$$

$$0.5\left(2.82nV / Hz^{1/2}\right) + 0.5\left(2.82nV / Hz^{1/2}\right) = 2.82nV / Hz^{1/2}$$

We would expect that the answers should be equal but get a difference. The reason is that we use linear calculus although we should have practiced squared calculus of the rms values.

We now perform the same calculation again but with the square of the noise voltage:

$$E_{no}{}^2 = \left(\frac{R_L}{R_{S1}+R_{S2}+R_L}\right)^2 E_{t1}^2 + \left(\frac{R_L}{R_{S1}+R_{S2}+R_L}\right)^2 E_{t2}^2$$

$$= (0.5)^2\left(2.82nV / Hz^{1/2}\right)^2 + (0.5)^2(2.82nV / Hz^{1/2})^2$$

$$= (0.5)\left(2.82nV / Hz^{1/2}\right)^2 = 4\times10^{-18} V^2 / Hz$$

Then we take the square root:

$$E_{no} = 2nV / Hz^{1/2}$$

which is the same as we did when we calculated for a).

When there are resistors in series or in parallel like in b) one should calculate the total resistance first and then calculate the noise for the resulting resistance.

Partly correlation

When some of the noise in the two noise voltages comes from the same source (cause), while some come from different sources, the sources are partly correlated. We may in this case use the expression:

$$E^2 = E_1^2 + E_2^2 + 2CE_1E_2$$

Here is *C* a correlation coefficient that can have any value between -1 and +1. When *C* is equal to 0 the voltages are uncorrelated and we have the relationship as discussed earlier. When *C* is equal to -1 the voltages are correlated but in opposite phase.

Typically the correlation is zero and this is considered as default. If one incorrectly assumes no correlation the maximum error will be when the two noise voltages are equal and completely correlated. For equally large signals (the same rms value) we will have:

- Two correlated signals $\Rightarrow$ 2 rms value.
- Two fully uncorrelated signals $\Rightarrow$ 1.4 of the rms value.

(i.e. the error will be 2/1.4 = 1.4 which gives that the noise level is 40% more than we assumed).

Example 3:

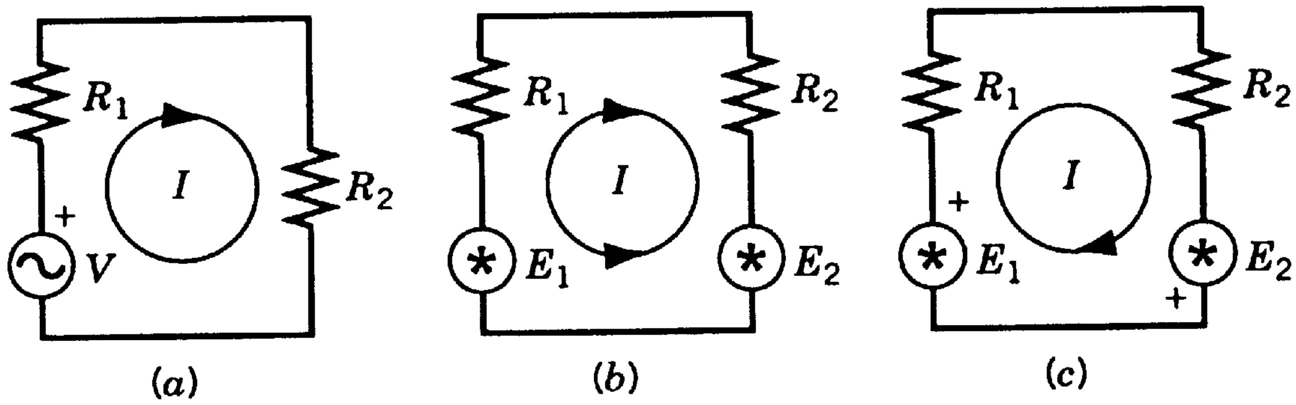


Figure 1-10 Circuits for analysis examples.

a)

We put up the following expression for a): $V = IR_1 + IR_2$

We square all terms and get: $V^2 = (IR_1)^2 + (IR_2)^2$

But this is not correct! Why?

Because it is the same current *I*! Hence all terms are 100% correlated. We must add the correlation expression:

$$V^2 = \left(IR_1\right)^2 + \left(IR_2\right)^2 + 2CIR_1IR_2$$

In this case C = 1 and we can write the expression:

$$V^2 = I^2(R_1 + R_2)^2$$

The rule of thumbs for serial resistors and impedances is that they should first be summed before they are squared!

b)

Two noise sources (or sine generators with different frequency) is in series with two noise-free resistors. It is also the same current that passes through both resistors. We therefore add the resistances before they are squared.

$$I^2 = \frac{E_1^2 + E_2^2}{(R_1 + R_2)^2}$$

The voltage sources are uncorrelated and hence squared before they are summed.

For the voltage source, there is no correlation term.

b) Calculated with the superposition principle:

The current *I* consists of I_1 and I_2. We use the super-position principle, which says: *In a linear network will the response from two or more sources be the sum of the response from each source alone with (the other) voltage sources short circuit, and the (other) current sources left open.*

That gives us: $I_1 = \frac{E_1}{R_1 + R_2}$ and $I_2 = \frac{E_2}{R_1 + R_2}$

The currents are uncorrelated and we add: $I^2 = I_1^2 + I_2^2$

When we insert for I_1 and I_2 so we get:

$$I^2 = \frac{E_1^2}{(R_1 + R_2)^2} + \frac{E_2^2}{(R_1 + R_2)^2} = \frac{E_1^2 + E_2^2}{(R_1 + R_2)^2}$$

which is what we found earlier.

c) Syntax for partial correlation

E1 and E2 have some correlation. On the figure this is marked with a plus sign. The position of these signs show that they support each other and that the correlation is positive i.e. 0 < 1.≤C

$$I^2 = \frac{E_1^2 + E_2^2 + 2CE_1E_2}{(R_1 + R_2)^2}$$

Example 4: Two uncorrelated sources

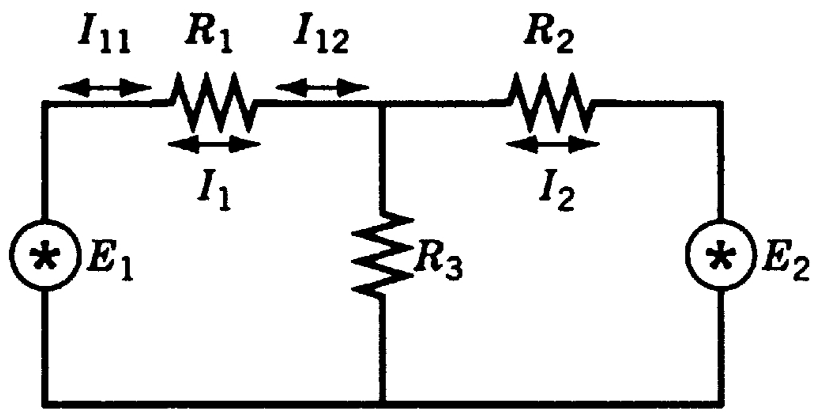


Figure 1-11 Two-loop circuit.

Target: Find the total current I_1 through R_1.
Method: Super-position
Syntax: I_1 consists of two contributions: I_{11} from E_1 and I_{12} from E_2. Notice! In the book is I_2 only the contribution from E_2.

We have:

$$E_1^2 = I_{11}^2 \left[R_1 + \frac{R_2 R_3}{(R_2 + R_3)} \right]^2$$

which we can write as:

$$I_{11}^2 = \frac{E_1^2 (R_2 + R_3)^2}{(R_1 R_2 + R_1 R_3 + R_2 R_3)^2}$$

We have also

$$E_2^2 = I_2^2\left[R_2 + \frac{R_1R_3}{(R_1 + R_3)}\right]^2$$

The part of I_2 that passes through R_1 is:

$$I_{12} = I_2R_3/(R_1 + R_3)$$

The last two expressions can be combined together to:

$$I_{12}^2 = \frac{E_2^2R_3{}^2}{(R_1R_2 + R_1R_3 + R_2R_3)^2}$$

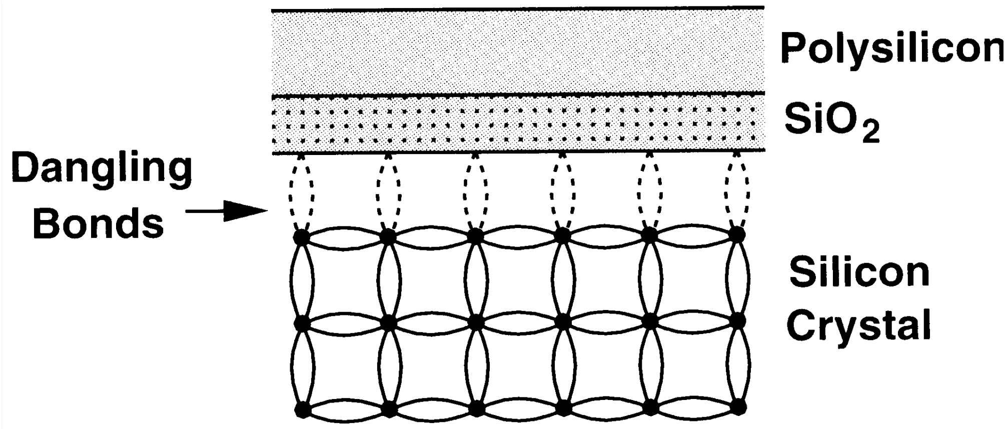
Since $I_1^2 = I_{11}^2 + I_{12}^2$

we can put together the two current contributions on the right side and we get:

$$I_1^2 = \frac{E_1^2(R_2 + R_3)^2 + E_2^2R_3^2}{(R_1R_2 + R_1R_3 + R_2R_3)^2}$$

Flicker noise

Flicker noise =1/f-noise = low frequency noise = pink noise



Flicker noise is a sort of noise that is related to irregularities at interfaces between different atom structures. One such place is the interface between the silicon crystal and silicon dioxide. The free electron pairs will collect charge that is trapped for a variable time.

Flicker noise is observed primarily in semiconductors but can also be seen in radio tubes and some sorts of resistors.

Flicker noise have a 1/f-characteristic i.e. it is weakest at the highest frequencies and grow to infinity when the frequency decreases.

To find the noise in a frequency band from f_l to f_h we can integrate as follows:

$$N_f = K_1 \int_{f_l}^{f_h} \frac{df}{f} = K_1 \ln \frac{f_h}{f_l}$$

N_f is the noise power in Watts.

K is a constant in Watt.

Example:
Let $f_h = 10f_l$.
Then we have: $N_f = 2.3K_1$
This shows that the noise level within each decade is constant. Thus the flicker noise between 0.01 Hz and 0.1 Hz is equal to the noise from 100kHz to 1MHz.

⇒When flicker is dominating, if possible, we should move the signalling frequency towards a higher frequency.

Most other types of noise is given pr √Hz and the multiplication with Δf is done at last. It is often practical to do the same with the flicker noise. If $\Delta f << f_l$ is true, we can make the approximation $\frac{\Delta f}{f_l} \approx \ln\left(\frac{f_h}{f_l}\right)$ and use Δf as for other types of noise.

Example: Flicker noise in MOSFET

$$I_f^2(f) = \frac{K_F I_{DS}^{AF}}{Cox \cdot L_{eff}^2} \cdot \frac{1}{f}$$

$$I_f^2(f_l, f_h) = \frac{K_F I_{DS}^{AF}}{Cox \cdot L_{eff}^2} \int_{f_l}^{f_h} \frac{1}{f} = \frac{K_F I_{DS}^{AF}}{Cox \cdot L_{eff}^2} \ln \frac{f_h}{f_l}$$

	AF	K_F	Cox
N	1.5	2.3e-26	2.2fF/µm²
P	1.3	6.3e-29	2.2fF/µm²

K_F: 2.3e-26 --- 6.3e-29
AF: 1.3 --- 1.8
Cox: 2.1fF/µm² --- 4.6fF/µm²

Shot-noise

Shot noise occurs in the pn-interfaces in the transistors and diodes. This type of noise describes fluctuations in the current running. It is expressed as:

$$I_{sh} = \sqrt{2qI_{DC}\Delta f}$$

where *q= 1,602e-19 Coulomb.*

We see that the noise level increases with the square root of the current. We also see that this type of noise is "white" i.e. it is constant and independent of the frequency (but not the bandwidth).

(Shot-noise in bipolar transistors)

From the equation for Shot-noise one could assume that the shot-noise level was almost zero when the power is zero. This is not correct. We will now study this further.

In bipolar transistors, we find most shot-noise in the emitter-base interface. The *VI* behaviour follows the familiar diode expression:

$$I_E = I_S(e^{qV_{BE}/kT} - 1)$$

where I_E is the emitter current, I_S is the reverse current and V_{BE} is the voltage between the base and emitter.

We split the current I_E in two parts ... $I_E = I_1 + I_2$

so that $I_1 = -I_S$ and

$I_2 = I_S \exp(V_{BE}/kT)$

I_1 is due to thermally generated minority carriers while I_2 is due to diffusion of majority carriers over the pn-interface.

NB! Both of these currents have full shot-noise even if the current itself eliminate each other at $V_{BE} = 0$ *Volt!*

(Shot-noise in bipolar transistors)

(During reversed bias voltage I_1 will dominate while under strong forward voltage I_2 will dominate.).
At $V_{BE} = 0$ we have that $I_E = 0$ while the noise is

$$I_{sh}^2 = 4qI_S\Delta f$$

Shot noise modell

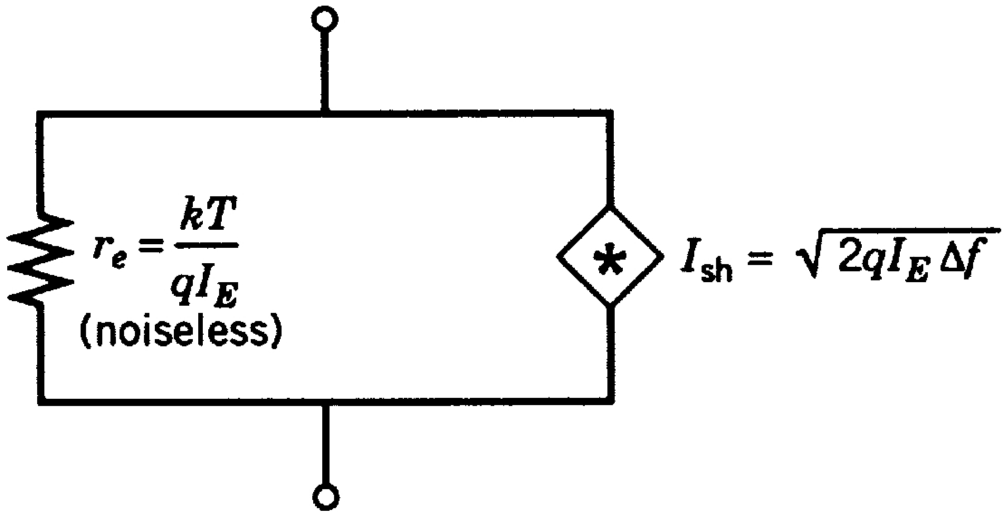
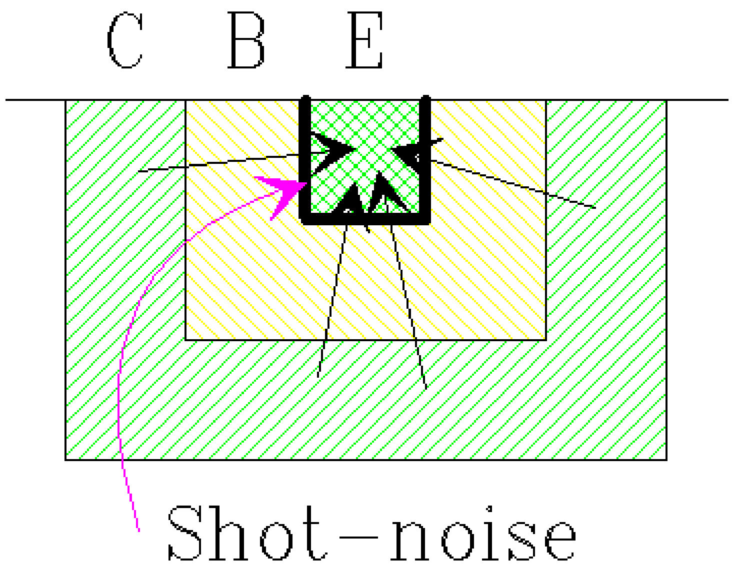


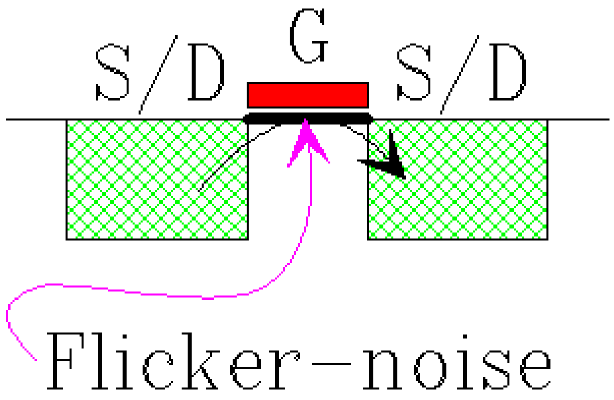
Figure 1-12 Shot noise equivalent circuit for forward-biased *pn* junction.

The circuit model for shot noise consists of a current source in parallel with a (noiseless) resistance.
We find the resistance value by deriving the expression for diode current by V_{BE}.
This derivation will give the conductivity. The resistance is found by finding the inverse of the conductivity.

$$r_e = kT / qI_E$$

A cut through BJT and MOSFET





Capacitive shunting of thermal noise: kT/C

The expression for thermal noise $E_e = \sqrt{4kTR\Delta f}$

indicates that an open circuit with infinite resistance will generate an infinite noise voltage. This will not be the case since there will always be (parasitic) capacitances between the terminals.

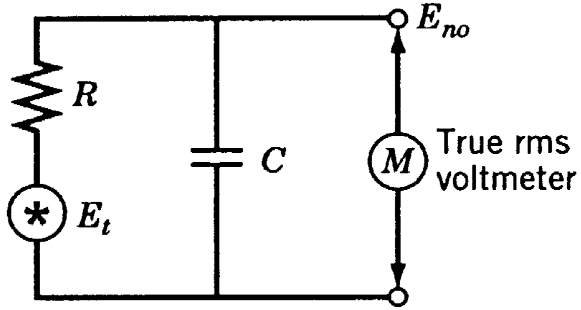


Figure 1-14 Thermal noise of a resistor shunted by a capacitance.

The resistance and capacitor will together act as a low pass filter. When the resistance grows, so do *Et*. But at the same time decreases the filter cut-off frequency and hence reduces the bandwidth.

The figure shows three curves with the same *C* but with three different *R*-values. The integral under the curves are equal.

First we calculate the integrated noise voltage:

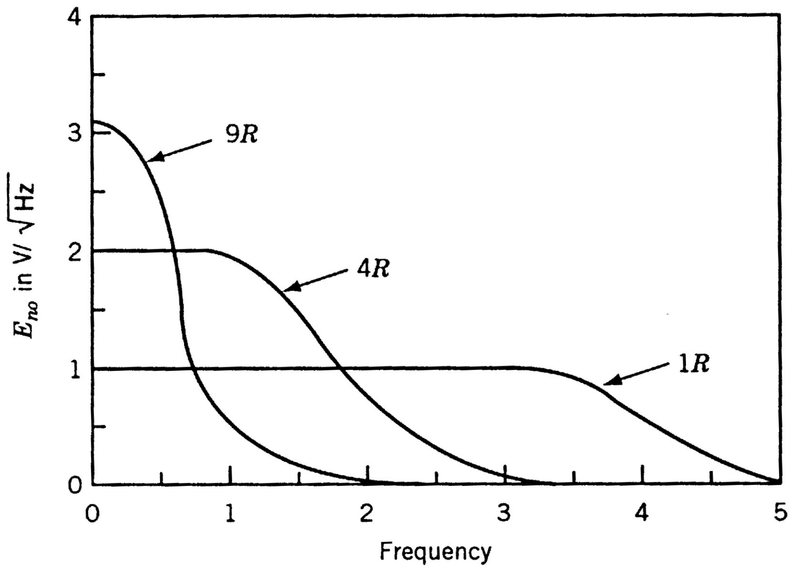


Figure 1-15 Noise spectral density for a resistance shunted by a capacitance.

$$E_{no}^2 = \int_0^\infty E_t^2 \left| \frac{1/j\omega C}{R + 1/j\omega C} \right|^2 df = \int_0^\infty \frac{E_t^2 df}{1 + (\omega RC)^2}$$

Then we need to replace some variables: $f = f_2 \tan\theta$, $f_2 = 1/(2\pi RC)$, $df = f_2 \sec^2\theta d\theta$

and change the upper limit to $\pi/2$. Then we get

$$E_{no}^2 = \int_0^{\pi/2} \frac{E_t^2 f_2 \sec^2\theta d\theta}{1 + \tan^2\theta} = \int_0^{\pi/2} E_t^2 f_2 d\theta = \int_0^{\pi/2} 4kTRf_2 d\theta = 2\pi kTRf_2$$

and when we insert for f_2 we get: $E_{no}^2 = kT/C$

C sets an upper limit for the thermal noise voltage

Example of a kT/C-calculation

Assume an input signal of 1μVrms amplified by 30dB. This gives an output signal of 31.6μVrms. The output will be sampled and measured and we would like to have a capacitance large enough to limit the noise voltage to -15dB below the signal level. With a 200pF capacitor and a temperature of 290° K the capacitor is limiting the noise to 4.5μVrms which is -17dB relative to the signal level.

$$E_n = \sqrt{\frac{kT}{C}} = \sqrt{\frac{1.38 \cdot 10^{-23} W / Ks \cdot 290^{\circ} K}{200\, pF}} = 4.5\, \mu Vrms$$