UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam:	**IN5520 / IN9520 – Digital image analysis**
Date:	**Wednesday December 12, 2018**
Exam hours:	**09.00-13.00 (4 hours)**
Number of pages:	**8 pages**
Enclosures:	**1 page Appendix**
Allowed aid:	**Calculator**

- Read the entire exercise text before you start solving the exercises. Please check that the exam paper is complete. If you lack information in the exam text or think that some information is missing, you may make your own assumptions, as long as they are not contradictory to the "spirit" of the exercise. In such a case, you should make it clear what assumptions you have made.

- You should spend your time in such a manner that you get to answer all exercises. If you get stuck on one question, move on to the next question.

- There are 6 exercises. They are weighted proportional to the number of sub-questions, except for Exercise 5, which is weighted as approximately 1/6 of the total.

- Your answers should be **short**, typically a few sentences and / or a sketch should be sufficient.

- Do not give just a numerical answer, but demonstrate your reasoning.

Good luck!!

Exercise 1: Texture analysis

Assume that you are given a gray level image of size MxN pixels with b bits per pixel.

a) Describe how a normalized Gray Level Coccurrence Matrix is computed, and which parameters this involves.

b) For a given inter-pixel distance and direction, how do we make a normalized GLCM symmetrical about the matrix diagonal without double counting?

c) Say that we have accumulated a normalized symmetrical GLCM for a given inter-pixel distance and direction.
How can we find what fraction of the pixel pairs in the image that have a difference of D gray levels? Please illustrate with a small sketch!

Exercise 2: Feature extraction from gradients

Histogram of Gaussians (HOG) is a commonly used method for feature extraction.

a) Let us assume that we use Sobel-filters to compute gradients

$$H_x(i,j) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}, \quad H_y(i,j) = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

How are these filters used to compute the gradient magnitude and direction?

b) HOG-features are computed in bins. In a given bin cell of size BxB pixels, describe briefly how the HOG-features are computed.
By bin cell we mean the cell where gradient information is aggregated.

c) For each bin you get 9 feature values. With a grid consisting of a high number of bins, the number of features will grow quickly.
Suggest one new feature characterizing the information in the 9 values, and discuss briefly why you think this is a good feature.

Exercise 3: Cartesian geometric moments

a) Give an expression for an ordinary geometric moment of order p+q of an object in an image, and explain the terms of the expression.

b) Describe, in words or using mathematical expressions, how to get from an ordinary moment of order p+q to a central moment.

c) What kind of invariance is obtained by such central moments, as compared to an ordinary moment? Please explain!

d) A third kind of moment is obtained when we normalize a central moment by

$$\eta_{pq} = \frac{\mu_{pq}}{(\mu_{00})^{\gamma}}, \quad \gamma = \frac{p+q}{2} + 1, \quad p+q \geq 2$$

What kind of invariance is obtained, and what is the condition for the expression above to be valid?

1st question for the PhD-students only:

e) The 7 Hu's moment combinations are position, scale, and rotation invariant, and may be useful to obtain features that will discriminate between different objects.

We have seen that for rectangles and ellipses, only the two first Hu moment combinations are different from 0.
The figure below gives a logarithmic plot of the first two Hu moments for binary rectangular objects of size 2a by 2b, and ellipses with semi-axes a and b, for a/b = [1,2,4,8,16].

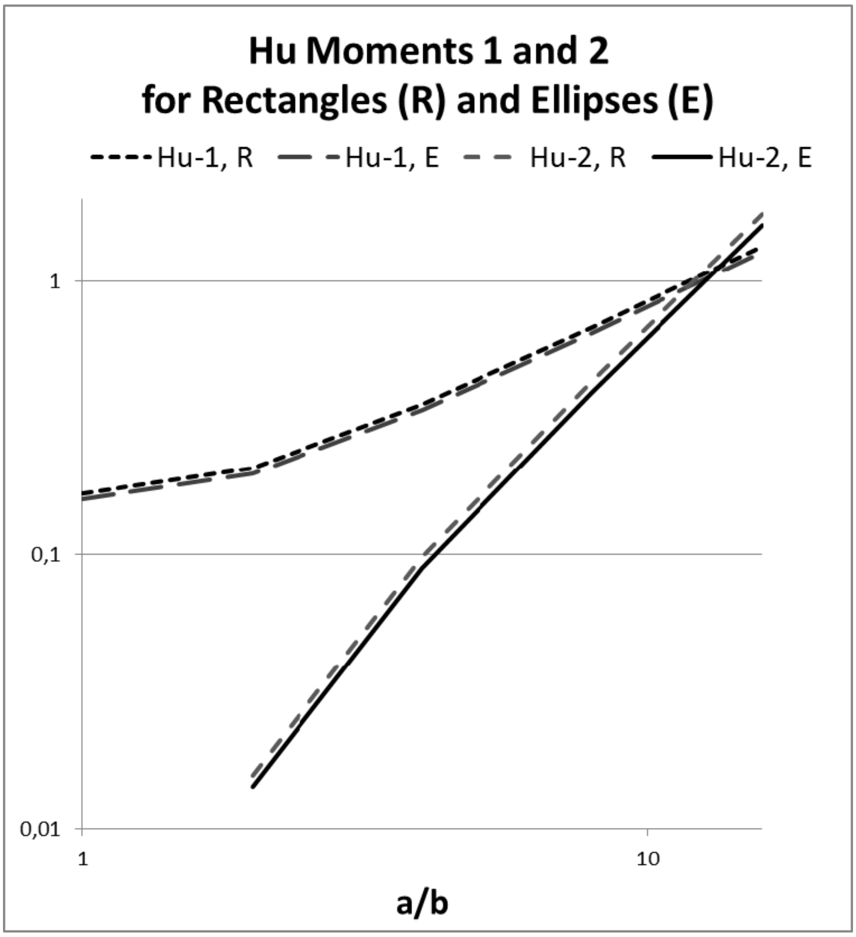
In the continuous case, the first two Hu moments of rectangles are given by

$$\phi_1 = \frac{1}{12}\left(\frac{a}{b} + \frac{b}{a}\right), \qquad \phi_2 = \left(\frac{1}{12}\right)^2 \left(\frac{a}{b} - \frac{b}{a}\right)^2$$

and for elliptical objects in binary images they are given by

$$\phi_1 = \frac{1}{4\pi}\left(\frac{a}{b} + \frac{b}{a}\right), \qquad \phi_2 = \left(\frac{1}{4\pi}\right)^2 \left(\frac{a}{b} - \frac{b}{a}\right)^2$$

In an object classification setting, what implication do you draw from the equations above and from the plot below?



2nd question for the PhD-students only:

f) We define object compactness $\gamma = P^2/(4\pi A)$, where P is the perimeter length and A is the area. For a circular disc, γ is minimum and equals 1, while γ attains a high value for both complex objects, and for very elongated simple objects, like rectangles and ellipses where the a/b ratio is high.

For ellipses and rectangles, the compactness measure in the continuous case is given by:

$$\gamma_{ellipse} = \frac{1}{4}\left(\frac{a}{b} + \frac{b}{a}\right), \qquad \gamma_{rectangle} = \frac{1}{\pi}\left(\frac{a}{b} + \frac{b}{a} + 2\right)$$

In an object classification setting, would you use both the given compactness measure and the first Hu moment to distinguish between ellipses of different a/b-ratio? And between rectangles of different a/b-ratio?
Please explain!

Exercise 4: Support vector machines

a) The basic optimization problem for a support vector machine classifier is:

$$\text{minimize} \quad J(w) = \frac{1}{2}\|w\|^2$$

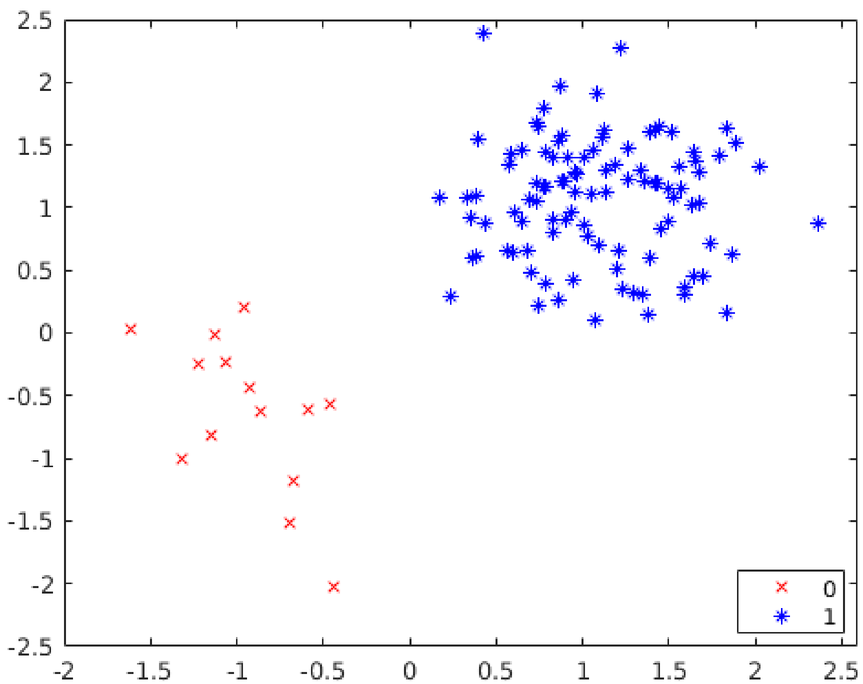
$$\text{subject to} \quad y_i(w^T x_i + w_0) \geq 1, \quad i = 1,2,\ldots N$$

What is the total margin for this problem?

b) Support vector machines are fundamentally different from Gaussian classifiers in terms of how the decision boundary is found – explain why.

c) Support vector machine classifiers can also be explained based on convex hulls. Explain the relationship between the convex hull of two regions and the hyperplane with maximum margin.

d) Given below is a scatter plot of a binary classification problem. Sketch the convex hulls on the figure in the Appendix and use this to find an approximate hyperplane.



e) In the general case the optimization problem is given as:

$$\max_{\lambda}\left(\sum_{i=1}^{N}\lambda_i - \frac{1}{2}\sum_{i,j}\lambda_i\lambda_j y_i y_j x_i^T x_j\right)$$

$$\text{subject to} \quad \sum_{\mathrm{i}=1}^{\mathrm{N}}\lambda_{\mathrm{i}} y_i = 0 \quad \text{and} \quad 0 \le \lambda_i \le C \ \ \forall i$$

Explain briefly which terms in the equation that can be computed using kernels in a high-dimensional space, and also explain what the kernels measure.

Exercise 5: Alternative approaches to object size estimation (w≈ 1/6)

The graylevel image to the right is from one of the chapters of the textbook used throughout this course.

It depicts a number of bright wood dowels of different brightness on a dark background.

There may be visible structures (small contrast variations) both inside each object (dowel) and in the background.

Please detail maximum two different approaches to obtain an estimate of the distribution of object sizes in the image.

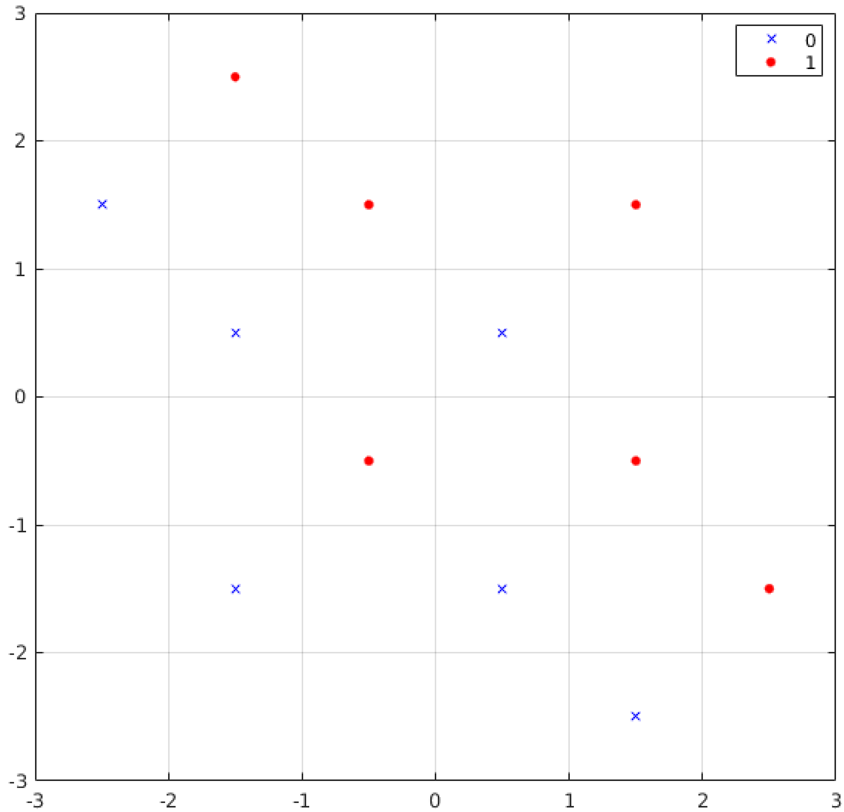
Exercise 6: PCA and classification

A set of samples from two classes are given as (a scatter plot is also given below):

- Class1: $\begin{bmatrix} -2.5 & 1.5 \\ -1.5 & 0.5 \\ 0.5 & -1.5 \\ 1.5 & -2.5 \\ 0.5 & 0.5 \\ -1.5 & -1.5 \end{bmatrix}$ $\mu_1 = \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}$ $\Sigma_1 = \begin{bmatrix} 2.4 & -1.6 \\ -1.6 & 2.4 \end{bmatrix}$ $\Sigma^{-1}{}_1 = \begin{bmatrix} 0.75 & 0.5 \\ -0.5 & 0.75 \end{bmatrix}$

- Class2: $\begin{bmatrix} -1.5 & 2.5 \\ -0.5 & 1.5 \\ 1.5 & -0.5 \\ 2.5 & -1.5 \\ -0.5 & -0.5 \\ 1.5 & 1.5 \end{bmatrix}$ $\mu_2 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$ $\Sigma_2 = \begin{bmatrix} 2.4 & -1.6 \\ -1.6 & 2.4 \end{bmatrix}$ $\Sigma^{-1}{}_2 = \begin{bmatrix} 0.75 & 0.5 \\ -0.5 & 0.75 \end{bmatrix}$

- Global mean: $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- Global covariance matrix: $\begin{bmatrix} 2.45 & -1.18 \\ -1.18 & 2.45 \end{bmatrix}$
- Eigenvalues 1.27 and 3.65 with eigenvectors: $\begin{bmatrix} -0.7 \\ -0.7 \end{bmatrix}$ and $\begin{bmatrix} -0.7 \\ 0.7 \end{bmatrix}$



a) Find the direction of the first principal component of this data set.

b) Sketch the first principal component on the plot

c) Compute the new feature values using this component for all samples.

d) Plot the points in the figure and discuss how the first principal component performs in this case.

Good Luck!

Candidat number:

Appendix, Exam IN5520/9520, 12. December, 2018

Please tear out this sheet and hand it in together with your answers.

Figure 1

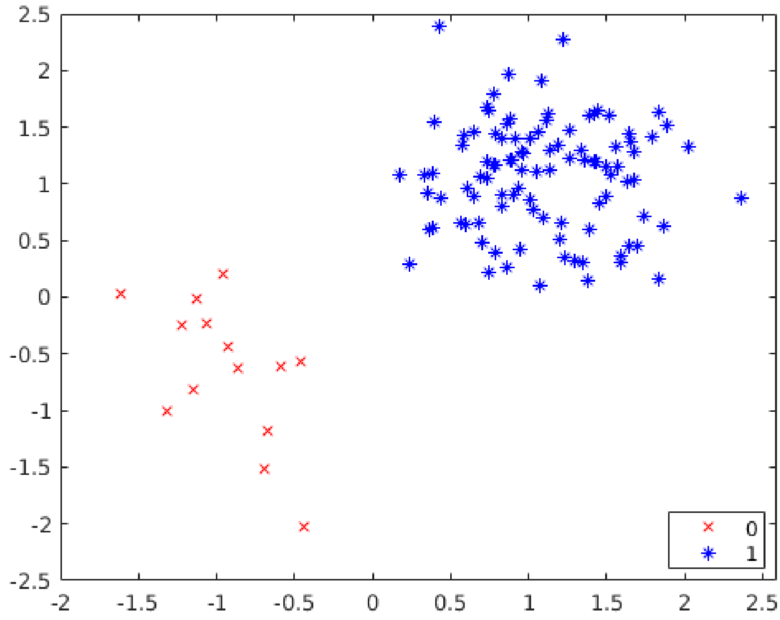


Figure 2