UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam:	**IN5520 / IN9520 – Digital image analysis**
Date:	**Wednesday December 9, 2019**
Exam hours:	**09.00-13.00 (4 hours)**
Number of pages:	**6 pages**
Enclosures:	**None**
Allowed aid:	**Calculator**

- Read the entire exercise text before you start solving the exercises. Please check that the exam paper is complete. If you lack information in the exam text or think that some information is missing, you may make your own assumptions, as long as they are not contradictory to the "spirit" of the exercise. In such a case, you should make it clear what assumptions you have made.

- You should spend your time in such a manner that you get to answer all exercises. If you get stuck on one question, move on to the next question.

- There are 5 exercises. They are weighted proportional to the number of sub-questions.

- Your answers should be **short**, typically a few sentences and / or a sketch should be sufficient.

- Do not give just a numerical answer, but demonstrate your reasoning.

Good luck!!

Exercise 1: Texture Analysis

Assume that you are given a gray level image of size MxN pixels with b bits per pixel.

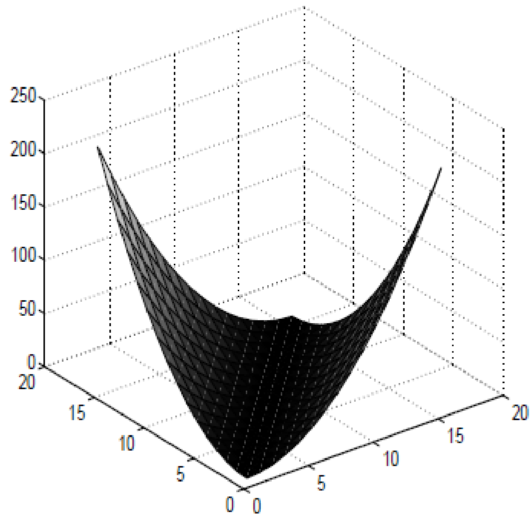
a) Describe how a normalized Gray Level Coccurrence Matrix (GLCM) is computed, and which parameters this involves.

b) For a given inter-pixel distance and direction, how do we make the normalized GLCM symmetrical about the matrix diagonal without double counting?

c) Assume that we have accumulated a normalized symmetrical GLCM for a given inter-pixel distance and direction.

Give the expression for a GLCM feature that has a weighting function equal to zero along the diagonal (i = j), and increases quadratically away from the diagonal, as illustrated for G=15.

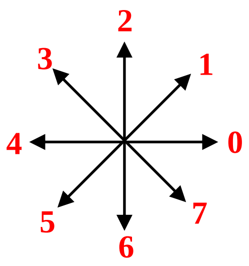
What will the effect of this weight function be, and what kind of images will get a high feature value?

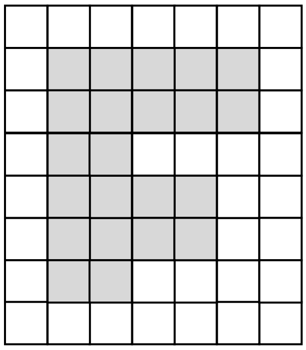


d) How can we find what fraction of pixel pairs at the given inter-pixel distance and direction in the image that have an absolute difference | i-j | ≥ D gray levels, while both i and j are in the upper ¼ of the gray scale? Please illustrate!

Exercise 2: Chain Codes

You are given the 8-directional chain code and the binary object below.



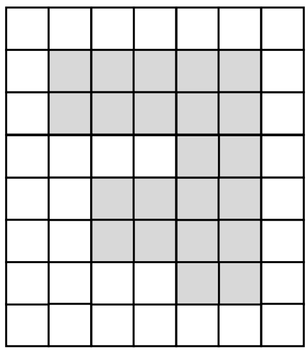


a) Find the <u>absolute</u> chain code of the boundary of the object clockwise from the upper left pixel.

b) Which technique, based on the 8-directional <u>absolute</u> chain code, can be used to make a description of the object that is independent of the start point? Demonstrate this by starting at the lower right pixel of the object, instead of the upper left.

c) Which technique, based on the clockwise <u>**relative**</u> chain code, will give you the same description of the object, independent of the start point? Demonstrate this by starting at the upper left and the lower right object pixel.

d) Rotation invariance is inherent in relative chain codes. But what if the object has been flipped horizontally. How can you then determine if it is the same object?



Exercise 3: Geometric Moments and Hough Transform

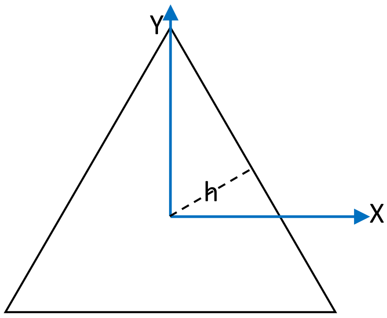
Assume that you have thresholded a gray level image into a binary image ***b(x,y)*** containing a solid object (pixel value = 1) and a background (pixel value 0).

a) Describe a moment-based approach to find the center of mass of the object.

b) Assume that the object is an equilateral triangle, located somewhere in the image. Give a definition of the object orientation, and describe a moment-based approach to estimate the orientation of the object. In this case, is the result unique?

c) Assume that we translate the origin to the center of mass of the triangle, then apply a gradient detector to the binary image, and use the "normal representation" Hough Transform.

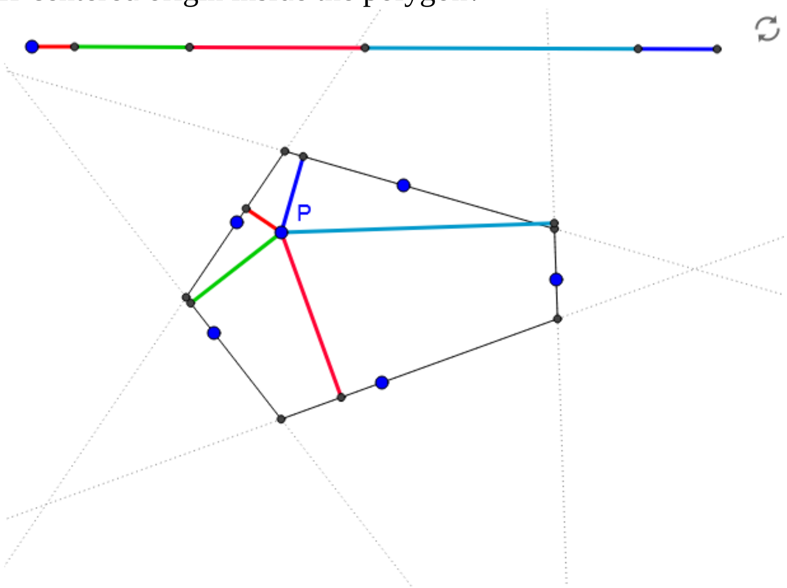
   Assume that one side of the triangle is parallel to the x-axis, describe the contents of the Hough space.



d) What happens in the Hough space if the origin is moved inside the triangle, so that it does not coincide with the center of mass?

The following two questions are intended for the PhD-students:

e) What will happen in the Hough domain, if an equi-angular polygon is rotated anti-clockwise around an off-centered origin inside the polygon?



f) Which polygon feature in the HT domain is invariant to the location of the origin inside an equiangular polygon, and may be useful when working with noisy images?

Exercise 4: Short questions on classification

a) Explain briefly the drawbacks of overfitting a classifier.

b) Explain briefly how you should split the available set of labelled data for a classification problem, and what the resulting subsets should be used for.

c) Given a classification problem, describe briefly in general how you use the discriminant functions to find the decision boundary between two classes.

d) When evaluating classifier performance, we often use precision, sensitivity and specificity:

Precision = TP/(TP+FP)
Sensitivity = TP/(TP+FN)
Specificity = TN/(TN+FP)

Give an example of a binary classification problem where specificity is more important than sensitivity.

e) Explain briefly which parameters that a SVM classifier has, and how they should be determined.

f) Assume that you use a Gaussian classifier with full covariance matrices. Discuss if you have some challenges working in high-dimensional feature space, e.g. 100 features.

Exercise 5: Classification

a) Give Bayes rule a classification problem.

b) Given a Gaussian classifier with d features. Consider a binary classification problem. The class-conditional probability density is given as:

$$p(x|\omega_s) = \frac{1}{(2\pi)^{d/2}|\Sigma_s|^{d/2}} exp\left[-\frac{1}{2}(x-\mu_s)^t\Sigma_s^{-1}(x-\mu_s)\right]$$

Write down the logarithmic discriminant function without any assumptions on the covariance matrix.

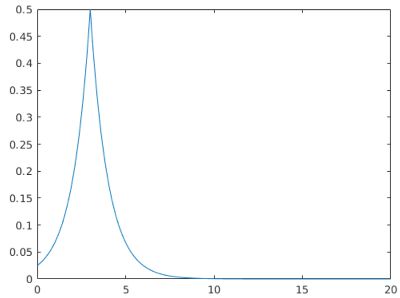
c) If we assume a 2-class problem with $P(\omega_1) = 0.75$ and $P(\omega_2) = 0.25$ and equal diagonal covariance matrices, describe by words how we can find the decision boundary.

d) If we assume a 2-class problem with $P(\omega_1)=0.5$ and $P(\omega_2)=0.5$, mean vectors $\mu_1=[1,3]^T$ and $\mu_2=[-1,1]^T$ and equal diagonal covariance matrices, compute the equation for the decision boundary.

e) Assume that you have a 1-d feature vector that follows a Laplace distribution given two parameters mean ω_s and shape b_s by:

$$p(x|\omega_s) = \frac{1}{2b_s} e^{-\frac{|x-\mu_s|}{b_s}}$$

$$p(x|\omega_s) = \begin{cases} \frac{1}{2b_s} e^{-\frac{\mu_s - x}{b_s}} & if\ x < \mu \\ \frac{1}{2b_s} e^{-\frac{x-\mu_s}{b_s}} & if\ x \geq \mu \end{cases}$$



The shape of this distribution is indicated in the figure. Assume two classes with equal prior probability and equal shape $b_s=1$. Compute the decision boundary for this 1-d case. For simplicity, you can assume $\mu_2 > \mu_1$.

f) For the binary problem in e), set up an expression for the total classification error.

Good Luck!