## UNIVERSITY OF OSLO

# Faculty of Mathematics and Natural Sciences 

Exam:<br>Date:<br>Exam hours:<br>Number of pages:<br>Enclosures:<br>Allowed aid:

INF 4300 / INF 9305 - Digital image analysis<br>Monday December 16, 2013<br>14.30-18.30 (4 hours)<br>6 pages plus 1 page enclosures<br>1 page med a plot<br>Calculator

- Read the entire exercise text before you start solving the exercises. Please check that the exam paper is complete. If you lack information in the exam text or think that some information is missing, you may make your own assumptions, as long as they are not contradictory to the "spirit" of the exercise. In such a case, you should make it clear what assumptions you have made.
- You should spend your time in such a manner that you get to answer all exercises shortly. If you get stuck on one question, move on to the next question.
- Two of the questions are based on sketches enclosed on an extra sheet at the end of the exam text. Please give your solution on these sheets, mark them with your candidate number, and include them in your solution.
- Your answers should be short, typically a few sentences and / or a sketch should be sufficient.

Good luck!!

## Exercise 1: Texture

You are given an image with a periodic texture in a sinusoidal pattern as given below. The image was created as a 1D sinus signal $f(x)=100 \sin (20 \pi x)+100$; so the amplitude varies gradually between 0 and 200 (not as abrupt as the image figure might indicate).


A profile of the gray levels along a horisontal line in the image is also given:

a) If you create a GLCM matrix with an offsets $\Delta x=0$ and $\Delta y=10$, what would the shape of the pattern that will appear in the GLCM matrix look like? Please make a simple sketch of the GLCM matrix including the axes of the coordinate system. Will it change with a different $\Delta y$ offset if $\Delta x=0$ ? Please explain!
b) This image is periodic in the horisontal direction with a period of $x=20$.

What will the shape of the GLCM matrix look like with $\Delta x=20$ and offset $\Delta y=0$ ? Explain!
c) Sketch the shape of the GLCM matrix with offsets $\Delta x=10$ and $\Delta y=0$, and explain your reasoning.
d) PHD students only:

As $\Delta \mathrm{x}$ goes from 0 to 10 , discuss how the shape of the GLCM matrix will change.

## Exercise 2: Texture

You are given an isotropic normalized Gray Level Cooccurrence Matrix computed with $\mathrm{G}=32$.

a) What can you say about the range of gray levels that the original image contains?
b) Discuss what you know about the texture in the image, and how do you think the original image looks?
c) There are several possible features that may be extracted from the GLCM.

Which of the two features below will give a high feature value for the given GLCM?
Entropy:

$$
E=-\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} P(i, j) \times \log (P(i, j))
$$

Inverse difference moment:

$$
I D M=\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} \frac{1}{1+(i-j)^{2}} P(i, j)
$$

Please explain your reasoning.

## Exercise 3: Discrimination of objects based on chain codes

You are given a set of objects. The absolute chain code for each object is given as:
Object 1: 174
Object 2: 1753
Object 3: 053
Object 4: 664422
Object 5: 2064
Object 6: 775522

a) Draw the objects in a simple sketch. Which shapes do the objects represent?
b) What kind of chain code would you use to best discriminate between the object types? Justify you answer. Can you use chain code to separate the two shapes classes perfectly?
c) Phd students only:

From the chain code, can you derive a new feature that is also invariant to object size?

## Exercise 4: Clustering and classification

You are given the following set of data points:
$(15,7),(11,11),(13,11),(8,10),(9,9),(7,7),(7,5),(13,5),(14,4),(9,3),(11,3)$.
The points are plotted in the scatter plot below.

a) Perform a K-means clustering of the points with $\mathrm{K}=3$ and starting with the cluster centers $(7,5),(9,9)$ and $(11,3)$. You can do the clustering either by computation or geometrically. Indicate the new cluster centers on figure 1 in the enclosure.
List the clusters that each point will belong to.
List the cluster centers after each iteration. How many iterations are necessary before no point will change cluster during a new iteration?
b) Assume now that you train a Gaussian classifier with equal diagonal covariance matrices with the end result from point a. Plot the resulting decision boundaries on figure 1 in the enclosure. Assume that the classes have equal prior probabilities.
c) What is the estimated classification accuracy on the training set if this is the training set for the Gaussian classifier with 3 classes and equal diagonal covariance matrices.
d) Consider now how the resulting classification would be if we removed one of the original cluster centers that you used in a). If you want a classification with only 2 remaining classes, and you want the resulting classes to be well described using a common covariance matrix, which cluster center would you remove?
e) Sketch the shape of the resulting two Gaussian classes with common covariance matrix on the plot in figure 2 in the enclosure.

## Exercise 5: Mathematical morphology

a) Explain morphological opening of a binary image.
b) Given the $10 \times 10$ binary image below

| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Show the necessary steps to arrive at the top-hat transform of that image, using the structuring element to the right, having a centered origin. What does the top-hat transform give you in this case?

## Exercise 6: Object features - moments

a) How do we define the moments of inertia of an object in a gray level image?
b) In terms of moments of inertia, what is the requirement for a 2 D object to exhibit a unique orientation?
c) Show the simple relationships between the two moments of inertia $\mu_{20}$ and $\mu_{02}$ of an image object $\mathrm{f}(\mathrm{x}, \mathrm{y})$ about axes through its center of mass and the same moments of inertia about the parallel image coordinate axes ( $\mathrm{x}=0$ and $\mathrm{y}=0$ ).

## Good Luck!

Enclosure 1, Exam, INF4300/INF9305, December 16, 2013 Please tear out this page, and hand it in!

Candidate number:

Figure 1


Figure 2


