UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam:	**INF 4300 / INF 9305 – Digital image analysis**
Date:	**Thursday December 1, 2016**
Exam hours:	**14.30-18.30 (4 hours)**
Number of pages:	**6 pages**
Enclosures:	**None**
Allowed aid:	**Calculator**

- Read the entire exercise text before you start solving the exercises.
  Please check that the exam paper is complete.
  If you lack information in the exam text or think that some information is missing, you may make your own assumptions,
  as long as they are not contradictory to the “spirit” of the exercise.
  In such a case, you should make it clear what assumptions you have made.

- You should spend your time in such a manner that you get to answer all exercises shortly. If you get stuck on one question, move on to the next question.

- Your answers should be **short**, typically a few sentences and / or a sketch should be sufficient.

Good luck!!

Exercise 1: Moments

a) Give the expression for an ordinary moment of order (p+q) of an object in a 2D digital image.

b) Describe, in words or using mathematical expressions, how you get from an ordinary to a central moment.

c) How can you get a central moment that is invariant to scale;
and what else are such moments invariant to?

d) In the table to the right, "+", "-", and "0" indicates a positive, negative, or zero value of seven scale-normalized central moments (η_{11}, η_{20}, η_{02}, η_{21}, η_{12}, η_{30}, η_{03}) for objects symmetric about the y-axis ("M"), the x-axis ("C"), and both ("O") in a continuous analog (non-discretized) image.

	η_{11}	η_{20}	η_{02}	η_{21}	η_{12}	η_{30}	η_{03}
M	0	+	+	-	0	0	-
C	0	+	+	0	+	+	0
O	0	+	+	0	0	0	0

Which of Hu's moment combinations in their simplified version (see below) may be useful for the task of obtaining rotation-invariant shape features of elliptical objects in a digital image? Please explain!

Hu's moments; a bit simplified

For second order moments (p+q=2), two invariants are used:

$\varphi_1 = \eta_{20} + \eta_{02}$

$\varphi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$

For third order moments, (p+q=3), we can use

$a = (\eta_{30} - 3\eta_{12})$, $b = (3\eta_{21} - \eta_{03})$,

$c = (\eta_{30} + \eta_{12})$, and $d = (\eta_{21} + \eta_{03})$

and simplify the five last invariants of the set:

$\varphi_3 = a^2 + b^2$

$\varphi_4 = c^2 + d^2$

$\varphi_5 = ac[c^2 - 3d^2] + bd[3c^2 - d^2]$

$\varphi_6 = (\eta_{20} - \eta_{02})[c^2 - d^2] + 4\eta_{11}cd$

$\varphi_7 = bc[c^2 - 3d^2] - ad[3c^2 - d^2]$

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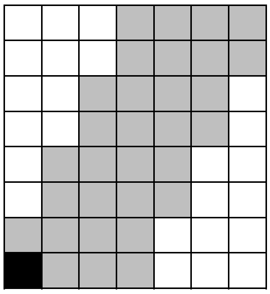
e) What may cause objects that corresponds to zero entries in the table above to show up as non-zero in digital images?

Exercise 2: Chain coding

a) Give the absolute and relative chain code of the binary object below, starting at the origin in the lower left corner having coordinates (0,0), using an 8-directional code, where 2 is up (forward).



b) How do we make a relative chain code start point invariant?

c) Discuss why the start point invariance is not unique in this case.

d) Given the 18 element relative code 021031313021031313 of a new object, how can you use a general property of chain codes to easily determine how it differs from the object in the figure above?

e) How do we normalize the absolute chain code in a) to get rotation invariance? Please perform the calculations!

<u>Next three questions for PhD-students only:</u>

f) An alternative way of obtaining a rotation invariant chain code would be to find the orientation of the object, rotate it, and then perform the chain coding. Please explain the steps of finding the orientation of the object in a) !

g) Compute the numerical values that are needed to find the object orientation.
hint: $(N+0.5)^2 = N(N+1)+0.25$

h) What does it take for this alternative rotation invariance to be valid?

Exercise 3: Classification

Consider a two-dimensional feature vector and a set of points from 2 classes in 2D feature space:
Class 1 has points: (3,0), (5,0), (7,0), (5,2), (5, -2)
Class 2 has points: (0,5), (0,3), (0,7), (-2, 5), (2,5)
The discriminant function for a Gaussian classifier is in the general form:

$$g(\mathbf{x}) = -\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_\mathbf{i})^T \Sigma_i^{-1}(\mathbf{x}-\boldsymbol{\mu}_\mathbf{i}) - \frac{d}{2}\ln 2\pi - \frac{1}{2}\ln|\Sigma_i| + \ln P(\omega_i)$$

a) Compute the mean for each class

b) Show that the covariance between feature 1 and 2 is 0 for both classes.

c) Show how the **discriminant function** can be simplified in this case with a classifier with equal diagonal covariance matrices.

d) Find an expression for the **decision boundary** using this simplified discriminant function.

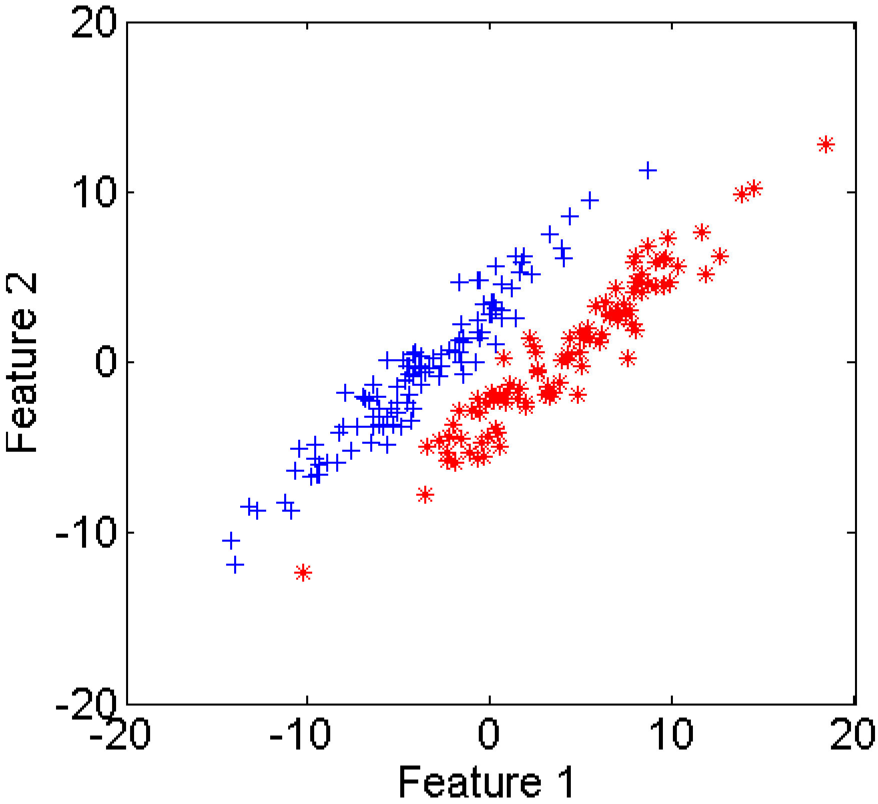
e) Compute the **decision boundary** if we assume equal prior probabilities.

f) Sketch the class means and the decision boundary in a plot if we assume that the two classes have equal prior probability.

g) If $P(\omega_1) = 0.75$, in which direction will the decision boundary change? Indicate this on the plot.

Exercise 4: Linear feature transforms

You are given a 2D dataset with 2 classes as plotted in the figure below.



The data has the following properties:

Covariance matrix C: $\begin{bmatrix} 36 & 21 \\ 21 & 19 \end{bmatrix}$

Eigenvectors of C: $v_1 = \begin{bmatrix} -0.55 \\ 0.83 \end{bmatrix}$ $\quad \mathrm{v}_2 = \begin{bmatrix} 0.83 \\ 0.55 \end{bmatrix}$

Eigenvalues of C: $\lambda_1 = 5, \quad \lambda_2 = 51$

a) Explain the criterion function that principal component analysis (PCA) optimizes

b) Explain if PCA requires any normalization of the input data

c) Which direction vector gives the first principal component?

d) PCA is a linear transform $y=A^Tx$ of the input data x. What is A for the data example?

e) How much of the variance in the data is explained by the first principal component?

f) Which geometrical relation is there between the first and the second principal component?

g) **<u>PhD Students only (subtask g only):</u>**
From the data listed above, we can construct the covariance matrix of the transform data y. What is the covariance matrix of y? No computation is needed.

h) In this exercise, the dominant direction is found by principal component analysis. Based on other topics in this course, could you suggest another method that could be used to find the dominant direction (unsupervised, no class labels used)?

i) An alternative to PCA is Fisher's linear discriminant. Which criterion function does Fisher use?

j) Do either PCA or Fisher have any limitations regarding how many features the transform can produce? Justify you answer.

Thank You for Your Attention!