UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam:	**IN5520 / IN9520 – Digital image analysis**
Date:	**Wednesday December 12, 2018**
Exam hours:	**09.00-13.00 (4 hours)**
Number of pages:	**10 pages of sketches to a solution**
Enclosures:	**1 page Appendix**
Allowed aid:	**Calculator**

- Read the entire exercise text before you start solving the exercises. Please check that the exam paper is complete. If you lack information in the exam text or think that some information is missing, you may make your own assumptions, as long as they are not contradictory to the "spirit" of the exercise. In such a case, you should make it clear what assumptions you have made.

- You should spend your time in such a manner that you get to answer all exercises. If you get stuck on one question, move on to the next question.

- There are 6 exercises. They are weighted proportional to the number of sub-questions, except for Exercise 5, which is weighted as an average exercise.

- Your answers should be **short**, typically a few sentences and / or a sketch should be sufficient.

- Do not give just a numerical answer, but demonstrate your reasoning.

Good luck!!

Exercise 1: Texture analysis

Assume that you are given a gray level image of size MxN pixels with b bits per pixel.

a) Describe how a normalized Gray Level Coccurrence Matrix is computed, and which parameters this involves.

Answer: Textbook stuff! ...

b) For a given inter-pixel distance and direction, how do we make the normalized GLCM symmetrical about the matrix diagonal without double counting?

Answer: Using the transpose. ...

c) Say that we have accumulated a normalized symmetrical GLCM for a given inter-pixel distance and direction.
How can we find what fraction of the pixel pairs in the image that have a difference of D gray levels? Please illustrate with a small sketch!

Answer: Matrix elements on the diagonal will all represent pixel pairs with no gray level difference.
Matrix elements that are one cell away from the diagonal represent pixel pairs with a difference of only one gray level.
So, the sum of all elements D cells away from the diagonal will represent the fraction of pixel pairs having a difference of D gray levels.

	J = 0	J = 1	J = 2	J = 3
I = 0	I – j = 0	| I – j | = 1	| I – j | = 2	| I – j | = 3
I = 1	| I – j | = 1	I – j = 0	| I – j | = 1	| I – j | = 2
I = 2	| I – j | = 2	| I – j | = 1	I – j = 0	| I – j | = 1
I = 3	| I – j | = 3	| I – j | = 2	| I – j | = 1	I – j = 0

Exercise 2: Feature extraction from gradients

Histogram of Gaussians (HOG) is a commonly used method for feature extraction.

a) Let us assume that we use Sobel-filters to compute gradients

$$H_x(i,j) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}, \quad H_y(i,j) = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

How are these filters used to compute the gradient magnitude and direction?

*Answer: Convolve image F with Hx to get Gx=F*Hx, and Gy=F*Hy, Then compute gradient magnitude as sqrt(Hx^2+Hy^2) and theta as atan2(Gy,Gx).*

b) HOG-features are compute in bins. In a given bin cell of size BxB pixels, describe briefly how the HOG-features are computed. By bin cell we mean the cell where gradient information is aggregated.

*Answer: Inside a bin cell, the gradient magnitude is weighted by a Gaussian window function (to assign most confidence to the center area in the bin), and accumulated into a gradient direction histogram. Gradient histograms are created in 9 directional histogram bins from 0-180. For a pixel with gradient magnitude M and gradient direction θ, the histogram bin according to θ is incremented by M*w(i,j), where w(i,j) is the window weight.*

c) For each bin you get 9 feature values. With a grid consisting of a high number of bins, the number of features will grow quickly. Suggest one new feature characterizing the information in the 9 values, and discuss briefly why you think this is a good feature.

Answer: After studying gradient histograms for simple symbols, we see that they are often characterized by either a dominant direction or the gradient is spread out. Suggested features can e.g. be the mean or median bin orientation, the normalized histogram value p(i) for the largest histogram peak, or the variance of the histogram. Other suggestions can also be provided as long as they are justified.

Exercise 3: Cartesian geometric moments

a) Give an expression for an ordinary geometric moment of order p+q of an object in an image, and explain the terms of the expression.

Answer: The expression for an ordinary moment is

$$m_{pq} = \sum_x \sum_y x^p y^q f(x,y)$$

where f(x,y) is the pixel value at the pixel coordinates (x,y). The summation is performed over the object.

b) Describe, in words or using mathematical expressions, how to get from an ordinary moment of order p+q to a central moment.

Answer: Shift the origin to the centre of mass of the object, and compute the moment as before.
Or: Find the coordinates of the center of mass of the object, and use the expression below.

$$\mu_{p,q} = \sum_x \sum_y (x-\bar{x})^p (y-\bar{y})^q f(x,y)$$

c) What kind of invariance is obtained by such central moments, as compared to an ordinary moment? Please explain!

Answer: Central moments are invariant to the position of the object in the image, since it is essentially an ordinary moment computed with the origin in its center of mass.

d) A third kind of moment is obtained when we normalize a central moment by

$$\eta_{pq} = \frac{\mu_{pq}}{(\mu_{00})^\gamma}, \quad \gamma = \frac{p+q}{2} + 1, \quad p+q \geq 2$$

What kind of invariance is obtained, and what is the condition for the expression above?

Answer: This is a scale invariant central moment (and thus also invariant to position), and the condition is that the scaling is the same in the x- and y direction.

1st question for the PhD-students only:

e) The seven Hu's moment combinations are position, scale, and rotation invariant, and may be useful to obtain features that will discriminate between different objects.

We have seen that for rectangles and ellipses, only the two first Hu moment combinations are different from 0.
The figure below gives a logarithmic plot of the first two Hu moments for binary rectangular objects of size 2a by 2b, and ellipses with semi-axes a and b, for a/b = [1,2,4,8,16].

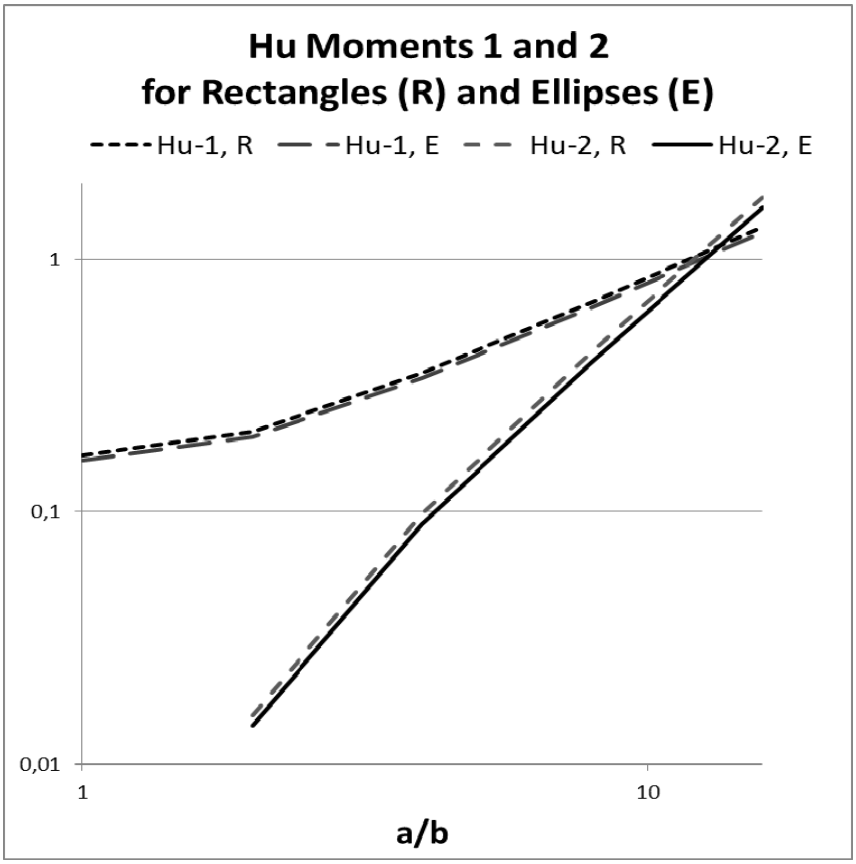
In the continuous case, the first two Hu moments of rectangles are given by

$$\phi_1 = \frac{1}{12}\left(\frac{a}{b} + \frac{b}{a}\right), \qquad \phi_2 = \left(\frac{1}{12}\right)^2 \left(\frac{a}{b} - \frac{b}{a}\right)^2$$

and for ellipses they are given by

$$\phi_1 = \frac{1}{4\pi}\left(\frac{a}{b} + \frac{b}{a}\right), \qquad \phi_2 = \left(\frac{1}{4\pi}\right)^2 \left(\frac{a}{b} - \frac{b}{a}\right)^2$$

What implication do you draw from the equations above and from this plot?



Answer: The relative difference between a rectangle and a same a/b ellipse feature is very small, only 4.5% for Hu-1 and 8.8% for Hu-2, regardless of eccentricity. Relative diferencef = [Hu(R) – Hu(E)]/Hu(R)
Thus, this will be useful only for large objects & little noise.
(For a circle and its bounding square, $\varphi_2 = 0$.)
Besides, features for a given rectangle (a/b) will be the same as for a slightly more eccentric ellipse.

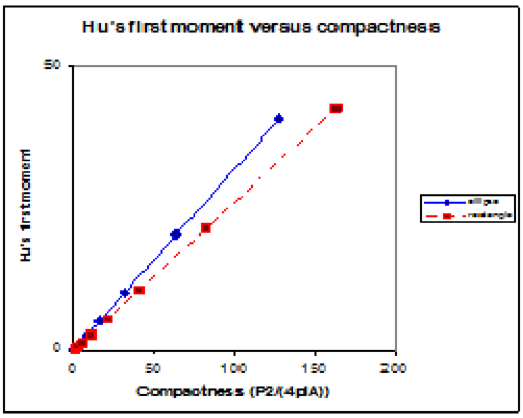
2nd question for the PhD-students only:

f) We define object compactness $\gamma = P^2/(4\pi A)$, where P is the perimeter length and A is the area. For a disc, γ is minimum and equals 1, while γ attains a high value for both complex objects, and for very elongated simple objects, like rectangles and ellipses where the a/b ratio is high.
For ellipses and rectangles, the compactness measure in the continuous case is given by:

$$\gamma_{ellipse} = \frac{1}{4}\left(\frac{a}{b}+\frac{b}{a}\right), \qquad \gamma_{rectangle} = \frac{1}{\pi}\left(\frac{a}{b}+\frac{b}{a}+2\right)$$

In an object classification setting, would you use both both the given compactness measure and the first Hu moment to characterize ellipses or rectangles? Please explain!

Answer: We notice that for ellipses, the first Hu moment is a simple linear function of its compactness measure, given by $\varphi_1 = \gamma/\pi$, while for rectangles the relationship is a little more complicated, but still approximately linear: $\varphi_1 = (\pi\gamma + 2)/12$, as illustrated in the plot to the right for rectangles (blue, top) and ellipses (red, bottom). So, using both the compactness measure and the first Hu moment to characterize ellipses or rectangles seems redundant, regardless of their size and a/b-ratio.



Exercise 4: Support vector machines

a) The basic optimization problem for a support vector machine classifier is:

$$\text{minimize} \quad J(w) = \frac{1}{2}\|w\|^2$$

$$\text{subject to} \quad y_i(w^T x_i + w_0) \geq 1, \quad i = 1,2,\ldots N$$

What is the total margin for this problem?

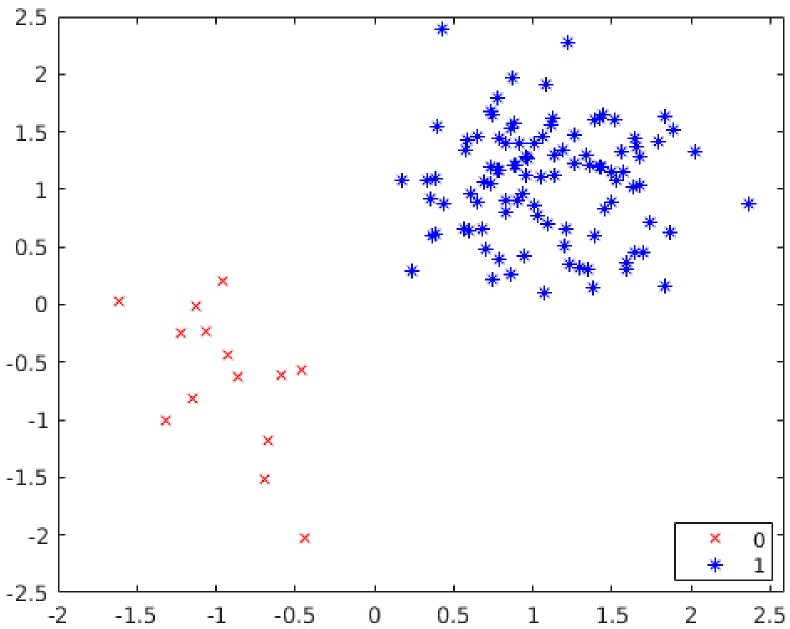
The margin that we maximize is 2/||w|| (1/||w|| on each side)

b) Support vector machines are fundamentally different from Gaussian classifiers in terms of how the decision boundary is found – explain why.

SVM uses the points closest to other classes to define the boundaries, and are thus sensitive to outliers. Gaussian classifiers use the class centres to define the boundaries.

c) Support vector machine classifiers can also be explained based on convex hulls. Explain the relationship between the convex hull of two regions and the hyperplane with maximum margin.
Answer: If the problem is linearly separable, the convex hulls for the two classes are non-overlapping. Furthermore, searching for the hyperplane is equivalent to searching for the two nearest points in the two convex sets.

d) Given below is a scatter plot of a binary classification problem. The plot is also copied to the appendix. Sketch the convex hulls and use this to find an approximate hyperplane.



e) In the general case the optimization problem is given as:

$$\max_{\lambda}\left(\sum_{i=1}^{N}\lambda_i - \frac{1}{2}\sum_{i,j}\lambda_i\lambda_j y_i y_j x_i^{\,T} x_j\right)$$

$$\text{subject to}\quad \sum_{\mathrm{i}=1}^{\mathrm{N}}\lambda_{\mathrm{i}} y_i = 0 \quad \text{and} \quad 0 \le \lambda_i \le C \quad \forall i$$

Explain briefly which of the terms in this equation that kernels are used to compute in a high-dimensional space,
and what the kernels measure.

Answer: Kernels are used to compute the inner product between pairs of samples xiTxj in a higher dimensional space. The inner product is a measure of similarity, the angle between two vectors can be expressed as the inner product. This is also seen in the RBF kernel.

Exercise 5: Alternative approaches to object size estimation

The graylevel image to the right is from one of the chapters of the textbook used throughout this course.

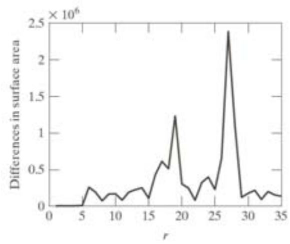
It depicts a number of bright wood dowels of different brightness on a dark background.

There may be visible structures both in each object (dowel) and in the background.

a) Please detail maximum two different approaches to obtain an estimate of the distribution of object sizes in the image.

Answer: 1) At least describe in detail the pattern matching approach from Chapter 9 of GW.

- This is termed “Granulometry”: determining the size distribution of particles in an image.
- Principle: perform a series of morphological *openings* (erosion followed by dilation), i.e., first creating an image containing the running local **minimum** gray level in the region defined by the structuring element, and based on that output image creating a second output image containing the running local **maximum** gray level in the region defined by the (reflected) structuring element.
- Let pixel (x,y) in the outimage have this value.
- Compute the sum of all pixel values after the opening.
- Repeat this with increasing radius r of structuring element
- Compute the difference in this sum between radius r and r-1, and plot this as a function of radius, see figure.



$$f \circ S = (f \ominus S) \oplus S$$
$$= \max(\min(\;f\,))$$

Gray level opening may have been described elsewhere already.

2) An obvious alternative is to do edge detection (gradient detection), followed by Hough transform to estimate circle radii. Since there are irrelevant structures both in the objects and in the background, Canny’s method may seem to be relevant, and that should be described.

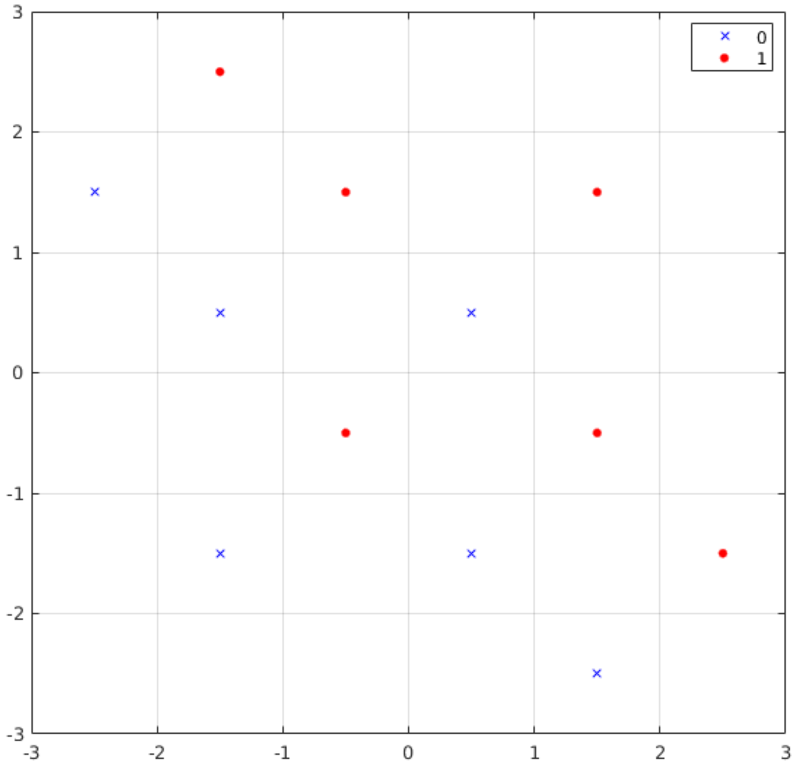
Sobel/Prewitt filtering should be mentioned anyway, since it occurs in Canny and since it gives gradient direction that is vey useful in the HT.

Exercise 6: PCA and classification

A set of samples from two classes are given as (a scatter plot is also given below):

Class1: $\begin{bmatrix} -2.5 & 1.5 \\ -1.5 & 0.5 \\ 0.5 & -1.5 \\ 1.5 & -2.5 \\ 0.5 & 0.5 \\ -1.5 & -1.5 \end{bmatrix}$ $\mu_1 = \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}$ $\Sigma_1 = \begin{bmatrix} 2.4 & -1.6 \\ -1.6 & 2.4 \end{bmatrix}$ $\Sigma^{-1}{}_1 = \begin{bmatrix} 0.75 & 0.5 \\ -0.5 & 0.75 \end{bmatrix}$

- Class2: $\begin{bmatrix} -1.5 & 2.5 \\ -0.5 & 1.5 \\ 1.5 & -0.5 \\ 2.5 & -1.5 \\ -0.5 & -0.5 \\ 1.5 & 1.5 \end{bmatrix}$ $\mu_2 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$ $\Sigma_2 = \begin{bmatrix} 2.4 & -1.6 \\ -1.6 & 2.4 \end{bmatrix}$ $\Sigma^{-1}{}_2 = \begin{bmatrix} 0.75 & 0.5 \\ -0.5 & 0.75 \end{bmatrix}$



- Global mean: $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- Global covariance matrix: $\begin{bmatrix} 2.45 & -1.18 \\ -1.18 & 2.45 \end{bmatrix}$
- Eigenvalues 1.27 and 3.65 with eigenvectors: $\begin{bmatrix} -0.7 \\ -0.7 \end{bmatrix}$ and $\begin{bmatrix} -0.7 \\ 0.7 \end{bmatrix}$

a) Find the direction of the first principal component of this data set.
This is the eigenvector associated with the largest eigenvalue [-0.7, 0.7]

b) Sketch the eigenvector on the plot

c) Compute the new feature values using this component for all samples.

*For class 1: Feature vector transpose*eigenvector:*

$$\begin{bmatrix} 2.5 * 0.7 + 1.5 * 0.7 = 2.8 \\ 1.5 * 0.7 + 0.5 * 0.7 = 1.4 \\ -0.5 * 0.7 - 1.5 * 0.7 = -1.4 \\ -1.5 * 0.7 - 2.5 * 0.7 = -2.8 \\ -1.5 * 0.6 - 1.5 * 0.7 = 0 \\ -0.5 * 0.7 + 0.5 * 0.7 = 0 \end{bmatrix}$$

For class 2:

$$\begin{bmatrix} 1.5 * 0.7 + 2.5 * 0.7 = 2.8 \\ 0.5 * 0.7 + 1.5 * 0.7 = 1.4 \\ -1.5 * 0.7 - 0.5 * 0.7 = -1.4 \\ -2.5 * 0.7 - 2.5 * 0.7 = -2.8 \\ -0.5 * 0.7 + 0.5 * 0.7 = 0 \\ -1.5 * 0.7 + 1.5 * 0.7 = 0 \end{bmatrix}$$

d) Plot the points in the figure and discuss how the first principal component performs in this case.
The two classes collapse to equal points.

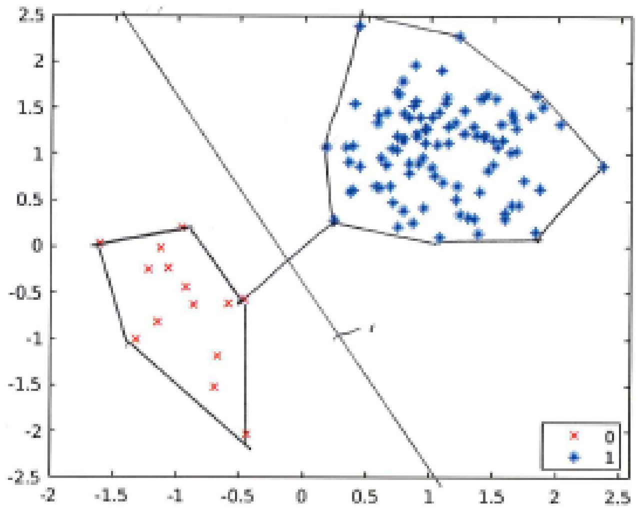
Good Luck!

Kandidatnummer:

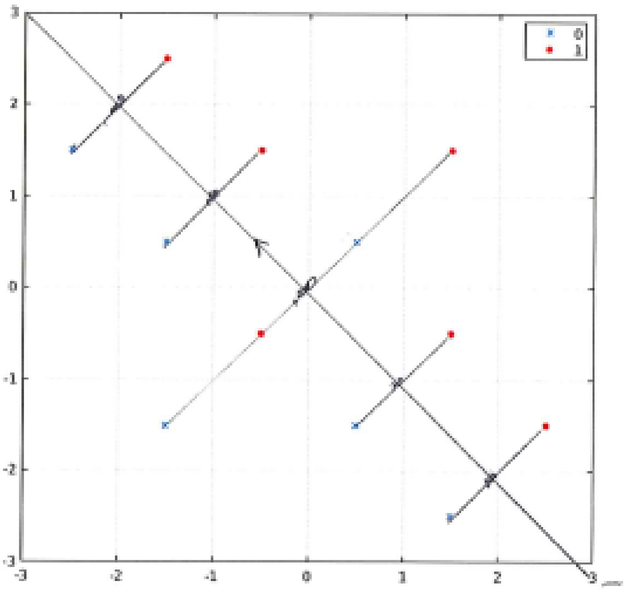
Vedlegg, Eksamen IN5520/9520, 12. desember, 2018

Vennligst riv av dette arket og lever det saman med svaret ditt.

Figur 1



Figur 2



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1. principal component