# UNIVERSITY OF OSLO

## Faculty of Mathematics and Natural Sciences

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| **Exam:** | **IN5520 / IN9520 – Digital image analysis** |
| **Date:** | **Wednesday December 9, 2019** |
| **Exam hours:** | **09.00-13.00 (4 hours)** |
| **Number of pages:** | **7 pages of sketches to a solution** |
| **Enclosures:** | **Nne** |
| **Allowed aid:** | **Calculator** |

- Read the entire exercise text before you start solving the exercises. Please check that the exam paper is complete. If you lack information in the exam text or think that some information is missing, you may make your own assumptions, as long as they are not contradictory to the "spirit" of the exercise. In such a case, you should make it clear what assumptions you have made.
- You should spend your time in such a manner that you get to answer all exercises. If you get stuck on one question, move on to the next question.
- There are 5 exercises. They are weighted proportional to the number of sub-questions.
- Your answers should be **short**, typically a few sentences and / or a sketch should be sufficient.
- Do not give just a numerical answer, but demonstrate your reasoning.

*Good luck!!*

# Exercise 1: Texture Analysis

Assume that you are given a gray level image of size MxN pixels with b bits per pixel.

a) Describe how a normalized Gray Level Coccurrence Matrix (GLCM) is computed, and which parameters this involves.

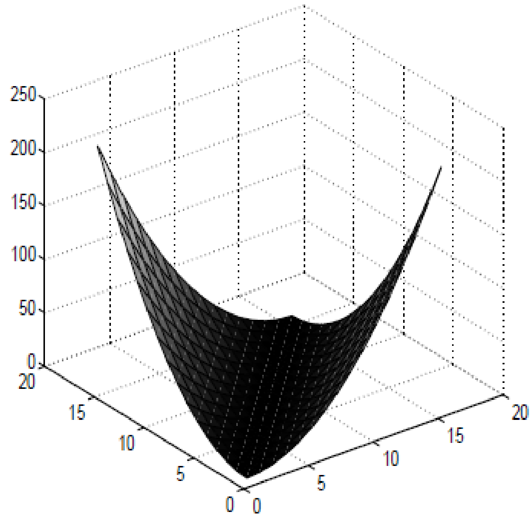
*Answer: Textbook stuff!*
*Eventual re-quantization of the input gray level image from $G = 2^b - 1$ to L gray levels should be motivated.*
*4 elements should be mentioned:*

1. *Initialize matrix of GxG (or LxL) with 0's.*
2. *Go through all MxN pixels where pixel pair of gray levels i and j a distance d pixels apart in direction $\theta$ are inside image, and add 1 at position (i,j) in GLCM.*
3. *Finally, normalize by integer sum of matrix entries.*
4. *Parameters: G (or eventually L),d, $\theta$ (or $\Delta x, \Delta y$).*

b) For a given inter-pixel distance and direction, how do we make the normalized GLCM symmetrical about the matrix diagonal without double counting?

*Answer: The GLCM in the opposite direction of $\theta$ is the transpose of the GLCM in the direction $\theta$.*
*So we have $P(d,\theta+\pi) = [P(d,\theta)]^T$ - see slide 38 of lecture 2.*
*Thus, double counting is avoided by adding the transpose of the GLCM to the GLCM before normalizing.*
*If both have been normalized, divide the sum by 2.*

c) Assume that we have accumulated a normalized symmetrical GLCM for a given inter-pixel distance and direction.
Give the expression for a GLCM feature that has a weighting function equal to zero along the diagonal (i = j), and increases quadratically away from the diagonal, as illustrated for G=15.




Answer:

$$\text{Inertia} = \sum_{i=1}^{G}\sum_{j=1}^{G}(i-j)^2 P(i,j)$$

$$W(i,j) = (i-j)^2$$

What will the effect of this weight function be, and what kind of images will get a high feature value?

*Answer: It will favor contributions from P(i, j) away from the diagonal ($i \neq j$), i.e., give higher values for images with high local contrast.*

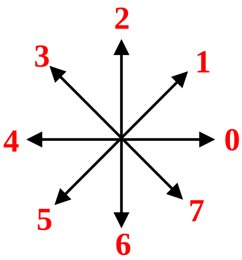
d) How can we find what fraction of pixel pairs at the given inter-pixel distance and direction in the image that have an absolute difference $|i-j| \geq D$ gray levels, while both i and j are in the upper ¼ of the gray scale? Please illustrate!

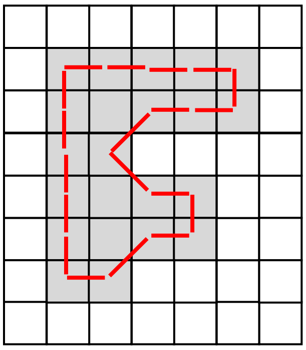
*Answer: Matrix elements on the diagonal will all represent pixel pairs with no gray level difference. Matrix elements that are one cell away from the diagonal (along I or j-direction) represent pixel pairs with a difference of only one gray level. So, the sum of all elements from D cells away from the diagonal (along i and j-axis) into the corners of the matrix will represent the fraction of pixel pairs having a difference of D gray levels or more. In yellow for D=2.Restricted to upper ¼ of gray scale: sum only outside dark grey.*

| | J = 0 | J = 1 | J = 2 | J = 3 |
|---|---|---|---|---|
| I = 0 | I – j = 0 | \| I – j \| = 1 | \| I – j \| = 2 | \| I – j \| = 3 |
| I = 1 | \| I – j \| = 1 | I – j = 0 | \| I – j \| = 1 | \| I – j \| = 2 |
| I = 2 | \| I – j \| = 2 | \| I – j \| = 1 | I – j = 0 | \| I – j \| = 1 |
| I = 3 | \| I – j \| = 3 | \| I – j \| = 2 | \| I – j \| = 1 | I – j = 0 |

# Exercise 2: Chain Codes

You are given the 8-directional chain code and the binary object below.



a) Find the absolute chain code of the boundary of the object clockwise from the upper left pixel.

Answer: The absolute code starting at the upper left point and moving clockwise is 19 digits:

0000644570645422222

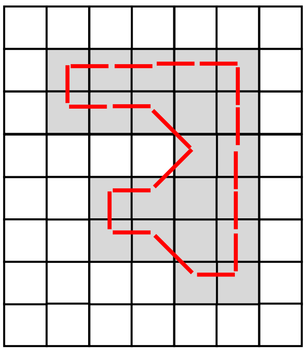
b) Which technique, based on the 8-directional absolute chain code, can be used to make a description of the object that is independent of the start point?
Demonstrate this by starting at the lower right pixel of the object, instead of the upper left.

Answer: A minimum circular shift of the clockwise absolute chain code gives start point invariance.
Demonstration: The absolute chain code starting at the lower right point is: 4222220000644457645.
The minimum circular shift of this is: 0000644457645422222, which is the same code as when we started at the upper left in a).

c) Which technique, based on the clockwise **relative** chain code, will give you the same description of the object, independent of the start point?
Demonstrate this by starting at the upper left and the lower right object pixel.

Answer: A minimum circular shift of the clockwise relative chain code gives a normalization for start point.
Demonstration:
The clockwise relative code, starting at the upper left pixel of the object, is the 19 digits:
0222002343003102222 . The minimum circular shift of this is: 00233430031022220222.
With start point at the lower right pixel we get 1022220222002343003 which has a minimum circular shift 0023430031022220222, which is the same as above.

d) Rotation invariance is inherent in relative chain codes.
But what if the object has been flipped horizontally.
How can you then determine if it is the same object?



Answer: The simplest way is to flip it back, select a starting point, and do as in the exercise above.
Alternatively, the anti-clockwise relative code, using a flipped code, and starting upper right, is 0222002343003102222, which has a minimum circular shift: 002343003102220222. So it is the same object!
As a vertical flip is equivalent to a rotation by 180 degrees followed by a horizontal flip, this works for vertical flips too.

# Exercise 3: Geometric Moments and Hough Transform

Assume that you have thresholded a gray level image into a binary image ***b(x,y)*** containing a solid object (pixel value = 1) and a background (pixel value 0).

a) Describe a moment-based approach to find the center of mass of the object.

Answer: We use first order moments to find the center of mass

$$m_{10} = \sum_x \sum_y x\, b(x,y) = \bar{x} m_{00} \quad \Rightarrow \bar{x} = \frac{m_{10}}{m_{00}}$$

$$m_{01} = \sum_x \sum_y y\, b(x,y) = \bar{y} m_{00} \quad \Rightarrow \bar{y} = \frac{m_{01}}{m_{00}}$$

b) Assume that the object is an equilateral triangle, located somewhere in the image. Give a definition of the object orientation, and describe a moment-based approach to estimate the orientation of the object. In this case, is the result unique?

Answer: We either use central moments defined by

$$\mu_{pq} = \sum_x \sum_y (x - \bar{x})^p \, (y - \bar{y})^q \, b(x,y)$$

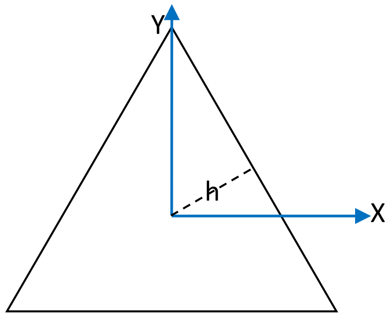
And use the three second order central moments $\mu_{11}$, $\mu_{20}$, and $\mu_{02}$ to estimate the orientation of the object by

$$\theta = \frac{1}{2} \tan^{-1} \left[ \frac{2\mu_{11}}{(\mu_{20} - \mu_{02})} \right]$$

Or we shift the origin to the center of mass, and use the ordinary moments to find the orientation. Since the orientation is defined as "the angle, relative to the X-axis, of an axis through the centre of mass that gives the lowest moment of inertia", we have three possible orientations of this triangle.

c) Assume that we translate the origin to the center of mass of the triangle, then apply a gradient detector to the binary image, and use the "normal representation" Hough Transform.

Assume that one side of the triangle is parallel to the x-axis, describe the contents of the Hough space.




Answer:

- *There will be three TH peaks, $\pi/3$ apart.*
- *The peaks will have the same height,*
  *since the three line segments have the same length, s.*
- *The normal onto the three sides are of the same length, so the all peaks will occur at $\rho$=h.*

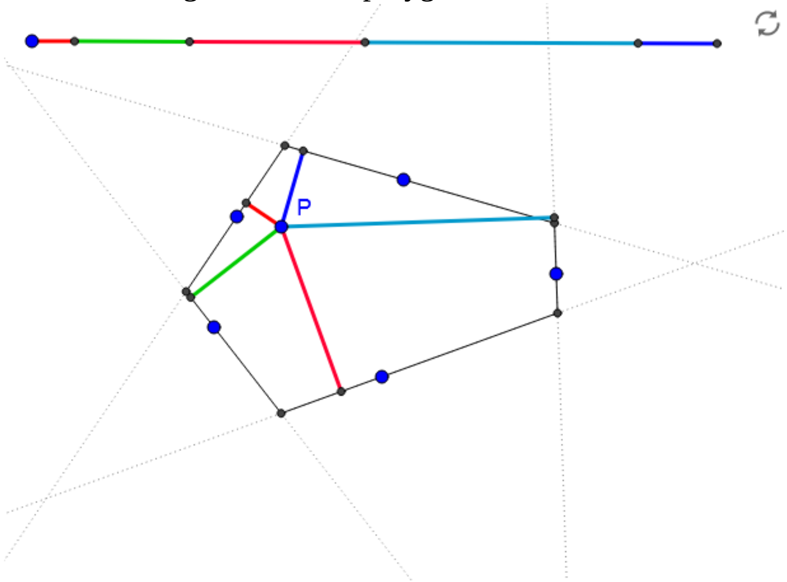
d) What happens in the Hough space if the origin ismoved inside the triangle, so that it does not coincide with the center of mass?

Answer:

- *The three TH peaks will still be $\pi/3$ apart.*
- *The peaks will still have the same height, but the normals onto the three sides are different.*

**The following two questions are intended for the PhD-students:**

e) What will happen in the Hough domain, if an equi-angular polygon is rotated anti-clockwise around an off-centered origin inside the polygon?




*Answer:*

- *The N peaks in the Hough space will be 180*(1-2/N)° apart on the θ-axis.*
- *The normals onto the N sides are in general of different length, so the peaks occur at different ρ-values.*
- *The sides of the polygon are in general different length, so the peaks have different heights.*
- *The N peaks in the (θ,ρ)-domain will slide in the positive θ-direction, keeping the distance between the maxima and the ρ-values, sliding out of the [-π/2,π/2]-domain at π/2 and reappearing at –π/2.*

f) Which polygon feature in the HT domain is invariant to the location of the origin inside an equiangular polygon, and may be useful when working with noisy images?

*Answer: "The sum of distances from an interior point to the sides of an equiangular polygon (∑ρ) does not depend on the location of the point, and is that polygon's invariant." Viviani's theorem. So a hint is that the length of the bar above the sketched polygon is constant when P moves.*

## Exercise 4: Short questions on classification

a) Explain briefly the drawbacks of overfitting a classifier.

*Keywords: will have poor generalization ability as it is fit too well to the training data. If the model is compex, it will also be difficult to explain the results.*

b) Explain briefly how you should split the available set of labelled data for a classification problem, and what the resulting subsets should be used for.

*Answer: it should be split into training, validation and test sets. The proportion into each subset will depend on the problem and the amount of data available. Training set: estimate classifier parameters. Validation: estimate hyperparameters. Test: estimate final classification accuracy, use only once.*

c) Given a classification problem, describe briefly in general how you use the discriminant functions to find the decision boundary between two classes.

*Answer: The decision boundary is where gi(x)=gj(x) for two classes i and j.*

d) When evaluating classifier performance, we often use precision, sensitivity and specificity:

Precision = TP/(TP+FP)
Sensitivity = TP/(TP+FN)
Specificity = TN/(TN+FP)

Give an example of a binary classification problem where specificity is more important than sensitivity.

*Answer: One example: Consider a system when fining cars based on licence plate recognition. Cars that have not prepaid should be fined. Due to dirt and imperfect imaging not all license plates are correctly recognized. High spesificity means that fewer cars that have paid are fined.*

e) Explain briefly which parameters that a SVM classifier has, and how they should be determined.

*Answer: Choice of kernel function, normally RBF with parameter $\sigma$, and the cost of misclassification C. A grid search on the validation data should be used to find the best values of $\sigma$ and C.*

f) Assume that you use a Gaussian classifier with full covariance matrices. Discuss if you have some challenges working in high-dimensional feature space, e.g. 100 features.

*Answer: The covariance matrix then has dimension 100x100, and the number of unique parameters to estimate for each class is 100x(100-1)/2. Firstly, you need a lot of training data to robustly estimate these as such a high dimensional space will be mostly empty. Secondly, computing the inverse is likely to be problematic due to singularity.*

## Exercise 5: Classification

a) Give Bayes rule for a classification problem .

*Answer:*

$$P(\omega_j|\boldsymbol{x}) = \frac{p(\boldsymbol{x}|\omega_j)P(\omega_j)}{p(\boldsymbol{x})}$$

b) Given a Gaussian classifier with d features. Consider a binary classification problem. The class-conditional probability density is given as:

$$\boldsymbol{p(x|\omega_s) = \frac{1}{(2\pi)^{d/2}|\Sigma_s|^{d/2}} exp\left[-\frac{1}{2}(x-\mu_s)^t\Sigma_s^{-1}(x-\mu_s)\right]}$$

Write down the logarithmic descriminant function without any assumptions on the covariance matrix.

*Answer:*

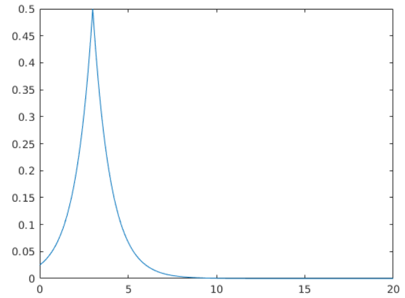
$$g_i(\mathbf{x}) = -\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_i)^t\Sigma_i^{-1}(\mathbf{x}-\boldsymbol{\mu}_i) - \frac{d}{2}\ln 2\pi - \frac{1}{2}\ln|\Sigma_i| + \ln P(\omega_i)$$

c) If we assume a 2-class problem with $P(\omega_1)=0.75$ and $P(\omega_2)=0.25$ and equal diagonal covariance matrices, describe by words how we can find the decision boundary.

*Answer: The boundary will be normal to the line connecting the mean values, but located at a distance 0.75 to class 1 and 0.25 to class 2 along that line.*

d) If we assume a 2-class problem with $P(\omega_1)=0.5$ and $P(\omega_2)=0.5$, mean vectors $\mu_1=[1,3]^T$ and $\mu_2=[-1,1]^T$ and equal diagonal covariance matrices, find the equation for the decision boundary.

*Answer: The boundary will be the line $x_2=2-x_1$*




e) Assume that you have a 1-d feature vector that follows a Laplace distribution given two parameters mean $\omega_s$ and shape $b_s$ by:

$$p(x|\omega_s) = \frac{1}{2b_s} e^{-\frac{|x-\mu_s|}{b_s}}$$

$$p(x|\omega_s) = \begin{cases} \frac{1}{2b_s} e^{-\frac{\mu_s - x}{b_s}} & if\ x < \mu \\ \frac{1}{2b_s} e^{-\frac{x-\mu_s}{b_s}} & if\ x \geq \mu \end{cases}$$

The shape of this distribution is indicated in the figure. Assume two classes with equal prior probability and equal shape $b_s=1$. Compute the decision boundary for this 1-d case. For simplicity, you can assume $\mu_2 > \mu_1$ .

*Answer: For the bondary between* $\mu_1$ and $\mu_2$:

$$\frac{1}{2} e^{-(x-\mu_1)} = \frac{1}{2} e^{-(\mu_2 - x)}$$

$$-x + \mu_1 = -\mu_2 + x$$

$$x = \frac{\mu_1 + \mu_2}{2}$$

*This is exacly the same solution as for the Gaussian, as the distributions are symmetrical around the mean and the prior probabilities and the shape parameters are equal. This could also be expected by looking at the plot.*

f) For the binary problem in e), set up an expression for the total classification error.

*Answer: The error will be the following integral:*

$$\int_{-\infty}^{T} \frac{1}{2} e^{\frac{\mu 2 - x}{1}} dT + \int_{T}^{\infty} \frac{1}{2} e^{\frac{x - \mu 1}{1}} dT$$

***Thank You for Your Attention!***