UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam:	**INF 4300 / INF 9305 – Digital image analysis**
Date:	**Monday December 16, 2013**
Exam hours:	**14.30-18.30 (4 hours)**
Number of pages:	**xx pages plus xx pages enclosures**
Enclosures:	**None**
Allowed aid:	**Calculator**

- Read the entire exercise text before you start solving the exercises. Please check that the exam paper is complete. If you lack information in the exam text or think that some information is missing, you may make your own assumptions, as long as they are not contradictory to the "spirit" of the exercise. In such a case, you should make it clear what assumptions you have made.

- You should spend your time in such a manner that you get to answer all exercises shortly. If you get stuck on one question, move on to the next question.

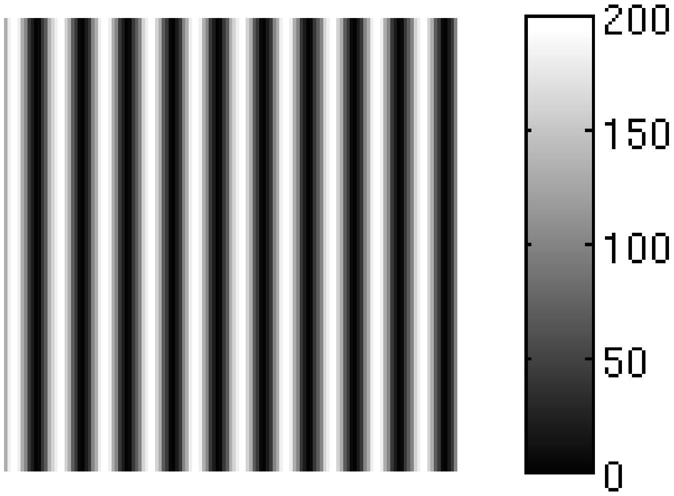
- Two of the questions are based on sketches enclosed two extra sheet at the end of the exam text. Please give your solution on these sheets, mark them with your candidate number, and include them in your solution.

- Your answers should be **short**, typically a few sentences and / or a sketch should be sufficient.

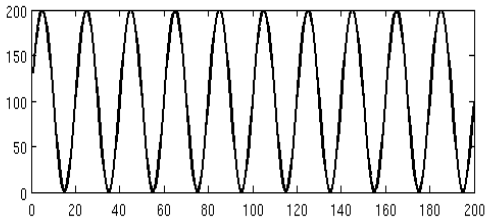
Good luck!!

Exercise 1: Texture

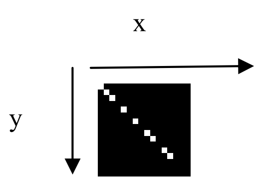
You are given an image with a periodic texture in a sinusoidal pattern as given below. The image was created as a 1D sinus signal $f(x)=100\sin(10*2\pi x/200)+100$; so the amplitude varies gradually between 0 and 200 (not as abrupt as the image figure might indicate).



A profile of the gray levels along a horisontal line in the in the image is also given:

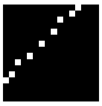


a) If you create a GLCM matrix with an offsets $\Delta x= 0$ and $\Delta y = 10$, what would the shape of the GLCM matrix look like? Please make a simple sketch of the GLCM matrix including the axes of the coordinate system.
   Will it change with a different Δy offset if $\Delta x = 0$? Please explain!
   It will be points on the diagonal $i = j$.
   The answer to the last sub-question is NO, since $i=j$ no matter what the value of Δy is.

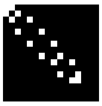
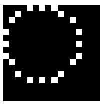


b) This image is periodic in the horisontal direction with a period of x=20.
   What will the shape of the GLCM matrix look like with $\Delta x=20$ and offset $\Delta y=0$? Explain!
   This will also give points on the diagonal $i=j$ because we hit exactly the period of the signal.

c) Sketch the shape of the GLCM matrix with offsets Δx=10 and Δy=0, and explain your reasoning.
This will give points on the off-diagonal, because we hit pixels half a period apart.



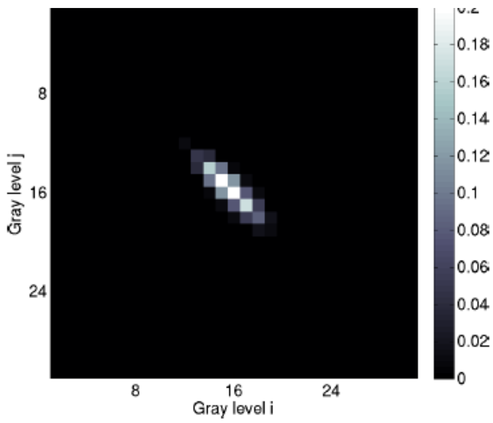
d) **PHD students only:**
As Δx goes from 0 to 10, discuss how the shape of the GLCM matrix will change.
It will gradually go from a diagonal, through an ellipse, and be a circle when Δx is 1/4T, where T is the period. From then it will go to an ellipse with orientation along the off-diagonal as Δx goes from T/4 towards T/2, and it will end on the off-diagonal.

 *Δx=1*  *Δx=5*

With Δx=9 it will be similar to Δx=1 but oriented on the off-diagonal.

Exercise 2: Texture

You are given an isotropic normalized Gray Level Cooccurrence Matrix computed with G=32.



a) What can you say about the range of gray levels that the image contains?
Only gray levels around 16 are present (13-18)

b) Discuss what you know about the texture in the image, and how do you think the original image looks?
Since it contains only similar gray levels in all directions, the image is fairly homogeneous with gray levels around 16.

c) There are several possible features that may be extracted from the GLCM.
Which of the two features below will give a high feature value for the given GLCM?
Entropy:

$$E = -\sum_{i=0}^{G-1}\sum_{j=0}^{G-1} P(i,j) \times log(P(i,j))$$

Inverse difference moment:

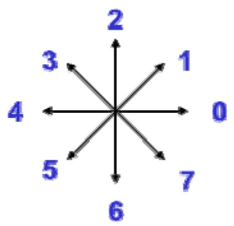
$$IDM = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} \frac{1}{1+(i-j)^2} P(i,j)$$

Please explain your reasoning.

One should notice that all probabilities are between 0 and 1, and that the weight $-log(P(i,j))$ is maximum for the small GLCM values and smaller for the larger GLCM values.
So inhomogeneous scenes have high entropy, while a homogeneous scene has a low entropy.
Inverse Difference Moment is also called "Homogeneity", as it gives high values for homogeneous images. Because of the weighting factor $(1+(i-j)^2)^{-1}$ IDM will get small contributions from inhomogeneous areas ($i \neq j$), and higher contributions from $i \approx j$.

Exercise 3: Discrimination of objects based on chain codes

You are given a set of objects. The absolute chain code for each object is given as:
Object 1: 1 7 4 4
Object 2: 1 7 5 3
Object 3: 0 5 3 0
Object 4: 6 6 4 4 2 2 0 0
Object 5: 2 0 6 4
Object 6: 7 7 5 5 2 2 2 2



a) Draw the objects in a simple sketch. Which shapes do the objects represent?
3 triangles and 3 squares of varying rotation and size

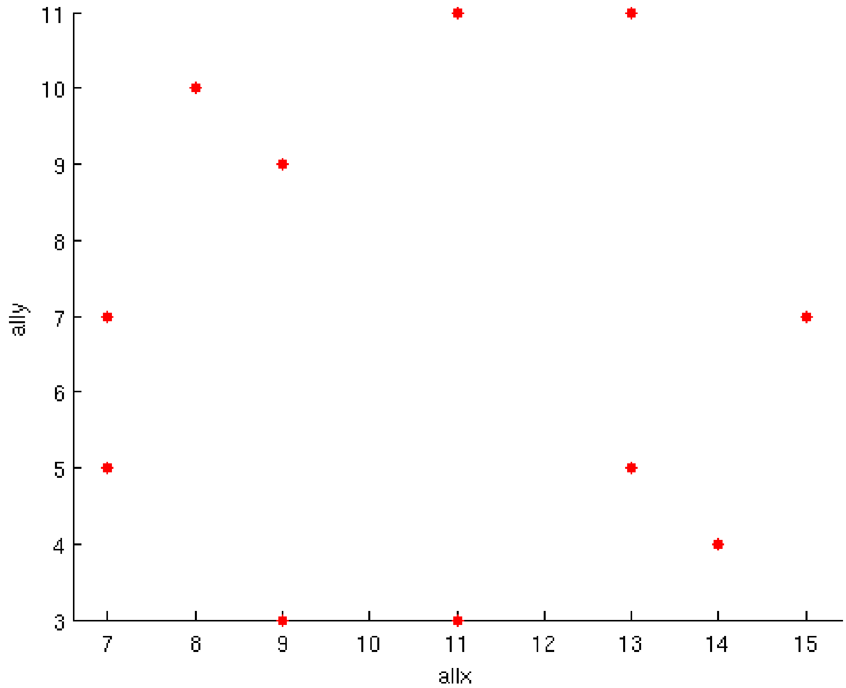
b) What kind of chain code would you use to best discriminate between the object types? Justify you answer. Can you use chain code to separate the two shapes classes perfectly?
First difference, minimum magnitude. Chain code alone cannot be used to give the same code for the larger vs. smaller triangles/squares.

c) **Phd students only:**
From the chain code, can you derive a new feature that is also invariant to object size?
There are many possibilities here, e.g only storing the changes in direction, or counting the number of direction changes in the relative chain code.

Exercise 4: Clustering and classification

You are given the following set of data points:
(15,7), (11,11), (13,11),(8,10),(9,9),(7,7), (7,5),(13,5),(14,4),(9,3), (11,3).

The points are plotted in the scatter plot below.



a) Perform a K-means clustering of the points with K=3 and starting with the cluster centers (7,5), (9,9) and (11,3). You can do the clustering either by computation or geometrically. Indicate the new cluster centers on figure 1 in the enclosure.
List the clusters that each point will belong to.

List the cluster centers after each iteration. How many iterations are necessary before no point will change cluster during a new iteration?

After 1 iteration, the means will be (7,6) (10.25,10.25) and (12.4,4.4).
In the second iteration point(9,3) wil change cluster. New cluster centers are (7.6,5) (10.25,10.25) (13.25,4.75)After this iterations no points will change cluster so these cluster centers are final.

b) Assume now that you train a Gaussian classifier with equal diagonal covariance matrices with the end result from point a. Plot the resulting decision boundaries on figure 1 in the enclosure. Assume that the classes have equal prior probabilities.

See figure above.

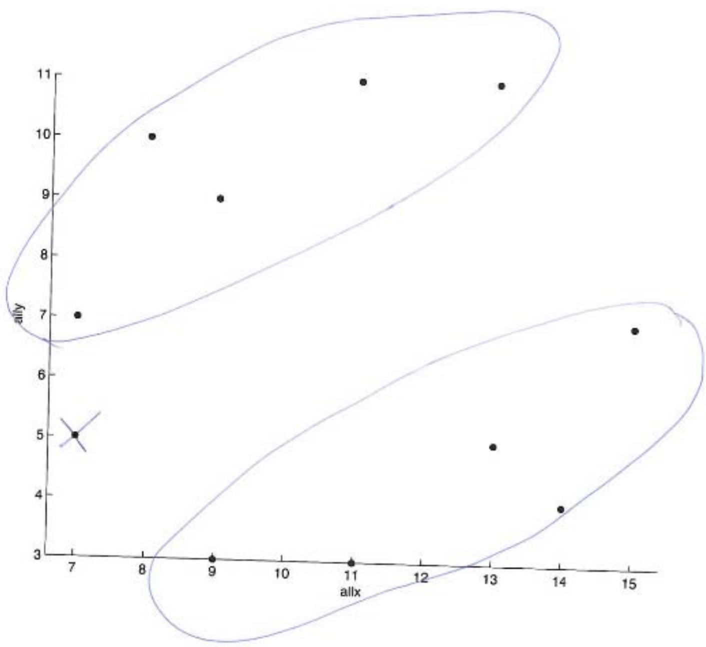
c) What is the estimated classification accuracy on the training set if this is the training set for the Gaussian classifier with 3 classes and equal diagonal covariance matrices.

This classifier will classify all training samples correctly, so the error will be 0.

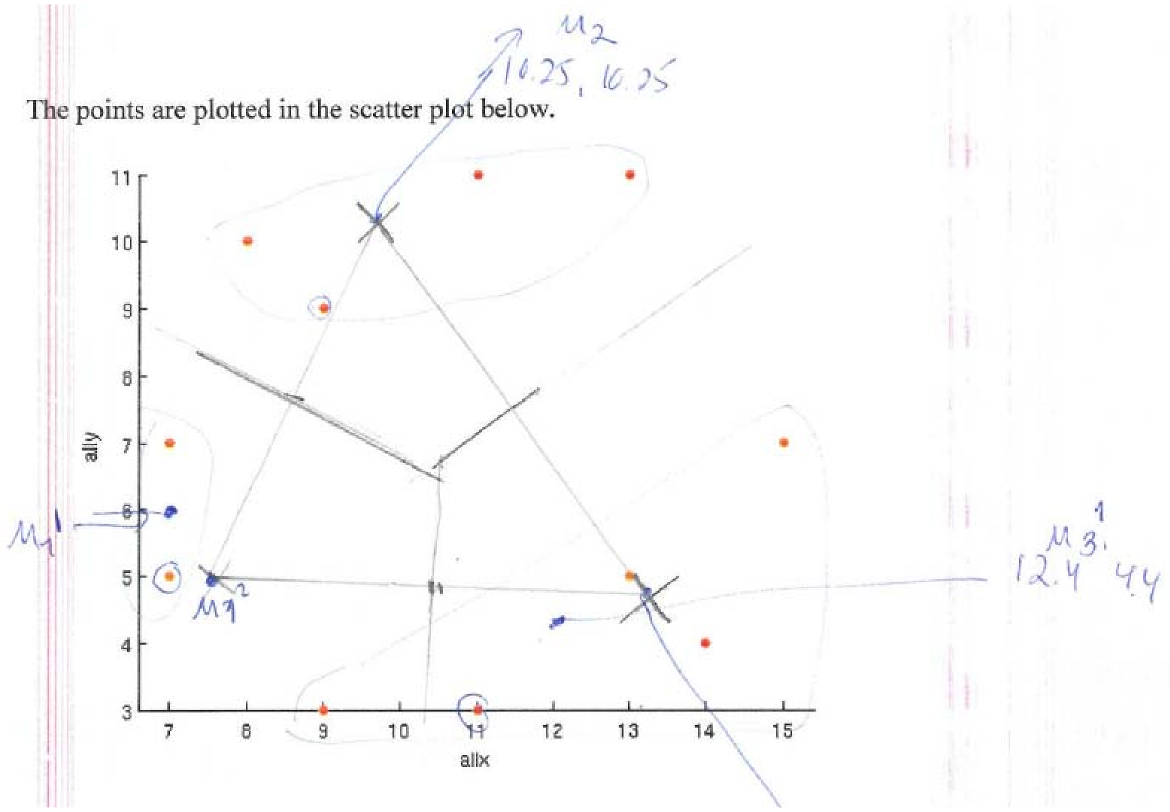
d) Consider now how the resulting classification would be if we removed one of the original cluster centers that you used in a). If you want a classification with only 2 remaining classes, and you want the resulting classes to be well described using a common covariance matrix, which cluster center would you remove?

If we remove cluster 1 with center (7.6,,5), both classes will be well described with a common elongated ellipse centered at the two new mean values. If we remove one of the other clusters, the classes will not be well modelled with a covariance matrix of the same orientation.

e) Sketch the shape of the resulting two Gaussian classes with common covariance matrix on the plot in figure 2 in the enclosure.



The points are plotted in the scatter plot below.



Exercise 5: Mathematical morpholog

a) Explain morphological opening of a binary image.

Answer: Opening consists of erosion followed by dilation. Erosion removes all structures that the structuring element can not fit inside, and shrinks all other structures.
Dilating the result of the erosion with the same structuring element,
the structures that survived the erosion (were shrunken, not deleted) will be restored.
The name “opening” hints that the operation can create an opening between
two structures that are connected only in a thin bridge, without shrinking the structures (as erosion would do).

$$f \circ S = (f \,\theta\, S) \oplus S$$

b) Given the 10x10 binary image below

0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	0	1	1	0	0	0	0
0	0	1	1	1	1	1	1	1	0
0	1	1	1	1	0	1	1	1	0
0	0	1	1	1	0	1	1	1	0
0	0	0	0	0	0	0	0	0	0

1	1	1
1	1	1
1	1	1

Show the necessary steps to arrive at the **top-hat** transform of that image, using the structuring element to the right, having a centered origin.
What does the top-hat transform give you in this case?

The eroded image:

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0
0	0	0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Opening = dilated result:

0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	0	1	1	1	0
0	0	1	1	1	0	1	1	1	0
0	0	1	1	1	0	1	1	1	0
0	0	0	0	0	0	0	0	0	0

TH = Original-Opening:

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	1	0
0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0
0	0	0	0	0	1	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

The top hat transform simply gives the binary 1’s that were removed during the opening, since top-hat is the difference between the image and its opening.

Exercise 6: Object features – moments

a) How do we define the moments of inertia of an intensity distribution f(x,y)?

Answer: The moments of inertia are the second order central moments, around the two axes that are parallel to the image axes x and y, passing through the center of mass of the object. Equations should also be given:

$$\mu_{20} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}(x-\bar{x})^2 f(x,y), \quad \mu_{02} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}(y-\bar{y})^2 f(x,y)$$

and

$$\mu_{11} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}(x-\bar{x})(y-\bar{y}) f(x,y)$$

b) In terms of inertial moments, what is the requirement for a 2D object to exhibit a unique orientation?

Answer: The requirement for a unique object orientation could simply be given as $\mu_{20} \neq \mu_{02}$.

c) Show the simple relationships between the two moments of inertia μ_{20} and μ_{02} of an image object f(x,y) about axes through its center of mass and the same moments of inertia about the parallel image coordinate axes (x=0 and y=0).

Answer: It is easy to show that $\mu_{20} = m_{20} - x' m_{10}$ and $\mu_{02} = m_{02} - y' m_{01}$
where x' and y' are the center of mass coordinates of the object in the x- and y-direction, and (m_{10}, m_{01}) are the first order ordinary moments.
Doing this, one also has to know that x' = m_{10}/m_{00} and y' = m_{01}/m_{00}:

$$\mu_{20} = \sum_x\sum_y (x-\bar{x})^2 f(x,y) = \sum_x\sum_y \left(x^2 - 2x\bar{x} + \bar{x}^2\right) f(x,y)$$

$$= m_{20} - 2\bar{x}\, m_{10} + 2\bar{x}\frac{m_{10}}{m_{00}} m_{00} = \underline{\underline{m_{20} - \bar{x}\, m_{10}}}$$

Good Luck!