UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam:	**INF 4300 / INF 9305 – Digital image analysis**
Date:	**Thursday December 4, 2015**
Exam hours:	**09.00-13.00 (4 hours)**
Number of pages:	**9 pages of sketches to a solution**
Enclosures:	**None**
Allowed aid:	**Calculator**

- Read the entire exercise text before you start solving the exercises. Please check that the exam paper is complete. If you lack information in the exam text or think that some information is missing, you may make your own assumptions, as long as they are not contradictory to the "spirit" of the exercise. In such a case, you should make it clear what assumptions you have made.

- You should spend your time in such a manner that you get to answer all exercises shortly. If you get stuck on one question, move on to the next question.

- Your answers should be **short**, typically a few sentences and / or a sketch should be sufficient.

Good luck!!

Exercise 1: Representation of binary shapes

One of the methods of representing the contour of binary objects is to traverse the boundary pixels clockwise and generate a chain of integer codes from 0 (=right) through 2 (=forward) to 7.

a) Given two chain codes 77553311 and 00664422, draw a simple sketch of the two shapes.

Answer: A diamond and a square.

b) Using Kulpas's 1977 expression for the length of the perimeter:

$$P_K = \frac{\pi}{8}\left(1+\sqrt{2}\right)\left(n_E + \sqrt{2}n_O\right)$$

where n_E is the number of even chain elements and n_O the number of odd chain elements, what is the ratio of the lengths of the two perimeters?

Answer: This is simpler than it looks, since the first (and largest) has only eight odd chain elements, while the square has only eight even elements. So the answer is square root of two.

c) Given two chains codes 77444411 and 00005533, how can we check whether these two codes represent the same shape?

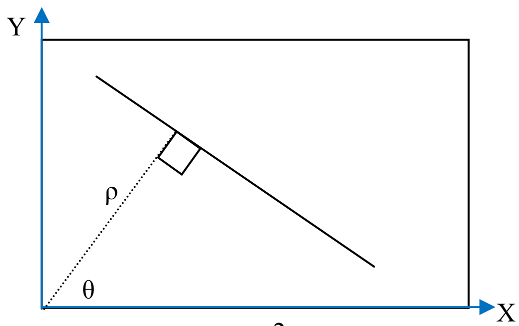
Answer: we could first make the representation start point independent by treating it as a circular/periodic sequence, and redefining the start point so that the resulting number is of minimum magnitude. But this is actually redundant, since we also have to take care of rotation by taking the first difference, followed by a circular shift:
First difference of 77444411 is 60500050, and minimum circular shift of this is 00050605.
First difference of 00005533 is 50005060, and minimum circular shift of this is 00050605.
So the two shapes are identical.

Exercise 2: Hough transform of lines

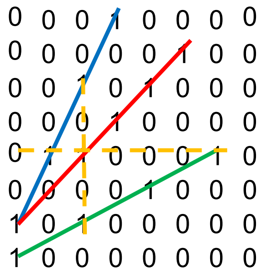
The Hough transform is often used to detect and find the parameters of line segments in gradient images.

a) Copy the sketch below of a line segment in a 2D image.
Explain the "normal representation" of a straight line in the Hough transform, and indicate the parameters involved.

Answer: The "normal representation" is the (θ,ρ)-representation of lines, where a normal to the line through the origin is drawn, and θ is the orientation of this normal with respect to the x-axis, and ρ is its length, as illustrated by the dotted line in the figure below.



b) Another representation of straight line segments in a 2D image is Y = aX + b. In the 8x8 pixel binary image below, three sets of co-linear pixels are present. Which of these sets that can be correctly detected by an (a,b)-accumulator matrix having integer indexes along both the a and b axis if the accumulator threshold is T = 4 ?
Please explain your reasoning without any computations!



Answer: We observe that there are three sets of pixels containing 4 or 5 pixels, as indicated by three solid line segments overlaid onto the figure above. In addition, there is a horizontal and a vertical set of three colinear pixels, indicated by a dashed line segment. The vertical line cannot be handled by this parametrization anyhow, but both are below the threshold.
Using the simple algorithm; for each value of 1 in the image, running through all values of a to compute corresponding values of b will produce a set of co-linear points in the (a,b)-plane, and pixels in the (x,y)-plane that are co-linear will give intersecting sets of points at an (a,b)-value that designates a line through this set of pixels in the (x,y)-plane.
The pixels along the red line will produce a peak at (a,b) = (1,1) and the blue at (2,1).
But the pixels along the green line are best represented by Y=0.5X, and the integer values of a in the accumulator will cause this line to go undetected.

c) Above we have seen the Hough transform as point-to-line mappings (PTLM).
How do we get from this to point-to-point mappings (PTPM)?

Answer: We use the gradient direction to pinpoint the orientation of the line.

For the PhD-students only :

d) The Hough transform has several weaknesses. Assuming an image of size *h x w*, Wallace introduced an alternative “Muff transform” 30 years ago:

As the basis for this parametrization, a bounding rectangle around the image is used. Each line in the image intersects this bounding box / perimeter at exactly two points. The distance of the first intersection (i.e. the nearest intersection from the origin along the perimeter) on the perimeter from the origin, and the distance between the first and second intersections are then used as the Muff parameters.

What is the size and resolution of this accumulator space, and which weakness(es) does this transform solve?

Answer: The maximum value of the shortest distance from the origin to an intersection is h+2w and the maximum distance between the intersections is 2h+2w.
The resolution is the same as the pixel resolution along the perimeter of the image.
Consequently, the main advantages are the automatically bounding of the Hough space and that the discretization error is directly connected to the discretization of the input image.
Therefore, it is possible to represent all necessary values as integers.

Exercise 3: Classification

a) Consider a two-dimensional feature vector and a set of points in 2D feature space:
(-3,6) (-2,4) (-1, 2) (0,0) (1, -2) (2, -4) (3, -6)
Show that the covariance matrix between the two features is :

$$\Sigma = \begin{bmatrix} 4 & -8 \\ -8 & 16 \end{bmatrix}$$

Show all your calculations.

Answer:

$$\mu_1 = (-3-2-1+0+1+2+3)/7 = 0$$

$$\mu 2 = (6+4+2+0+2+4+7)/7 = 0$$

$$\sigma_{12} = \frac{1}{N}\sum_i (x_1 - \mu_1)(x_2 - \mu_2) =$$

$$\frac{1}{7}[(-3-0)(6-0)+(-2-0)(4-0)+(-1-0)(2-0)+(0-0)(0-0)+(-1)(2)+(-2)(4)+-3)(6)] =$$

$$-56/6 = -8$$

$$\sigma_1{}^2 = \frac{1}{N}\sum_i (x_1 - \mu_1)^2 = \frac{1}{7}[3*3+2*2+1*1+0+1*1+2*2+3*3] = 28/7 = 4$$

$$\sigma_2{}^2 = \frac{1}{N}\sum_i (x_2 - \mu_2)^2 = \frac{1}{7}[-6*-6+-4*-4+1*1+0+1*1+4*4+6*6] = 16$$

b) Given the two features defined above, would you base you classification on 1 or 2 features? Justify your answer.

Answer: We note that the points line on a straight line, thus the two features are linear dependent, and there is no need to use more than one of them (and the 2D covariance matrix is singular).

c) The discriminant functions for a multivariate Gaussian classifier are given as:

$$g_i(\mathbf{x}) = -\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_i)^T \Sigma_i^{-1}(\mathbf{x}-\boldsymbol{\mu}_i) - \frac{n}{2}\ln 2\pi - \frac{1}{2}\ln|\Sigma_i| + \ln P(\omega_i)$$

Consider two classes with equal prior probability and

$$\boldsymbol{\mu}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \boldsymbol{\mu}_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad \Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \Sigma_1{}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \quad \Sigma_2{}^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \quad |\Sigma_1| = 1 \quad |\Sigma_2| = 1$$

Can the discriminant function be simplified in this case?

Answer: Somewhat,, we can avoid the ln2π and lnP(ωi) and also the determinant of the covariance matrices, which is equal.

d) Classify the point $x = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ by computing the value of the discriminant functions and assign it to the class corresponding to the highest probability.

Answer:

$$g_1(x) = -\frac{1}{2}[3-1, 0-0]^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3-1 \\ 0 \end{bmatrix} - 0.5\ln 1 = -2$$

...

$$g_2(x) = -\frac{1}{2}[3-0, 0-2]^T \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} - 0.5\ln 1 = -5/2$$

Here we have ignore the two class-independent terms. We should assign it to the class with maximum value, so the pattern will be assigned to class 1.

e) Explain how classifier sensitivity and specitivity are computed, and discuss their importance on a medical classification problem.

Answer: Very short: Sensitivity: TP/(TP+FN) The probability that the test is positive given that the patient is sick. Specifitivity: TN/(TN+FP). The probability that the test is negative given that the patient is not sick. For a sick/healthy scenario we want all sick to be diagnosed as sick, so sensitivity is important and we can allow a lower specitivity.

For the PhD-students only:

f) Let us assume that we have a 2-class classification problem with a 1-dimensional feature vector f(x) which is exponentially distributed given the class-conditional parameter λ_i:

$$f(x) = \lambda_i \exp^{-\lambda_i x}$$

Find an expression for the decision boundary for this classification problem.

Answer:

The decision boundary is the point T where

$$\lambda_1 e^{-\lambda_1 T} = \lambda_2 e^{-\lambda_2 T} \Leftrightarrow$$

$$\frac{\lambda_1}{\lambda_2} = e^{(\lambda_1 - \lambda_2)T} \Leftrightarrow$$

$$T = \frac{\ln\left(\frac{\lambda_1}{\lambda_2}\right)}{\lambda_1 - \lambda_2}$$

Exercise 4: Clustering

a) Describe how the K-means clustering algorithm work and which parameters it has.

Answer: Textbook material

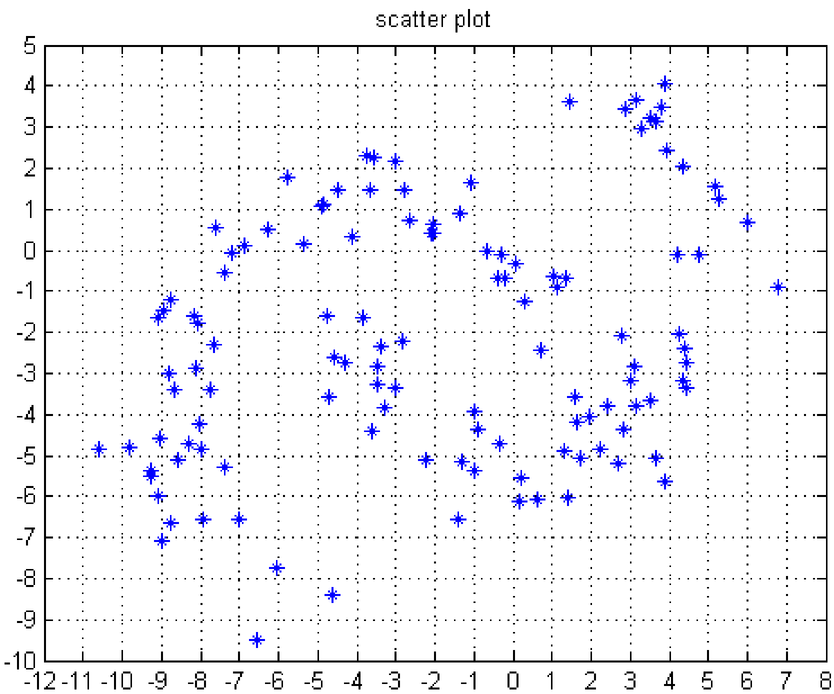
b) Kmeans-clustering with K=2 is done on the data points given in the scatter plot below. Three different strategies for initializing the clustering is to assign the initial cluster centers to 1) minimum/maximum among the points in the data set 2) the first K points 3) K random data points. In the table below the resulting cluster centers after each iteration in Kmeans-clustering are given.
Discuss which of these two methods you would choose for this particular data set, and how different the clustering result would be.

Initialization	Cluster 1 mean	Cluster 2 mean
Iteration 0	(-10.6, -4.8)	(6.8, -0.9)
Iteration 1	(-6.4, -2.9)	(1.8, -1.4)
Iteration 2	(-6.4, -2.9)	(1.7, -1.5)
Iteration 3	(-6.4, -2.9)	(1.7, -1.5)
Iteration 4	(-6.4, -2.9)	(1.7, -1.5)
Iteration 5	(-6.4, -2.9)	(1.7, -1.5)
Iteration 6	(-6.4, -2.9)	(1.7, -1.5)

Table 1 Clustering, initialization using minmax points

Initialization	Cluster 1 mean	Cluster 2 mean
Iteration 0	(1.6, -3.6)	(-1.3, -5.1)
Iteration 1	(2.2, -0.9)	(-5.6, -3.1)
Iteration 2	(2.1, -1.5)	(-6.0, -2.8)
Iteration 3	(2.0, -1.6)	(-6.2, -2.7)
Iteration 4	(1.9, -1.7)	(-6.3, -2.6)
Iteration 5	(1.9, -1.7)	(-6.3, -2.6)
Iteration 6	(1.9, -1.7)	(-6.4, -2.6)

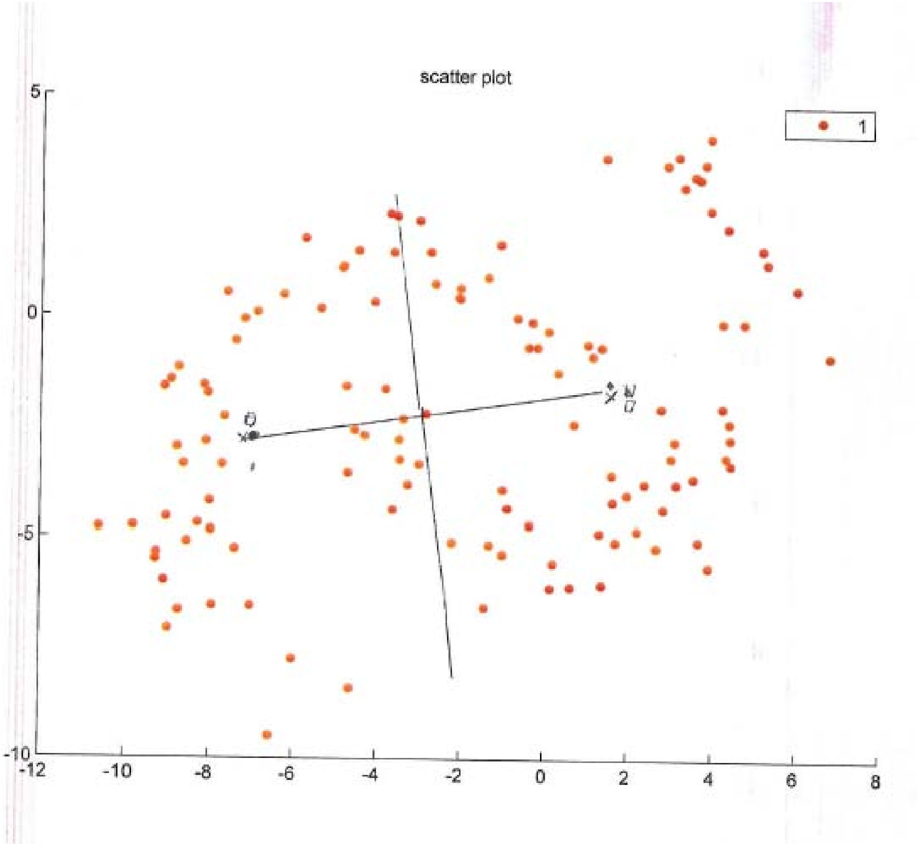
Table 2 - Cluster means after initialization by the first points in the data set



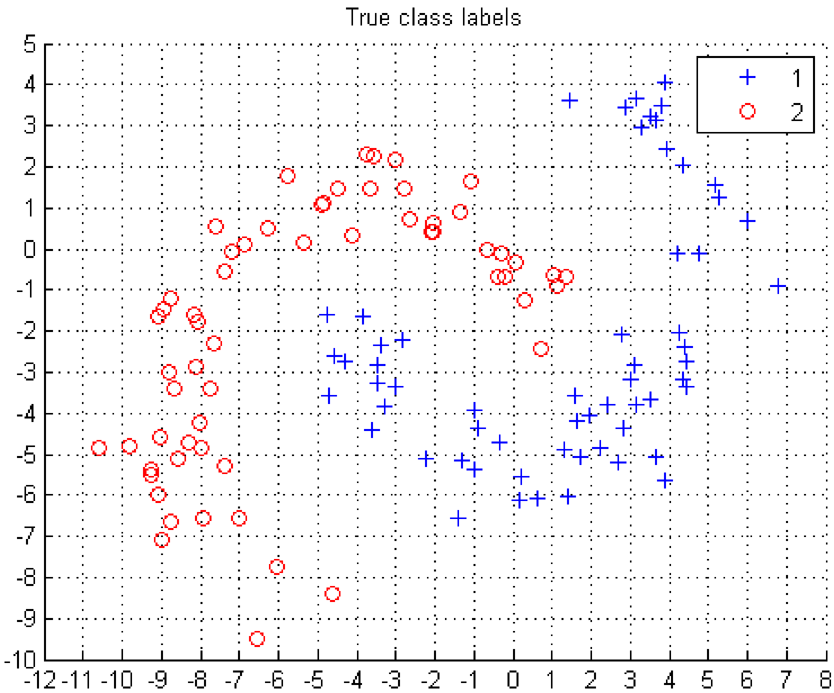
Answer: The means after 6 iterations are only slightly different so the resulting clustering shoul be fairly similar (only few data points will be assigned to different clusters). Note of course that cluster1method 1 corresponds to cluster 2 for method 2, but this does not matter for clustering.

c) Select one of the initializations and indicate the decision boundaries the clustering would result in on the scatter plot in enclosure 1.

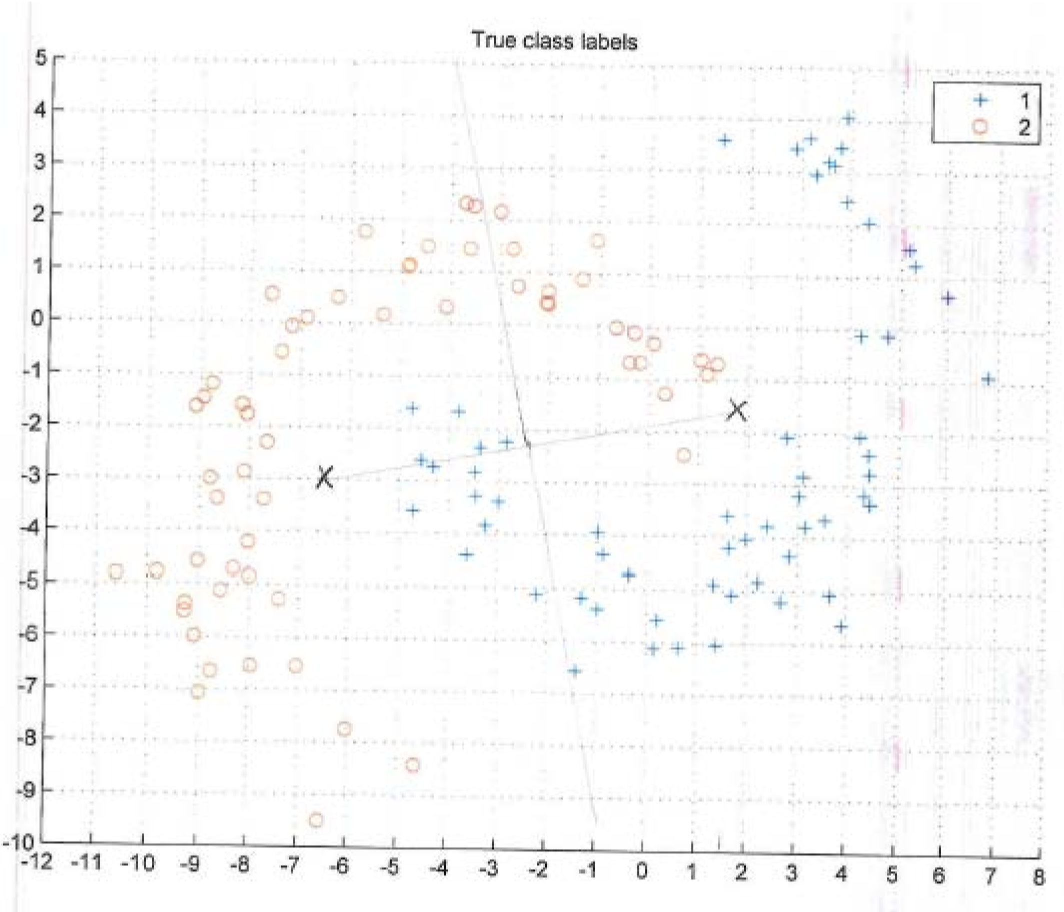
Answer: Kmeans-clustering corresponds to a Gaussian classifier with equal diagonal covariance matrix so the decision boundary will be perpendicular to the line connecting the cluster means after the last iteration.



d) This data originates from a classification data set with known class labels. A scatter plot with class labels is given in the figure below.
Plot your estimated decision boundary from the clustering on the labelled scatter plot. Use the scatter plot to compute the confusion matrix for the clustering result.



Answer: The ecact confusion matrix will depend on the drawing for points close to the boundary. Based on the scanned drawing we get:



	Estimated	*Estimated 1*	*Estimated 2*
True class	*1*	*42*	*18*
	2	*13*	*47*

e) Discuss if a Gaussian classifier with full class-conditional covariance matrix would perform well on this data set given the known class labels.

Answer: a Gaussian classifier would not work very well on these banana shaped classes.

Exercise 5: Mathematical morphology on binary images

a) Explain what erosion and dilation do to a binary image.

Answer: …

b) Perform an erosion-based edge detection on the binary image given below, using a 3x3 plus-shaped structuring element that is symmetric around its origin.

```
0 0 0 1 1 0 1 1 0 0 0
0 0 1 1 1 1 1 1 1 0 0
0 1 1 1 1 1 1 1 1 1 0
```

1	1	1	1	0	1	1	1	1	1	1
0	1	1	1	1	0	1	1	1	1	1
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	1	0	1	0	0	0

Answer: The result of the erosion is given below,
and the edge-image is the difference between the original and the eroded image.

0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	1	1	0	0	0
0	0	1	1	0	1	1	1	1	0	0
0	1	1	0	0	0	1	1	1	1	0
0	0	1	1	0	0	0	1	1	1	0
0	0	1	1	1	0	1	1	1	0	0
0	0	0	0	0	1	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0

0	0	0	1	1	0	1	1	0	0	0
0	0	1	0	0	1	0	0	1	0	0
0	1	0	0	1	0	0	0	0	1	0
1	0	0	1	0	1	0	0	0	0	1
0	1	0	0	1	0	1	0	0	0	1
0	1	0	0	0	1	0	0	0	1	0
0	1	1	1	1	0	1	0	1	0	0
0	0	0	0	0	1	0	1	0	0	0

c) Give the expression for the Bottom Hat operation, and apply this to the image given below. Illustrate each step of the operation and describe the effect of the operators.

0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	0	1	1	0	0
0	0	1	0	1	0	1	1	1	1	0
0	0	1	1	1	0	1	1	0	0	0
0	0	1	1	1	0	0	1	1	0	0
0	0	0	1	1	0	0	0	1	0	0
0	1	1	1	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0

Answer: The Bottom Hat is given by a closing minus the original. The closing consists of two steps: first a dilation and then an erosion of the result of the dilation.
So we will dilate, erode and subtract.

Dilation

0	1	0	0	1	0	0	1	1	0	0
1	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	1
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	0	1	1	1	0
1	1	1	1	1	0	0	0	1	1	1
0	1	1	1	0	0	0	0	0	1	0

Erosion

0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	0	1	1	0	0
0	0	1	1	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	0	1	1	0	0
0	0	1	1	1	0	0	0	1	0	0
0	1	1	1	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0

Difference = Bottom Hat

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

- *The dilation expands the object borders - both inside and outside. It fills holes in the object and smooths out the object contour.*
- *The erosion removes pixels on the border of the dilated object.*
- *The difference from the original will mark inlets and gaps in the original object.*

Thank You for Your Attention!