UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam:	**INF 4300 / INF 9305 – Digital image analysis**
Date:	**Thursday December 21, 2017**
Exam hours:	**09.00-13.00 (4 hours)**
Number of pages:	**5 pages of exercises**
Enclosures:	**1 page Appendix**
Allowed aid:	**Calculator**

- Read the entire exercise text before you start solving the exercises. Please check that the exam paper is complete. If you lack information in the exam text or think that some information is missing, you may make your own assumptions, as long as they are not contradictory to the "spirit" of the exercise. In such a case, you should make it clear what assumptions you have made.

- You should spend your time in such a manner that you get to answer all exercises shortly. If you get stuck on one question, move on to the next question.

- Your answers should be **short**, typically a few sentences and / or a sketch should be sufficient.

Good luck!!

Exercise 1: Representation of binary objects

Given a concave object outlined by its one pixel wide perimeter in a binary image.

a) What do we mean by *projections* of such an object?

b) What do we mean by the *signature* of the object perimeter,
and what is the relation between signature and projection?

c) Describe and sketch the *convex hull* of such a concave object,
and list some object features based on the *convex deficiency*.

d) Describe an algorithm to obtain a simplifying polygon from the given object,
so that the resulting vertexes are a subset of the original perimeter pixels.

e) Assume that you have a large set of binary images like the one produced in d),
each containing only one object. The objects are all polygons, some concave,
some convex. The number of polygon edges is unknown, but may vary.
Explain the steps to obtain the Hough transform of each of these images,
using the normal representation of lines.

For the PhD-students only :

f) Based on the information given in e) above, please explain how a number of projections *less than or equal to* the number of peaks in the Hough domain can easily distinguish concave from convex objects. Why ≤ ?

Exercise 2: Filtering and mathematical morphology

Assume that you have a binary image containing both large objects and small noise elements.
The noise elements vary in shape, but are much smaller than the real objects.
The latter have small details that are important for the subsequent image analysis.

a) What filters would you consider for removal of the noise elements,
and what are the pros and cons of these filters?

b) The basic morphological operators erosion and dilation, and the simplest
combinations of these, are often used for this task.
How would you use them, and what are the pros and cons?

c) Please give one more advanced combination of morphological operators for this task,
and discuss the pros and cons.

For the PhD-students only :

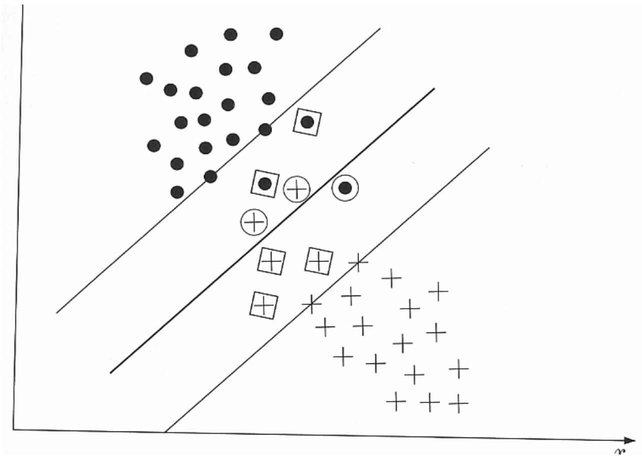
d) It is possible to remove the noise elements completely without altering the shape
of the objects. Please explain how.

Exercise 3: Support vector machine classifiers

a) Consider a linear SVM with decision boundary $g(x) = w^T x + w_0$.
   In SVM classification, explain why it is useful to assign class labels -1 and 1 for a binary classification problem.

b) The basic SVM optimization problem is to minimize $J = ½ ||w||^2$
   What are the additional constraints for this optimization problem?
   Ideally, you should answer both by math and explain what this expression means.

c) Explain how slack variables ξ_i are used to solve a non-separable case like the one below:



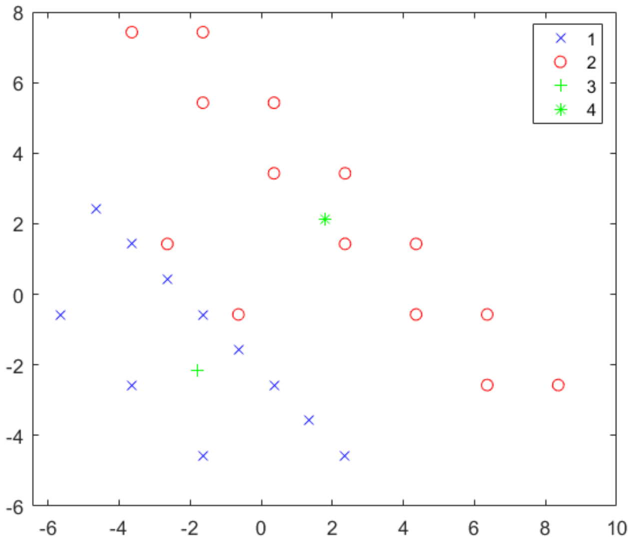
d) Discuss how likely a Gaussian classifier and a SVM classifier are to overfit to the training data.

e) Explain how a SVM can be used for a classification problem with M classes.

f) Explain briefly how SVM parameters should be determined.

Exercise 4: Feature selection, dimensionality reduction and classification

A two-class dataset is sketched below.



The mean values for the two classes are indicated with the symbols + and *, respectively.
In this exercise, the following values are given:

The mean for the entire data set is zero. $\mu 1 = \begin{bmatrix} -1.8 \\ -2.1 \end{bmatrix}$ $\mu 2 = \begin{bmatrix} 1.8 \\ 2.1 \end{bmatrix}$

Covariance $R = \begin{bmatrix} 12.4 & -2.4 \\ -2.4 & 13.1 \end{bmatrix}$

Eigenvalues of R : $\lambda_1 = 10.4 \quad \lambda_2 = 15.2$

Eigenvectors of R $v_1 : \begin{bmatrix} -0.76 \\ -0.65 \end{bmatrix}$ $v_2 : \begin{bmatrix} -0.65 \\ 0.76 \end{bmatrix}$

a) In Figure 1 in the appendix, indicate the decision boundary for this problem if we use the two original features and a Gaussian classifier with equal diagonal covariance matrix.

b) Based on this decision boundary, compute the confusion matrix for the training data.

c) Principal component analysis projects the data based on a criterion function. What is this criterion function?

d) How do we find the first principal component of the data?

e) Indicate in Figure 2 in the appendix the direction of the first principal component of the data.

f) Consider sequential forward feature selection on this data set, with average Euclidean distance between the classes as the criterion function.
Set up the value of the criterion function for the two original features independently.

g) Select the best feature with this criterion function, and draw the decision boundary for this feature set on Figure 1 in the appendix.

h) Perform a projection down onto the principal component by sketching the projection (no computation needed) on Figure 2 in the appendix.

i) Do a classification of the first principal component based on Euclidean distance and count the number of misclassified samples.

Exercise 5: Gray level texture analysis

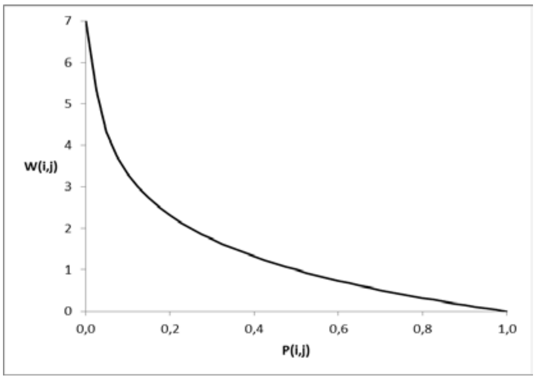
The Gray Level Cooccurrence Matrix method is a frequently used texture analysis method.
In 2D gray level images we often use isotropic GLCMs based on symmetric matrices obtained for a given pixel distance, *d*, in a limited number of directions θ.

a) How do you obtain an isotropic symmetric GLCM for a given *(d,θ),* and how can you normalize it ?

b) Assume – without knowing what the image looks like – that you have obtained a normalized GLCM as described above. Then there are several possible formulae describing features that can be extracted from the GLCM. One of them is

$$E = -\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} P(i,j) \times log(P(i,j))$$

where the weight function W(i,j) = - log(P(i,j)) is given in the graph below



Discuss, based on the figure above, what the output value E will be (high or low) for homogeneous and inhomogeneous images, respectively.

For the PhD-students only :

c) Is the feature above invariant to a linear scaling of <u>*the pixel values*</u> in the image? Please explain, and also discuss another feature that behaves oppositely!

Good Luck!

Candidate number:

Appendix, Exam INF4300/9305, Desember 21, 2017

Please hand in this page with your solution.

Figure 1

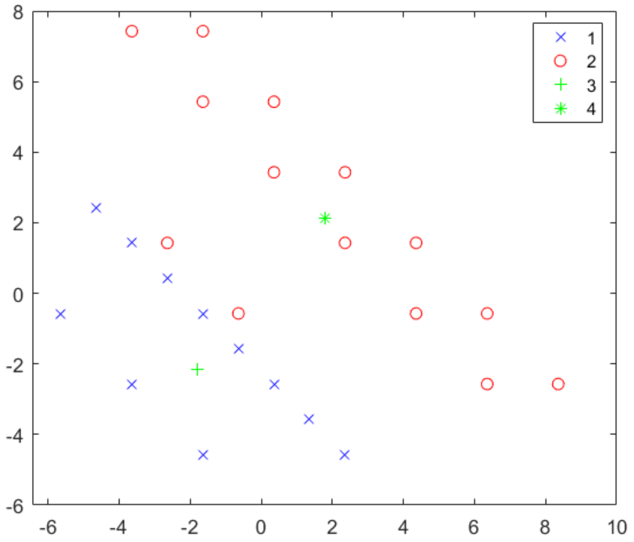


Figure 2