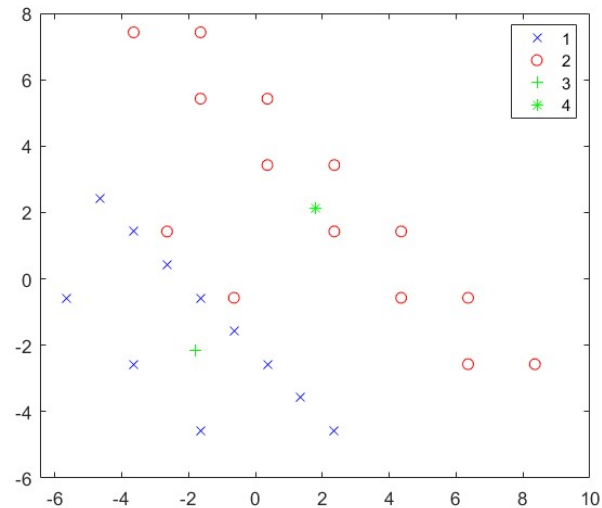# IN 5520   2020
Classification/PCA Lecture 28.10.20

Exercise 1: Feature selection, dimensionality reduction and classification (from 2017 exam)

A two-class dataset is sketched below.

The mean values for the two classes are indicated with the symbols + and *, respectively.
In this exercise, the following values are given:

The mean for the entire data set is zero. $\mu1 = \begin{bmatrix} -1.8 \\ -2.1 \end{bmatrix}$ $\mu2 = \begin{bmatrix} 1.8 \\ 2.1 \end{bmatrix}$

Covariance $R = \begin{bmatrix} 12.4 & -2.4 \\ -2.4 & 13.1 \end{bmatrix}$

Eigenvalues of R: $\lambda_1 = 10.4$ $\lambda_2 = 15.2$

Eigenvectors of R $v_1 : \begin{bmatrix} -0.76 \\ -0.65 \end{bmatrix}$ $v_2 : \begin{bmatrix} -0.65 \\ 0.76 \end{bmatrix}$

a) In the plot in the appendix, indicate the decision boundary for this problem if we use the two original features and a Gaussian classifier with equal diagonal covariance matrix.

b) Based on this decision boundary, compute the confusion matrix for the training data.

c) Principal component analysis projects the data based on a criterion function. What is this criterion function?

d) How do we find the first principal component of the data?

e) Indicate in the plot in the appendix the direction of the first principal component of the data.

f) Consider sequential feature selection on this data set, with average Euclidean distance between the classes as the criterion function.
Set up the value of the criterion function for the two original features independently.

g) Select the best feature with this criterion function, and draw the decision boundary for this feature set on the plot.

h) Perform a projection down onto the principal component by sketching the projection (no computation needed) on the plot.

i) Do a classification based on Euclidean distance of the first principal component and count the number of misclassified samples.

Exercise 2.

You are given data from two classes with means and covariance as given below:

$$\mu_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix} \quad \Sigma_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix} \quad \Sigma_1^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix}$$
$$\mu_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \quad \Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad \Sigma_1^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

a) Compute the eigenvectors and eigenvalues of the covariance matrices (use Matlab if you want) and use them to sketch the ellipses for the covariance matrices in a plot.

b) Show that the decision boundary (g1(x)=g2(x)) in this case with features x_1 and x_2 can be expressed as $x_2 = 3.514 - 1.125x_1 + 0.1875x_1^2$

c) Plot the resulting decision boundary in Matlab.

d) Create a synthetic image with 2 bands with samples that span the feature space from e.g. -10 to 10 for both features. For simplicity, let us just consider a coarse grid of
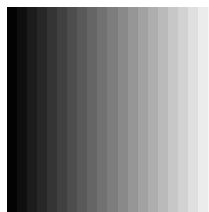
samples on integer values (-10,-9,….,0,….9,10). Feature image 1 should look like a horisontal ramp from -10 to 10, and feature image 2 like a vertical ramp from 10 to -10:
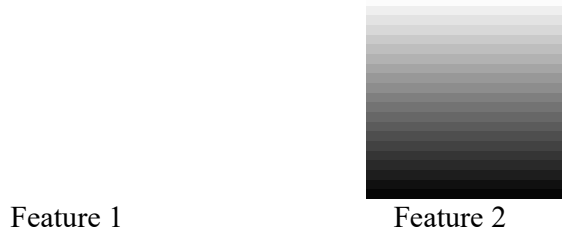
Feature 1   Feature 2

This corresponds to creating feature vectors that span the entire feature space (from -10 to 10). If we later classify all these feature vectors, the resulting classification map should have the same decision boundary as the plot we computed in b) and plotted c). This is just a way to create a visualization of the decision boundary without computing it analytically.

e) Classify this image, and verify that the shape of the decision boundary you got in c) is the same as you get after classifying this image.

Exercise 3: Exercises in linear transforms
This lab will use matlab to implement and explore principal component transform on a small example.

Principal Component Analysis:
1.First we work with a 6 - band satellite image from Kjeller.
Load the file tm.mat

2. Put all the image data into a long pattern vector nx6-matrix (one row per pixel with 6 feature values).

3. Compute the covariance matrix of the data vector.

4. Use eig() to find the eigenvectors and eigenvalues of the covariance matrix.

5. Form the A matrix by letting each column be an eigenvector of the covariance matrix.

6. Form the 6 principal components of the pattern vector using pca1(i)=x(i)*a(i).

7. Convert the 6 principal component nx6-vectors back to the 2D image geometry yielding 6 bands of the image.

8. Display the principal component images.
Looking at them, how many do you think are useful for classification?

9. Plot the eigenvalues scaled by the sum of all eigenvalues.

Exercise 4 (from 2018 exam): PCA and classification

A set of samples from two classes are given as (a scatter plot is also given below):

Class1: $\begin{bmatrix} -2.5 & 1.5 \\ -1.5 & 0.5 \\ 0.5 & -1.5 \\ 1.5 & -2.5 \\ 0.5 & 0.5 \\ -1.5 & -1.5 \end{bmatrix}$ $\mu_1 = \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}$ $\Sigma_1 = \begin{bmatrix} 2.4 & -1.6 \\ -1.6 & 2.4 \end{bmatrix}$ $\Sigma^{-1}_1 = \begin{bmatrix} 0.75 & 0.5 \\ -0.5 & 0.75 \end{bmatrix}$

• Class2: $\begin{bmatrix} -1.5 & 2.5 \\ -0.5 & 1.5 \\ 1.5 & -0.5 \\ 2.5 & -1.5 \\ -0.5 & -0.5 \\ 1.5 & 1.5 \end{bmatrix}$ $\mu_2 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$ $\Sigma_2 = \begin{bmatrix} 2.4 & -1.6 \\ -1.6 & 2.4 \end{bmatrix}$ $\Sigma^{-1}_2 = \begin{bmatrix} 0.75 & 0.5 \\ -0.5 & 0.75 \end{bmatrix}$

• Global mean: $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

• Global covariance matrix: $\begin{bmatrix} 2.45 & -1.18 \\ -1.18 & 2.45 \end{bmatrix}$

• Eigenvalues 1.27 and 3.65 with eigenvectors: $\begin{bmatrix} -0.7 \\ -0.7 \end{bmatrix}$ and $\begin{bmatrix} -0.7 \\ 0.7 \end{bmatrix}$
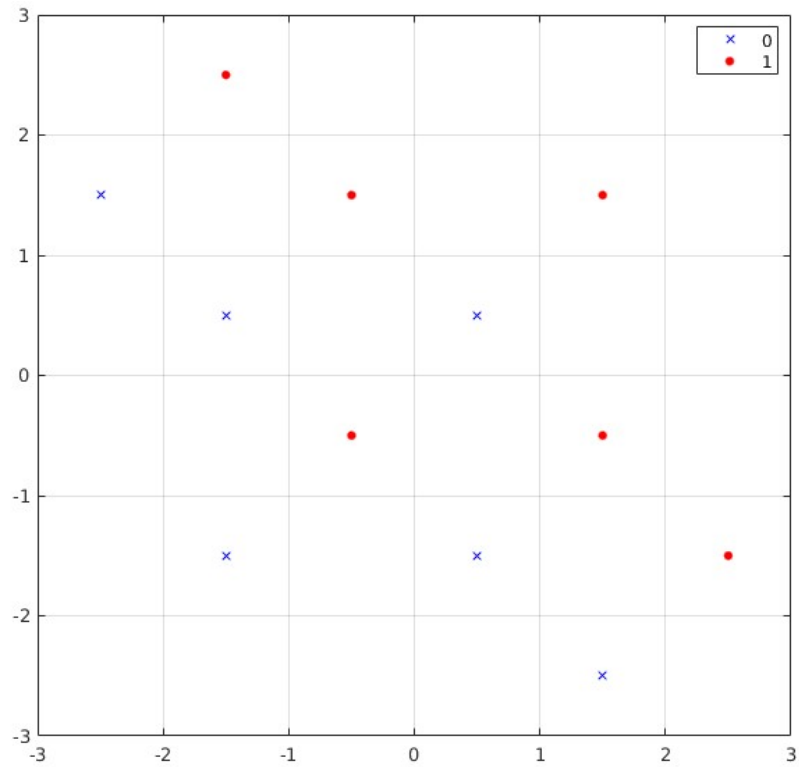
a) Find the direction of the first principal component of this data set.

b) Sketch the first principal component on the plot

c) Compute the new feature values using this component for all samples.

d) Plot the points in the figure and discuss how the first principal component performs in this case.