

(classification 3)

Ex 1a)

$$\Sigma_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\mu_1 = [3 \ 6]^T$$

$$\mu_2 = [3 \ -2]^T$$

Finding the eigenvalues of Σ_1 :

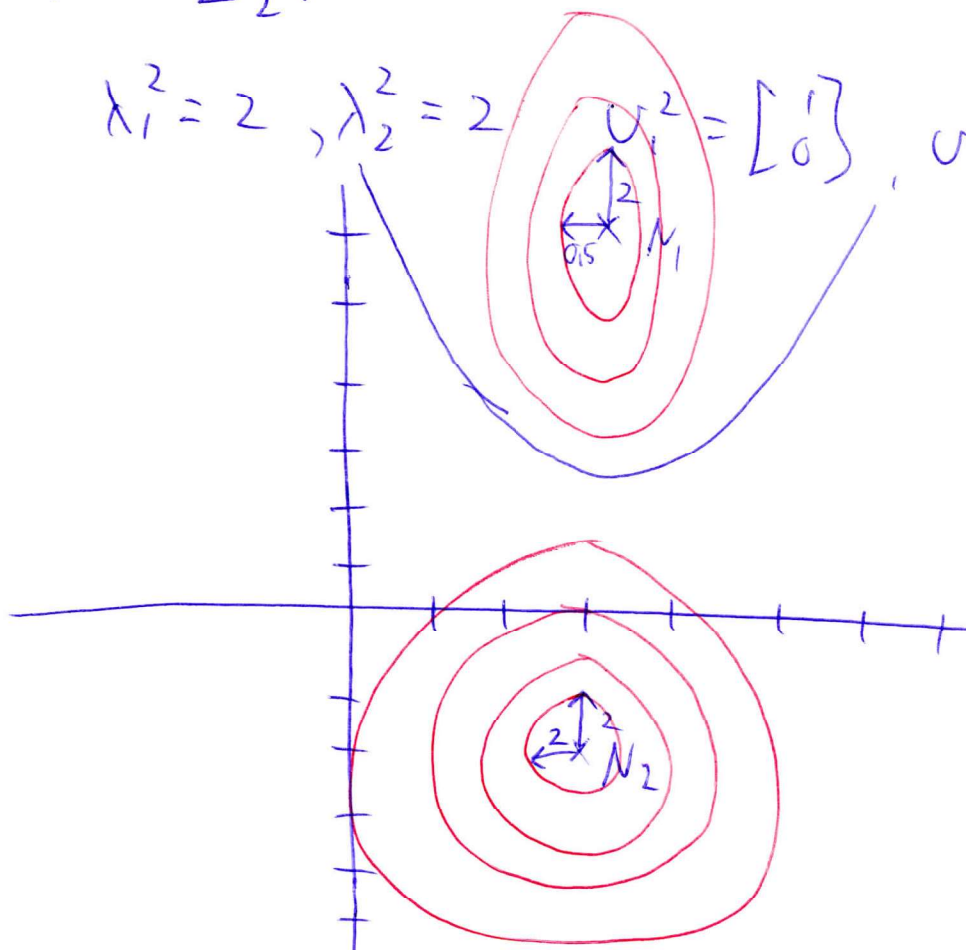
$$\det \begin{bmatrix} 0.5 - \lambda & 0 \\ 0 & 2 - \lambda \end{bmatrix} = (0.5 - \lambda)(2 - \lambda)$$

So the eigenvalues are $\lambda'_1 = 0.5$, $\lambda'_2 = 2$.

And the eigenvectors must be $v'_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v'_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

For Σ_2 :

$$\lambda_1^2 = 2, \lambda_2^2 = 2 \quad v_1^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, v_2^2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



Classification 3

Ex 1 b) Show that $g_1(x) = g_2(x)$ in the case with features x_1 and x_2 can be expressed as:

$$x_2 = 0,1875x_1^2 - 1,125x_1 + 3,5142$$

We have the general case, so $g_j(\bar{x}) = -\frac{1}{2}(\bar{x} - \bar{\mu}_j)^T \Sigma_j^{-1}(\bar{x} - \bar{\mu}_j) - \frac{1}{2} \ln |\Sigma_j| + \ln P_j$

$$g_1(\bar{x}) = -\frac{1}{2} [x_1 - 3 \quad x_2 - 6] \begin{bmatrix} 2 & 0 \\ 0 & 0,5 \end{bmatrix} \begin{bmatrix} x_1 - 3 \\ x_2 - 6 \end{bmatrix} - \frac{1}{2} \ln 1$$

$$= -\frac{1}{2} [2x_1 - 6 \quad \frac{x_2}{2} - 3] \begin{bmatrix} x_1 - 3 \\ x_2 - 6 \end{bmatrix}$$

$$= -\frac{1}{2} (2x_1^2 - 6x_1 - 6x_1 + 18 + \frac{x_2^2}{2} - 3x_2 - 3x_2 + 18)$$

$$= -x_1^2 + 6x_1 - \frac{x_2^2}{4} + 3x_2 - 18$$

$$g_2(\bar{x}) = -\frac{1}{2} [x_1 - 3 \quad x_2 + 2] \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1 - 3 \\ x_2 + 2 \end{bmatrix} - \frac{1}{2} \ln 4$$

$$= -\frac{1}{2} [\frac{x_1}{2} - \frac{3}{2} \quad \frac{x_2}{2} + 1] \begin{bmatrix} x_1 - 3 \\ x_2 + 2 \end{bmatrix} - \frac{1}{2} \ln 4$$

$$= -\frac{1}{2} (\frac{x_1^2}{2} - \frac{3}{2}x_1 - \frac{3}{2}x_1 + \frac{9}{2} + \frac{x_2^2}{2} + x_2 + x_2 + 2) - \frac{1}{2} \ln 4$$

$$= -\frac{x_1^2}{4} + \frac{3}{2}x_1 - \frac{x_2^2}{4} - x_2 - \frac{13}{4} - \frac{1}{2} \ln 4$$

So $g_1(x) = g_2(x)$ is:

$$4x_2 = \frac{3}{4}x_1^2 - \frac{9}{2}x_1 + \frac{59}{4} - \frac{1}{2} \ln 4$$

$$x_2 = \frac{3}{16}x_1^2 - \frac{9}{8}x_1 + \frac{59}{16} - \frac{1}{8} \ln 4$$

$$x_2 = 0,1875x_1^2 - 1,125x_1 + 3,5142$$

QED.