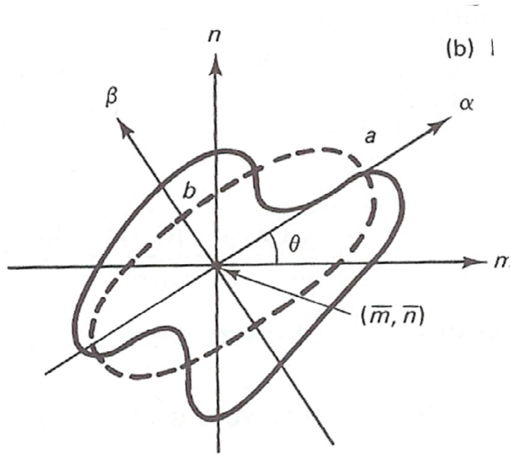
INF 4300 – Digital Image Analysis

OBJECT DESCRIPTION – SHAPE FEATURE EXTRACTION



(c) Best fit ellipse

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Today

From the textbook:	DIP3E	DIP4E
Boundary (Feature) Descriptors	11.2 (815-822)	11.3 (831-840)
Regional (Feature)Descriptors	11.3 (822-842)	11.4 (840-859)

Albregtsen: "Object shape descriptors - Area, perimeter, compactness, and spatial moments"

Curriculum also includes these lecture notes.

Today we cover the following :

1. Introduction
2. Topological features
3. Projections
4. Geometric features
5. Statistical shape features
6. Moment-based geometric features
7. Finding the best feature subset

What is feature extraction?

- Devijver and Kittler (1982):

  "Extracting from the raw data the information which is most relevant for classification purposes, in the sense of minimizing the within-class pattern variability while enhancing the between-class variability".

  - Within-class pattern variability: variance between objects belonging to the same class.

  - Between-class pattern variability: variance between objects from different classes.

A B C D E F G H I J

Feature extraction

- We will discriminate between different object classes based on a set of features.
- The features are often chosen given the application.
- Normally, a large set of different features is investigated.
- Classifier design also involves feature selection - selecting the best subset out of a larger feature set.
- Given a training data set of a certain size, the dimensionality of the feature vector must be limited.
- Careful selection of an optimal set of features is the most important step in image classification!

Feature extraction methods

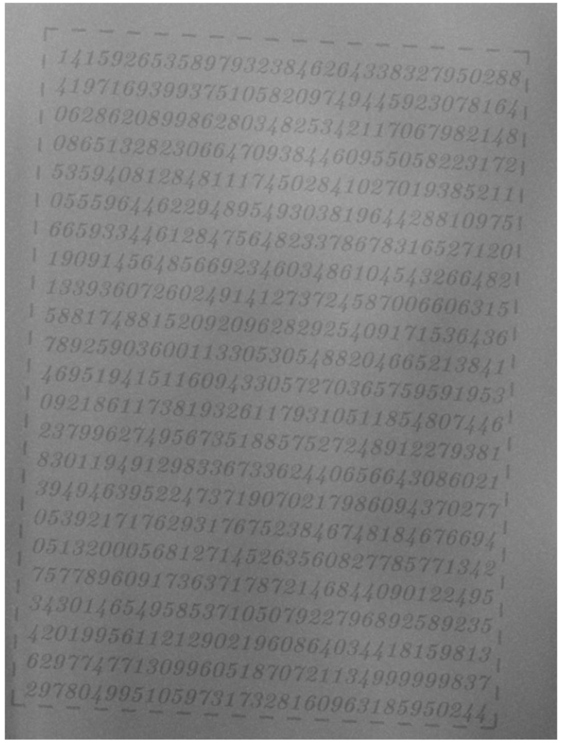
- There are a lot of different feature extraction methods, you will only learn some in this course.
- The focus of this lecture is on **features for describing the shape of an object**.
- Features can also be extracted in local windows around each pixel, e.g.
  - texture descriptors,
  - colour features,
  - or other metods.
- The features will later be used for object recognition and/or classification.

Example: Recognize printed numbers

- Goal: get the series of digits,
  e.g. 14159265358979323846……

Steps in the program:

1. Segment the image to find digit pixels.
2. Find angle of rotation and rotate back.
3. Create region objects – one object pr. digit or connected component.
4. Compute features describing shape of objects
5. Train a classifier on many objects of each digit.
6. Assign a class label to each new object, i.e., the class with the highest probability.



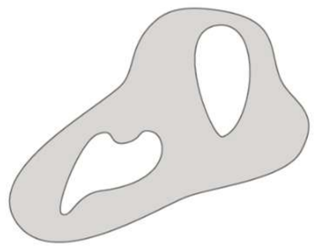
Focus of this lecture

Typical image analysis tasks

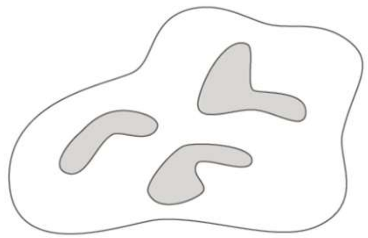
- Preprocessing/noise filtering
- Segmentation
- Feature extraction
  - Are the original image pixel values sufficient for classification, or do we need additional features?
  - What kind of features do we use in order to discriminate between the object classes involved?
- Exploratory feature analysis and selection (next lecture)
  - Which features separate the object classes best?
  - How many features are needed?
- Classification (following three/four lectures)
  - From a set of object examples with known class, decide on a method that separates objects of different types.
  - For new objects: assign each object/pixel to the class with the highest probability
- Testing and validation of classifier accuracy

Topologic features

- This is a group of invariant <u>integer</u> object shape features
  - Invariant to position, rotation, scaling, warping
- Features based on the object skeleton
  - Number of terminations (one line from a point)
  - Number of breakpoints or corners (two lines from a point)
  - Number of branching points (three lines from a point)
  - Number of crossings (> three lines from a point)
- Region features:
  - Number of holes in the object (H)
  - Number of components (C)
  - Euler number, E = C - H
    - Number of connected components – number of holes
  - Symmetry



Region with two holes



Regions with three connected components

1D Projection histograms

- For each row in the region,
  count the number of object pixels.

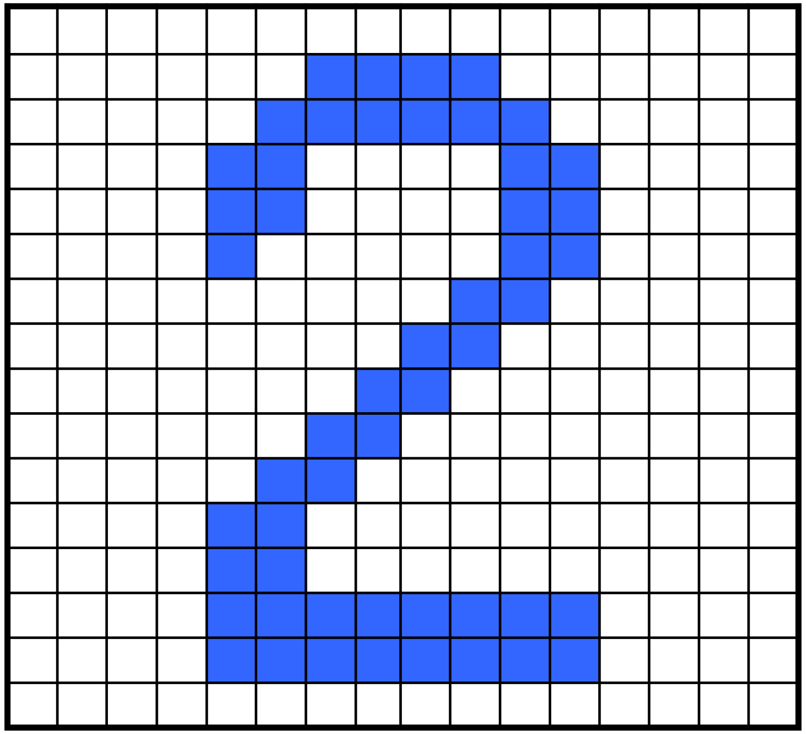
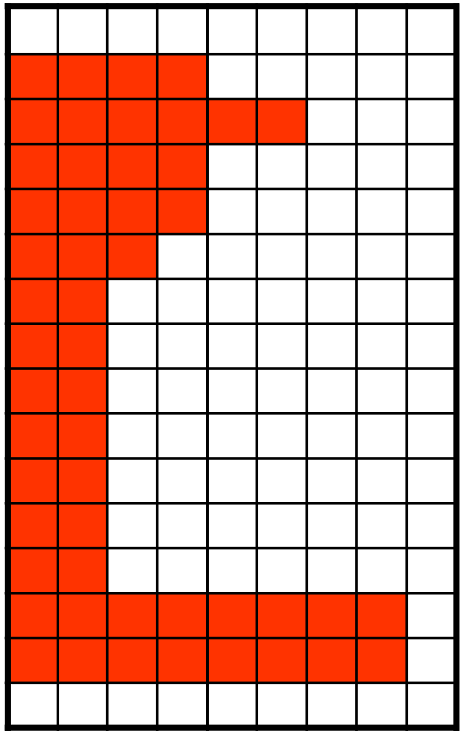


Image – binary region pixels



Row histogram

Projections

- 1D horizontal projection of the region (project on the x-axis):

$$p_h(x) = \sum_y f(x, y)$$

- 1D vertical projection of the region (project on the y-axis):

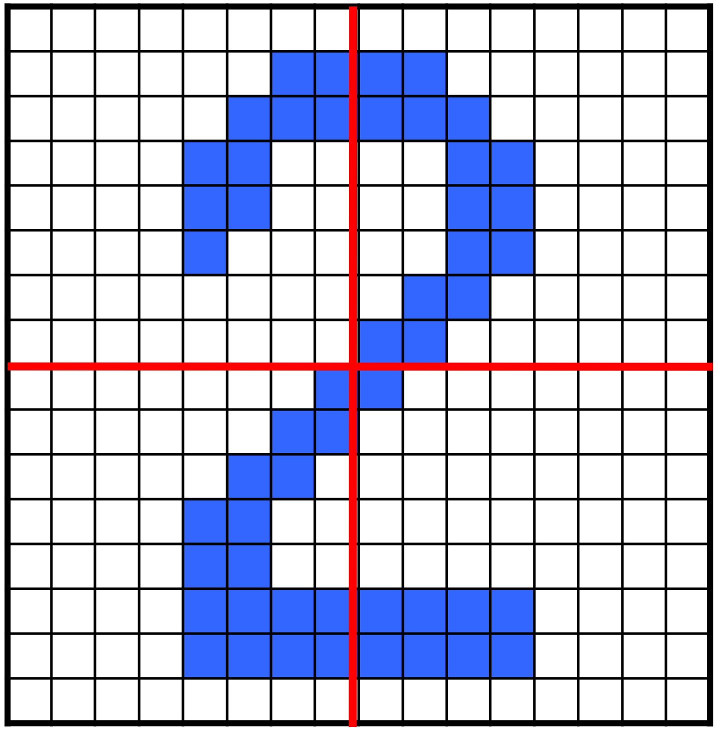
$$p_v(y) = \sum_x f(x, y)$$

- f(x,y) is normally the binary segmented image
- Can be made scale independent by using a fixed number of bins and normalizing the histograms.

- Radial projection in reference to centroid -> ”signature”, see previous lecture.

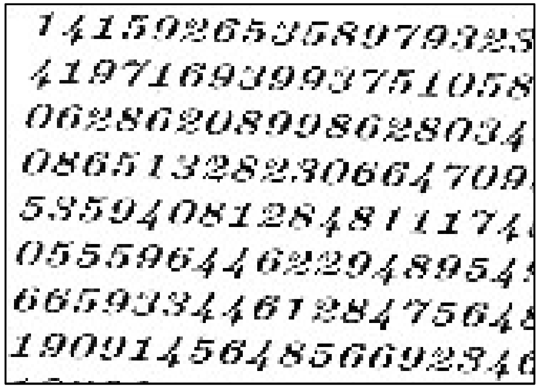
Use of projection histograms

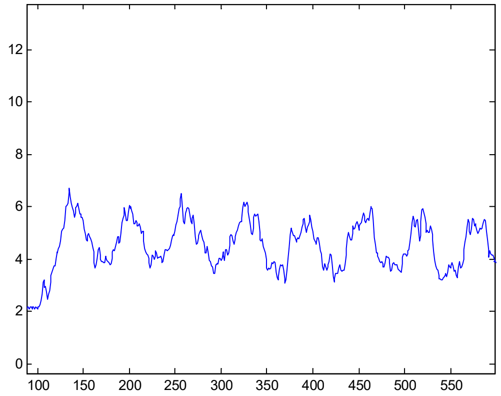
- Divide the object into different regions and compute projection histograms for each region.
  - How can we use this to separate 6 and 9?
- Compute features from the histograms.
  - E.g. mean and variance of the histograms.
- The histograms can also be used as features directly.
- Projections are also useful for preprocessing (e.g., finding and correcting angle of rotation).



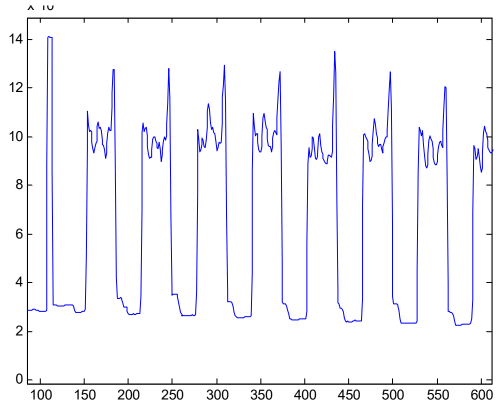
Use of projection histograms

- Check if a page with text is rotated



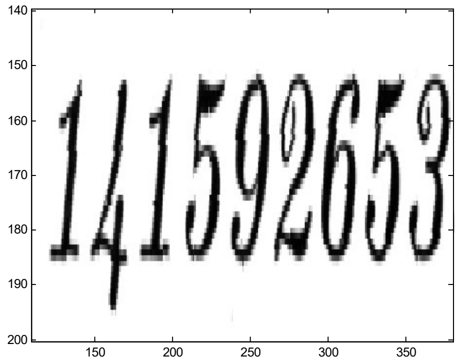


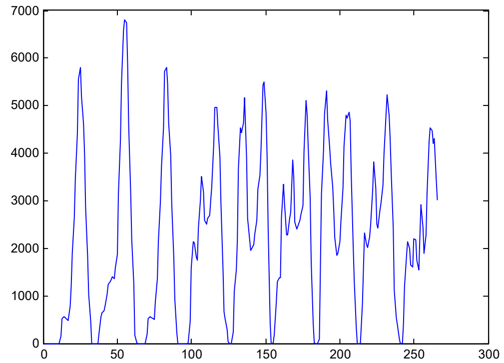
Y-axis projection of page



Y-axis projection after rotation correction

- Then detect lines, connected objects, single symbols, …





x-axis projection of page

Object area

- Generally, the area is defined as: $A = \int_X \int_Y I(x,y)dxdy$

  I(x,y) = 1 if the pixel is within the object, and 0 otherwise.

- In digital images: $A = \sum_X \sum_Y I(x,y)\,\Delta A$

  ΔA = area of one pixel. If ΔA = 1, area is simply measured in pixels.

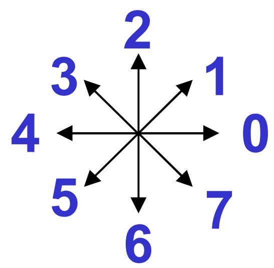
- Area changes if we change the scale of the image
  - change is not perfectly linear, because of the discretization of the image.

- Area ≈ invariant to rotation (except small discretization errors).

Geometric features from contours

- Boundary length/perimeter
- Area
- Curvature
- Diameter/major/minor axis
- Eccentricity
- Bending energy
- Basis expansion (Fourier – last week)

Perimeter length from chain code



- Distance measure differs when using 8- or 4-neighborhood
- Using 4-neighborhood, measured length ≥ actual length.
- In 8-neighborhood, fair approximation from chain code by:

$$P_F = n_E + n_O\sqrt{2}$$

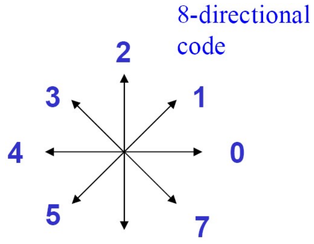
where $\mathbf{n_E}$ and $\mathbf{n_O}$ are the number of even / odd chain elements .

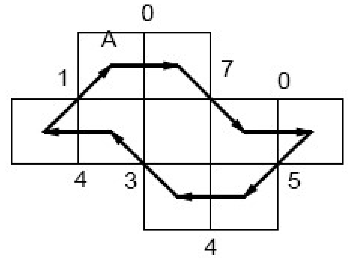
- This overestimates real perimeters systematically.
- Freeman (1970) computed the area and perimeter of the chain by

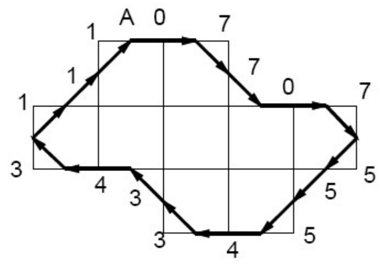
$$A_F = \sum_{i=1}^{N} c_{ix}\left(y_{i-1} + {c_{iy}}/{2}\right), \quad P_F = n_E + n_O\sqrt{2}$$

- where **N** is the length of the chain, $\mathbf{c_{ix}}$ and $\mathbf{c_{iy}}$ are the **x** and **y** components of the **i**th chain element $\mathbf{c_i}$ ($\mathbf{c_{ix}}$, $\mathbf{c_{iy}}$ = {1, 0, -1} indicate the change of the **x**- and **y**-coordinates), $\mathbf{y_{i-1}}$ is the **y**-coordinate of the start point of the chain element $\mathbf{c_i}$.

Perimeter from chain code







- Vossepoel and Smeulders (1982) improved perimeter length estimate by a corner count n_C, defined as the number of occurrences of unequal consecutive chain elements:

$$P_{VS} = 0.980\, n_E + 1.406 n_O - 0.091\, n_C$$

- Kulpa (1977) gave the perimeter as

$$P_K = \frac{\pi}{8}\left(1+\sqrt{2}\right)\left(n_E + \sqrt{2} n_O\right)$$

Pattern matching - bit quads

- Let n{Q} = number of matches between image pixels and pattern Q.
- Then area and perimeter of 4-connected object is given by:

$$A = n\{1\}, \quad P = 2n\{0 \quad 1\} + 2n\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

"Bit Quads" can handle 8-connected images:

$$Q_0 : \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$Q_1 : \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$Q_2 : \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$Q_3 : \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Q_4 : \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$Q_D : \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- Gray (1971) gave area and the perimeter as

$$A_G = \frac{1}{4}\left[n\{Q_1\} + 2n\{Q_2\} + 3n\{Q_3\} + 4n\{Q_4\} + 2n\{Q_D\}\right], \quad P_G = n\{Q_1\} + n\{Q_2\} + n\{Q_3\} + 2n\{Q_D\}$$

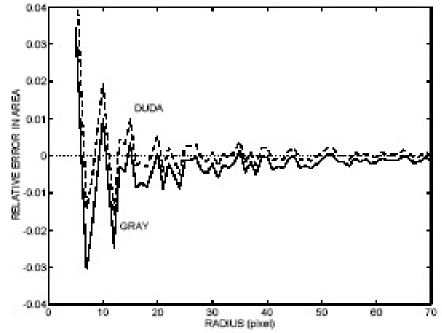
- More accurate formulas by Duda :

$$A_D = \frac{1}{4}\left[n\{Q_1\} + 2n\{Q_2\} + \frac{7}{2}n\{Q_3\} + 4n\{Q_4\} + 3n\{Q_D\}\right], \quad P_D = n\{Q_2\} + \frac{1}{\sqrt{2}}\left[n\{Q_2\} + n\{Q_3\} + 2n\{Q_D\}\right]$$

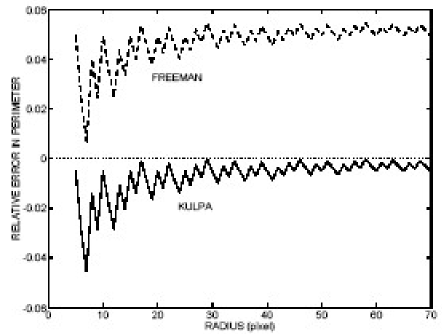
-

A comparison of methods

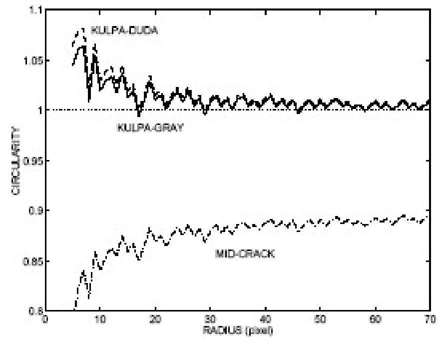
- We have tested the methods on circles, R ={5,…,70}.
- Area estimator :
  - Duda is slightly better than Gray.
- Perimeter estimator :
  - Kulpa is more accurate than Freeman.
- Circularity :
  - Kulpa’s perimeter and Gray’s area gave the best result.
- Errors and variability largest when R is small.
  - Test this yourself on this image: $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$
- Best area and perimeter not computed simultaneously.
- Gray’s area can be computed using discrete Green’s theorem, suggesting that the two estimators can be computed simultaneously during contour following.



(a)



(b)



Object area from contour

- The surface integral over S (having contour C) is given by Green's theorem:

$$A = \iint_S dxdy = \int_C xdy$$

```
s := 0.0;
n := n + 1;

pkt[n].x := pkt[1].x;
pkt[n].y := pkt[1].y;
for i:=2 step 1 until n do
begin
        dy := pkt[i].y - pkt[i-1].y
        s := s + (pkt[i].x + pkt[i-1].x)/2 * dy;
end;
area := if (s > 0) then s else -s;
```

- The region can also be represented by n polygon vertices (see previous lecture)

$$\hat{A} = \frac{1}{2}\left| \sum_{k=0}^{N-1} \left(x_k \, y_{k+1} - x_{k+1} \, y_k \right) \right|$$

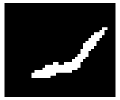
where the sign of the sum reflects the polygon orientation.

Compactness and circularity

- Compactness (very simple measure)
  - $\gamma = P^2/(4\pi A)$, where P = Perimeter, A = Area,
  - For a circular disc, γ is minimum and equals 1.
  - Compactness attains high value for complex object shapes, but also for very elongated simple objects, like rectangles and ellipses where a/b ratio is high.

  => Compactness is not correlated with complexity!

- G&W defines
  - Compactness = P^2/A
  - Circularity ratio = $4\pi A/P^2$



Y=3.4
Circ=0.28

Y=10.1
Circ=0.09

Circularity and irregularity

- Circularity may be defined by $C = 4\pi A/P^2$.
- $C = 1$ for a perfect continuous circle; betw. 0 and 1 for other shapes.
- In digital domain, C takes its smallest value for a
  - digital octagon in 8-connectivity perimeter calculation
  - digital diamond in 4-connectivity perimeter calculation
- Dispersion may be given as the major chord length to area
- Irregularity of an object of area A can be defined as:

$$D = \frac{\pi \max\left((x_i - \bar{x})^2 + (y_i - \bar{y})^2\right)}{A}$$

  - where the numerator is the area of the centered enclosing circle.
- Alternatively, ratio of maximum and minimum centered circles:

$$I = \frac{\max\left(\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}\right)}{\min\left(\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}\right)}$$

Curvature

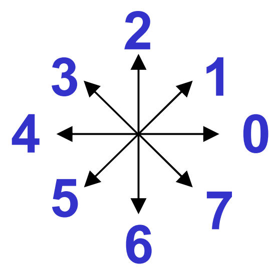
– In the continous case, curvature is the rate of change of slope.

$$|\kappa(s)|^2 = \left[\frac{d^2x}{ds^2}\right]^2 + \left[\frac{d^2y}{ds^2}\right]^2$$

– In the discrete case, difficult because boundary is locally ragged.

– Use difference between slopes of adjacent boundary segments to describe curvature at point of segment intersection.

– Curvature can be calculated from chain code.

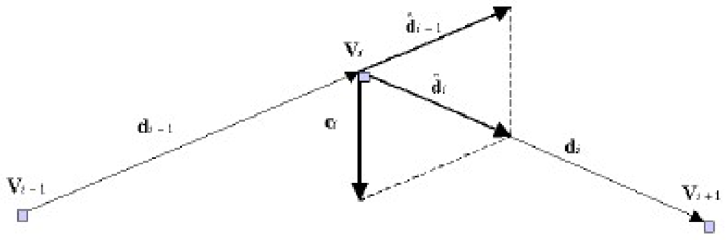


How to get from chain code to curvature, a simple example:

0, 0, 2, 0, 1, 0, 7, 6, 0, 0 --> 0, 2, -2, 1, -1, -1, -1, 2, 0

then square: 0, 4, 4, 1, 1, 1, 1, 4, 0

Discrete computation of curvature



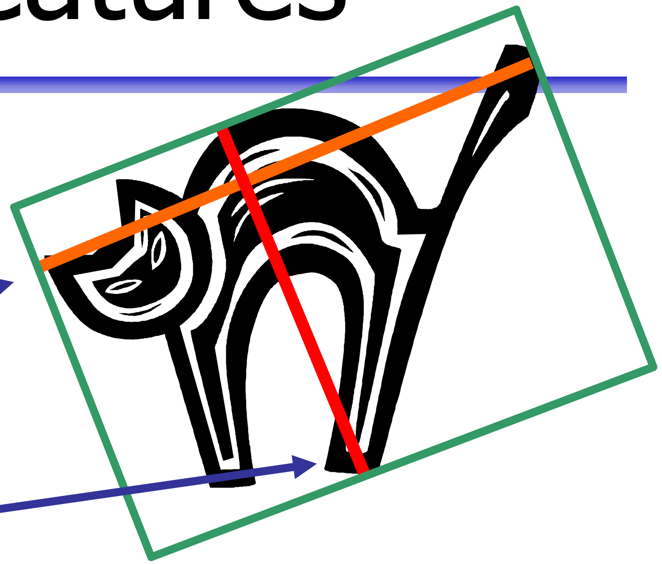
v_i: edge segment i
$d\hat{}_{t-1}$: unit vectors of edge segments d_{t-1} and dt
c_i: local curvature at point i

- Trace the boundary and insert vertices, at a given distance (e.g. 3 pixels apart), or by polygonization (previous lecture).
- Compute local curvature c_i as the difference between the directions of two edge segments joining a vertex:

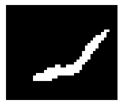
$$c_i = \vec{d}_i - \vec{d}_{i-1}$$

- Curvature feature: sum all local curvature measures along the border.
- More complex regions get higher curvature.

Contour based features



- Diameter = Major axis (a)
  Longest distance of a line segment
  connecting two points on the perimeter
- Minor axis (b)
  Computed along a direction perpendicular to the major axis. Largest length possible between two border points in the given direction.
- So called “Eccentricity” of the contour (a/b)



Eccentricity: 4.7

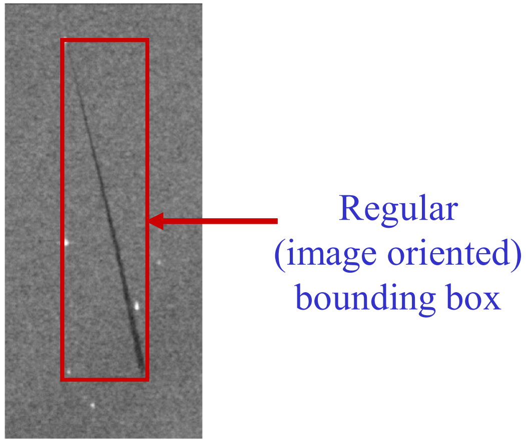
Eccentricity: 1.8

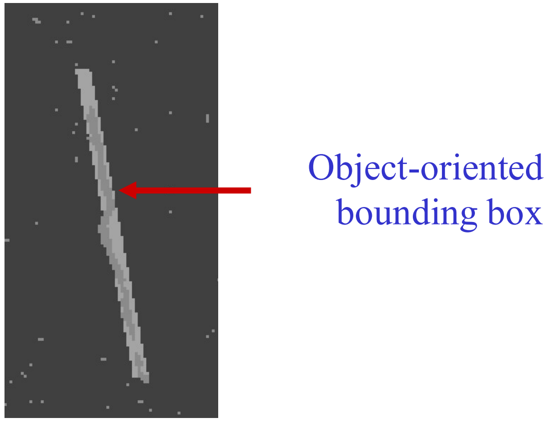
Bounding box and CH features

- Regular bounding box features:
  - Width/height of bounding box
  - Centre of mass position in box

- If the object's orientation is known, a bounding box can also be oriented along this direction. <u>More meaningful!</u>

- Extent = Area/(Area of bounding box)
  - But which type of bounding box?
- Solidity = Area/(Area of Convex Hull) (also termed "convexity")





Moments

- Borrows ideas from physics and statistics.
- For a given continuous intensity distribution *g(x, y)* we define moments m_{pq} by

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q g(x, y) dx dy$$

- For sampled (and bounded) intensity distributions f(x, y) over a region R

$$m_{pq} = \sum_x \sum_y x^p y^q f(x, y)$$

- A moment m_{pq} is said to be of *order* p + q.

Moments from binary images - area

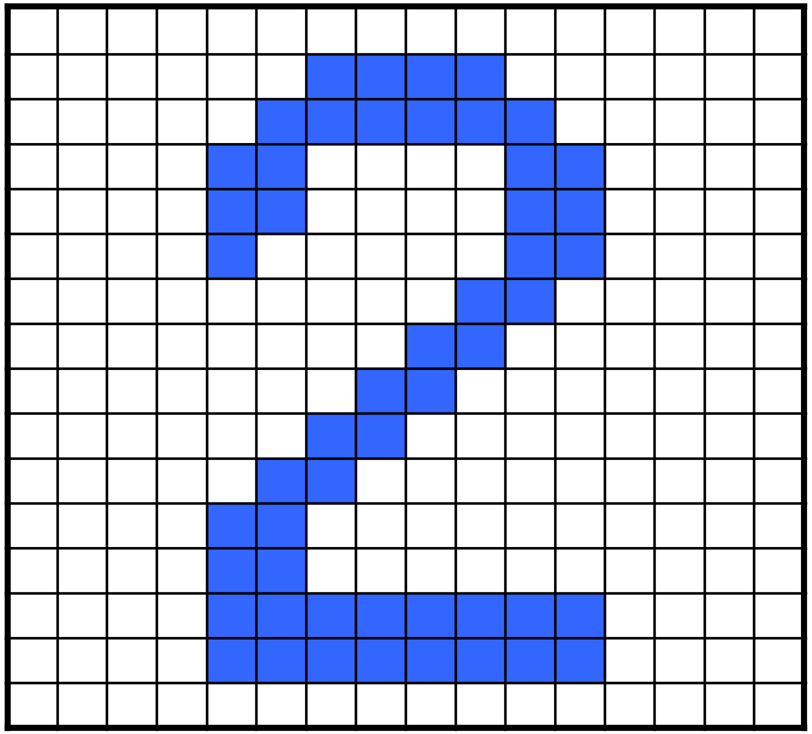
- For binary images, where

  f (x, y) = 1 ⇒ object pixel

  f (x, y) = 0 ⇒ background pixel

- Area

$$m_{00} = \sum_x \sum_y f(x, y)$$



Centroid/center of mass from moments

- For binary images, where

  f (x, y) = 1 $\Rightarrow$ object pixel

  f (x, y) = 0 $\Rightarrow$ background pixel

- Center of mass /"tyngdepunkt"

$$m_{10} = \sum_x \sum_y x f(x,y) = \bar{x} m_{00} \qquad \Rightarrow \qquad \bar{x} = \frac{m_{10}}{m_{00}}$$

$$m_{01} = \sum_x \sum_y y f(x,y) = \bar{y} m_{00} \qquad \Rightarrow \qquad \bar{y} = \frac{m_{01}}{m_{00}}$$

gives the position of the object

Grayscale moments

- In gray scale images, we may regard *f(x,y)* as a discrete 2-D probability distribution over (x,y)

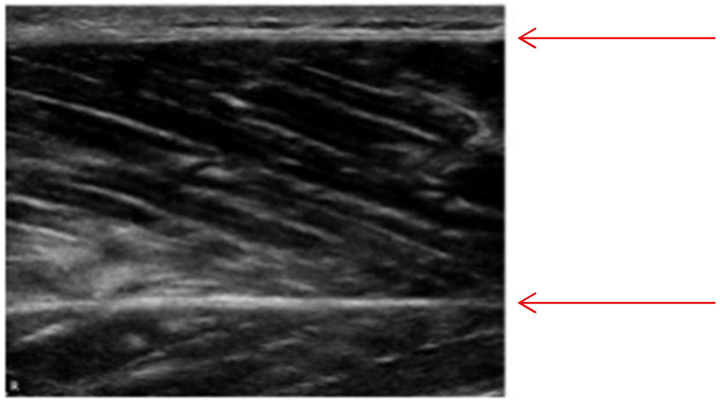
- For probability distributions, we should have

$$m_{00} = \sum_x \sum_y f(x,y) = 1$$

  - And if this is not the case we can normalize by

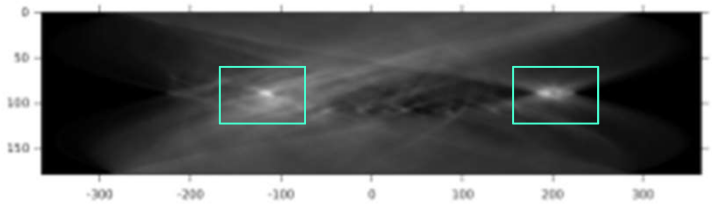
$$F(x,y) = f(x,y)/m_{00}$$

Example use of centroid of grayscale image



(a) Image before transformation.

Ultrasound image of muscle fibers
Want to find the near-horisontal lines
using Radon-transform
(generalized Hough to grayscale images)



Find peaks in the Radon domain
How do we robustly find the location
of a «peak» in the marked areas?

Central moments

- These are position invariant moments, defined by

$$\mu_{p,q} = \sum_x \sum_y (x - \bar{x})^p (y - \bar{y})^q f(x,y)$$

- where

$$\bar{x} = \frac{m_{10}}{m_{00}}, \qquad \bar{y} = \frac{m_{01}}{m_{00}}$$

- The total mass, and the center of mass coordinates are given by

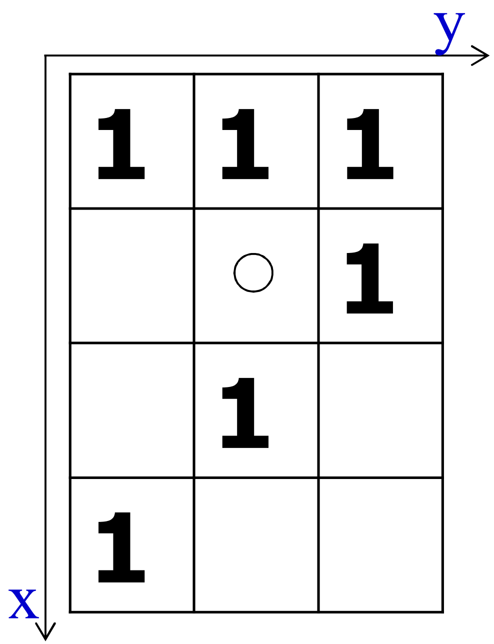
$$\mu_{00} = \sum_x \sum_y f(x,y), \qquad \mu_{10} = \mu_{01} = 0$$

- This corresponds to computing ordinary moments after having translated the object so that center of mass is in origo.

- Central moments are independent of position, but are not scaling or rotation invariant.

- Q: What is μ_{00} for a binary object?

A simple example: center of mass



$$m_{00} = \sum_x \sum_y f(x,y) \quad \mathbf{=6}$$

$$m_{10} = \sum_x \sum_y x f(x,y) = \bar{x} m_{00} \quad \Rightarrow \quad \bar{x} = \frac{m_{10}}{m_{00}} \quad \mathbf{=2}$$

$$m_{01} = \sum_x \sum_y y f(x,y) = \bar{y} m_{00} \quad \Rightarrow \quad \bar{y} = \frac{m_{01}}{m_{00}} \quad \mathbf{=2}$$

2D Central moments μ_{pq} from ordinary moments m_{pq}

- Moments μ_{pq} ($p + q \leq 3$) are given by m_{pq} by:

$$\boxed{\begin{aligned}
&\mu_{00} = m_{00},\ \ \mu_{10} = 0,\ \ \mu_{01} = 0\\
&\mu_{20} = m_{20} - \bar{x} m_{10}\\
&\mu_{02} = m_{02} - \bar{y} m_{01}\\
&\mu_{11} = m_{11} - \bar{y} m_{10}\\
&\mu_{30} = m_{30} - 3\bar{x} m_{20} + 2\bar{x}^2 m_{10}\\
&\mu_{12} = m_{12} - 2\bar{y} m_{11} - \bar{x} m_{02} + 2\bar{y}^2 m_{10}\\
&\mu_{21} = m_{21} - 2\bar{x} m_{11} - \bar{y} m_{20} + 2\bar{x}^2 m_{01}\\
&\mu_{03} = m_{03} - 3\bar{x} m_{02} + 2\bar{y}^2 m_{01}
\end{aligned}}$$

Generalization to 3D moments from m_{pq}

- The 3D μ_{pqr}, are expressed by m_{pqr}:

$$\boxed{\mu_{pqr} = \sum_{S=0}^{p}\sum_{t=0}^{q}\sum_{u=0}^{r} -1^{[D-d]} \binom{p}{s}\binom{q}{t}\binom{r}{u} \Delta x^{p-s}\ \Delta y^{q-t}\ \Delta z^{r-u}\ m_{stu}}$$

- where

  D = (p + q + r); d = (s + t + u)

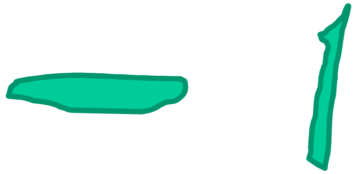
- and the binomial coefficients are given by

$$\boxed{\binom{v}{w} = \frac{v!}{w!\ (v-w)!}, \quad w < v}$$

Moments of inertia or Variance

- The two second order central moments measure the spread of points around the y- and x-axis through the centre of mass

$$\mu_{20} = \sum_x \sum_y (x - \bar{x})^2 f(x, y)$$



$$\mu_{02} = \sum_x \sum_y (y - \bar{y})^2 f(x, y)$$

- From physics: moment of inertia about an axis:
  how much energy is required to rotate the object about this axis:
  - Statisticans like to call this variance.

- The cross moment of intertia is given by

$$\mu_{11} = \sum_x \sum_y (x - \bar{x})(y - \bar{y}) f(x, y)$$

  - Statisticians call this covariance or correlation.

- Orientation of the object can be derived from these moments.
  - **This implies that they are not invariant to rotation.**

A simple example

$$\mu_{20} = \sum_x \sum_y (x - \bar{x})^2 f(x,y) \quad = 5$$

$$\mu_{02} = \sum_x \sum_y (y - \bar{y})^2 f(x,y) \quad = 5$$

$$\mu_{11} = \sum_x \sum_y (x - \bar{x})(y - \bar{y}) f(x,y) \quad = -5$$

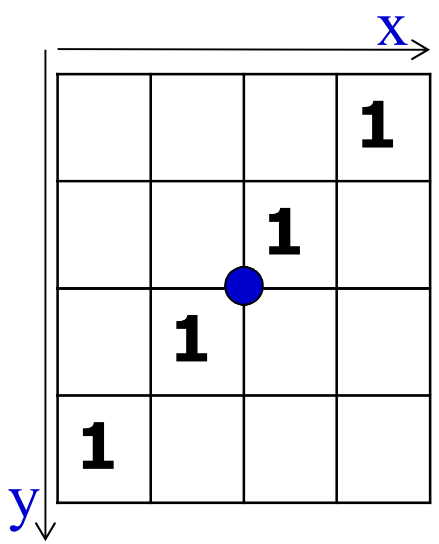


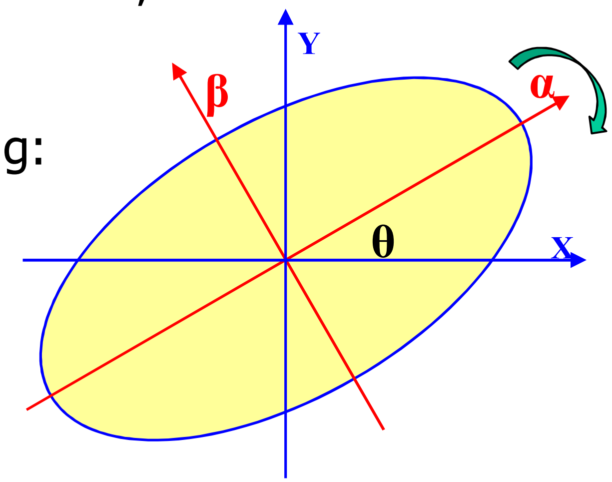
Image coordinates

$$\begin{bmatrix} 4,1 \\ 3,2 \\ 2,3 \\ 1,4 \end{bmatrix}$$

(Note that image coordinates are swapped)

Object orientation - I

- Orientation is defined as the angle, relative to the X-axis, of an axis through the centre of mass that gives the lowest moment of inertia.
- Orientation θ relative to X-axis found by minimizing:



$$I(\theta) = \sum_{\alpha} \sum_{\beta} \beta^2 f(\alpha, \beta)$$

where the rotated coordinates are given by

$$\alpha = x \cos\theta + y \sin\theta, \quad \beta = -x \sin\theta + y \cos\theta$$

- The second order central moment of the object around the α-axis, expressed in terms of x, y, and the orientation angle θ of the object is:

$$I(\theta) = \sum_{x} \sum_{y} [y \cos\theta - x \sin\theta]^2 f(x, y)$$

- We take the derivative of this expression with respect to the angle θ
- Set derivative equal to zero, and find a simple expression for θ :

Object orientation - II

- Second order central moment around the α-axis:

$$I(\theta)=\sum_x\sum_y[y\cos\theta - x\sin\theta]^2\, f(x,y)$$

- Derivative w.r.t. Θ = 0 =>

$$\frac{\partial}{\partial\theta}I(\theta)=\sum_x\sum_y 2f(x,y)[y\cos\theta - x\sin\theta][-y\sin\theta - x\cos\theta]=0$$

$$\Downarrow$$

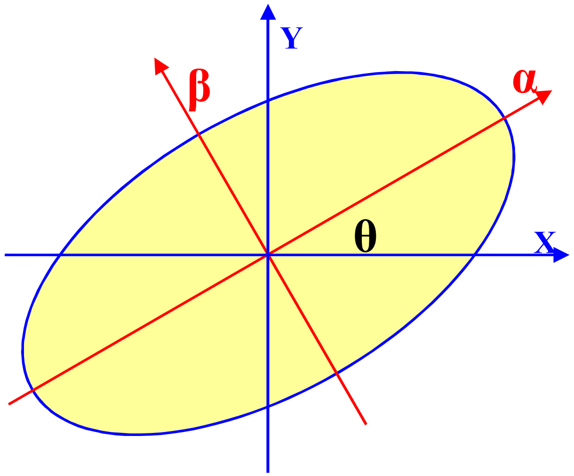
$$\sum_x\sum_y 2f(x,y)\left[xy\left(\cos^2\theta-\sin^2\theta\right)\right]=\sum_x\sum_y 2f(x,y)\left[x^2-y^2\right]\sin\theta\,\cos\theta$$

$$\Downarrow$$

$$2\mu_{11}\left(\cos^2\theta-\sin^2\theta\right)=2(\mu_{20}-\mu_{02})\sin\theta\cos\theta$$

$$\Downarrow$$

$$\frac{2\mu_{11}}{(\mu_{20}-\mu_{02})}=\frac{2\sin\theta\cos\theta}{\left(\cos^2\theta-\sin^2\theta\right)}=\frac{2\tan\theta}{1-\tan^2\theta}=\tan(2\theta)$$



- So the object orientation is given by:

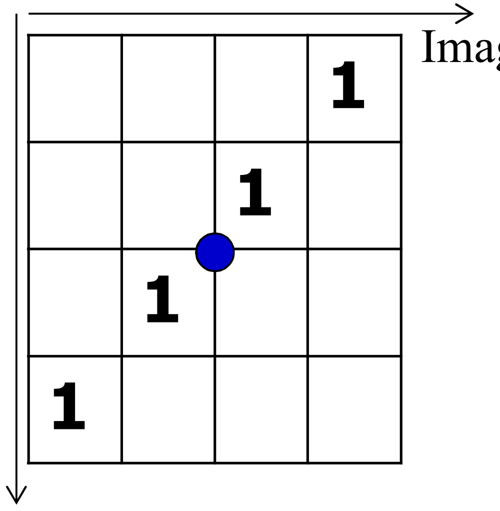
$$\underline{\underline{\theta=\frac{1}{2}\tan^{-1}\left[\frac{2\mu_{11}}{(\mu_{20}-\mu_{02})}\right]}},\qquad \textit{where}\ \ \theta\in\left[0,\pi/2\right] \textit{if}\ \mu_{11}>0,\ \ \theta\in\left[\pi/2,\pi\right] \textit{if}\ \mu_{11}<0$$

A simple example

$$\mu_{20} = \sum_x \sum_y (x - \bar{x})^2 f(x,y)$$ **=5**

$$\mu_{02} = \sum_x \sum_y (y - \bar{y})^2 f(x,y)$$ **=5**

$$\mu_{11} = \sum_x \sum_y (x - \bar{x})(y - \bar{y}) f(x,y)$$ **= - 5**



$$\begin{bmatrix} 4,1 \\ 3,2 \\ 2,3 \\ 1,4 \end{bmatrix}$$

$$\theta = \frac{1}{2}\tan^{-1}\left[\frac{2\mu_{11}}{(\mu_{20} - \mu_{02})}\right], \quad \textit{where} \;\; \theta \in \left[0, \pi/2\right] \textit{if} \; \mu_{11} > 0, \;\; \theta \in \left[\pi/2, \pi\right] \textit{if} \; \mu_{11} < 0$$ **= - 45degrees**

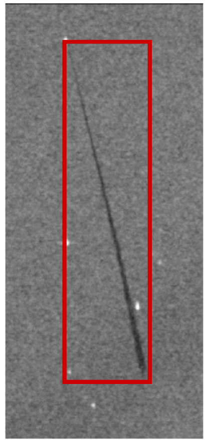
(Note that image coordinates are swapped)

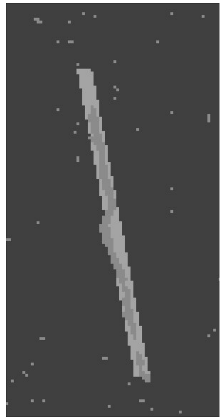
Bounding box - again

- Image-oriented bounding box:
  - The smallest rectangle around the object, having sides parallell to the edges of the image.
  - Found by searching for min and max x and y within the object (*xmin, ymin, xmax, ymax*)
- Object-oriented bounding box:
  - Smalles rectangle around the object, having one side parallell to the orientation of the object (θ).
  - The transformation

$$\alpha = x\cos\theta + y\sin\theta, \quad \beta = y\cos\theta - x\sin\theta$$

    is applied to all pixels in the object (or its boundary).
  - Then search for α_{min}, β_{min}, α_{max}, β_{max}





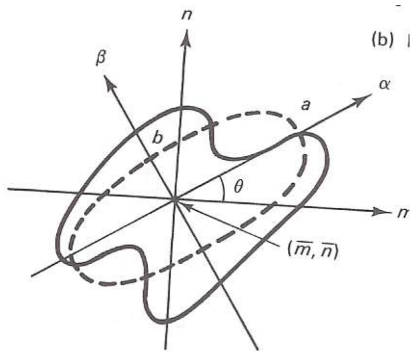
The best fitting ellipse

- Object ellipse is defined as the ellipse whose least and greatest moments of inertia equal those of the object.
- Semi-major and semi-minor axes are given by

$$\left(\hat{a},\hat{b}\right)=\sqrt{\frac{2\left[\mu_{20}+\mu_{02}\pm\sqrt{\left(\mu_{20}+\mu_{02}\right)^{2}+4\mu_{11}^{2}}\right]}{\mu_{00}}}$$

- Numerical eccentricity is given by

$$\hat{\varepsilon}=\sqrt{\frac{\hat{a}^{2}-\hat{b}^{2}}{\hat{a}^{2}}}$$



(c) Best fit ellipse

- Orientation invariant object features.
- Gray scale or binary object.

Radius of gyration, K

- The radius of a circle where we could concentrate all the mass of an object without altering the moment of inertia about its center of mass.

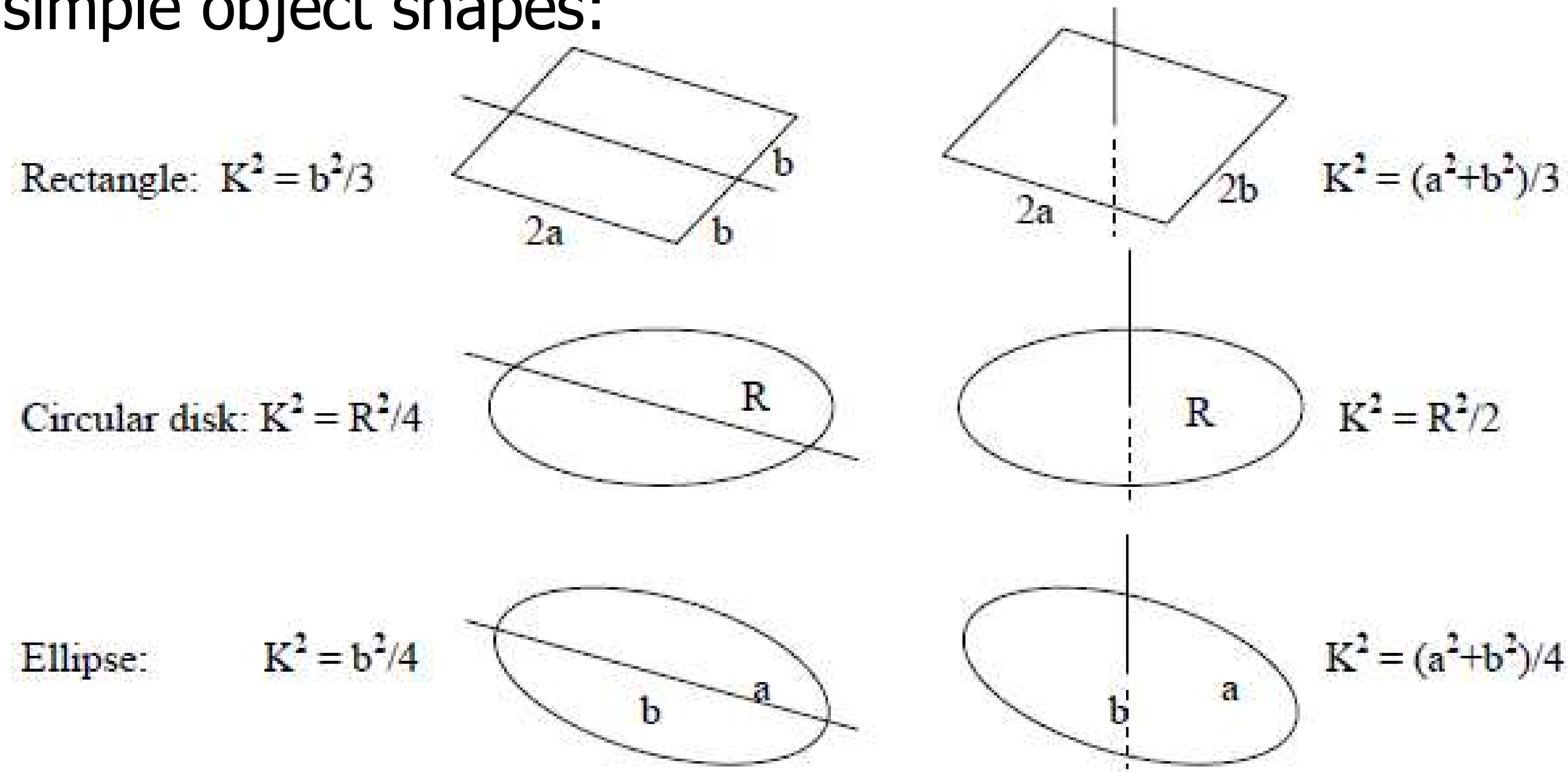
- For arbitrary object having a mass $\mu_{\mathbf{00}}$ and a moment of inertia around the Z-axis, we may write

$$I = \mu_{00}\,\hat{K}^2 \quad \Rightarrow \hat{K} = \sqrt{\frac{I_Z}{\mu_{00}}} = \sqrt{\frac{I_X + I_Y}{\mu_{00}}} = \sqrt{\frac{\mu_{20} + \mu_{02}}{\mu_{00}}}$$

- This feature is obviously invariant to rotation.

Radius of gyration, K

- For homogeneous objects, only determined by geometry.
- Thus, the squared radius of gyration may be tabulated for simple object shapes:



What if we want scale-invariance?

- Changing the scale of *f(x,y)* by (α,β) gives a new image:

$$f'(x,y) = f\left(x/\alpha, y/\beta\right)$$

- The transformed central moments

$$\mu'_{pq} = \alpha^{1+p}\beta^{1+q}\mu_{pq}$$

- If $\alpha=\beta$, scale-invariant central moments are given by the normalization:

$$\eta_{pq} = \frac{\mu_{pq}}{(\mu_{00})^{\gamma}}, \quad \gamma = \frac{p+q}{2}+1, \quad p+q \geq 2$$

Symmetry

- To detect symmetry about center of mass, use central moments.

- For invariance of scale, use scale-normalised central moments
  - $(\boldsymbol{\eta_{11}, \eta_{20}, \eta_{02}, \eta_{21}, \eta_{12}, \eta_{30}, \eta_{03}})$.
- Objects symmetric about either x or y axis will produce $\boldsymbol{\eta_{11} = 0}$.
- Objects symmetric about y axis will give $\boldsymbol{\eta_{12} = 0}$ and $\boldsymbol{\eta_{30} = 0}$.
- Objects symmetric about x axis will give $\boldsymbol{\eta_{21} = 0}$ and $\boldsymbol{\eta_{03} = 0}$.
- X axis symmetry: $\boldsymbol{\eta_{pq} = 0}$ **for all p = 0, 2, 4, … ; q = 1, 3, 5, …**

	η_{11}	η_{20}	η_{02}	η_{21}	η_{12}	η_{30}	η_{03}
'M'	0	+	+	-	0	0	-
'C'	0	+	+	0	+	+	0
'O'	0	+	+	0	0	0	0

Rotation invariant moments

Method 1:

Find principal axes of object, rotate and compute moments.

This can break down if object has no unique principal axes.

Rotation invariant moments

Method 2 : Hu moments

The method of absolute moment invariants:

This is a set of normalized central moment combinations, which can be used for scale, position, and rotation invariant pattern identification.

- For second order (p+q=2), there are two invariants/Hu moments:

$$\varphi_1 = \eta_{20} + \eta_{02} \qquad \varphi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

Third order Hu moments

- For third order moments, (p+q=3), the invariants are:

$$\varphi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\varphi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

$$\varphi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]$$

$$\varphi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})$$

$$\varphi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] - (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]$$

φ_7 is skew invariant, and may help distinguish between mirror images.

- These moments are not independent, and do not comprise a complete set.

Hu's moments; a bit simplified notation

For second order moments (p+q=2), two invariants are used:

$\varphi_1 = \eta_{20} + \eta_{02}$

$\varphi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}{}^2$

For third order moments, (p+q=3), we can use

$a = (\eta_{30} - 3\eta_{12})$, $b = (3\eta_{21} - \eta_{03})$,

$c = (\eta_{30} + \eta_{12})$, and $d = (\eta_{21} + \eta_{03})$

and simplify the five last invariants of the set:

$\varphi_3 = a^2 + b^2$

$\varphi_4 = c^2 + d^2$

$\varphi_5 = ac[c^2 - 3d^2] + bd[3c^2 - d^2]$

$\varphi_6 = (\eta_{20} - \eta_{02})[c^2 - d^2] + 4\eta_{11}cd$

$\varphi_7 = bc[c^2 - 3d^2] - ad[3c^2 - d^2]$

Hu moments of simple objects

- In the continuous case, the two first Hu moments of a binary rectangular object of size 2a by 2b, are given by

$$\phi_1 = \frac{1}{12}\left(\frac{a}{b}+\frac{b}{a}\right), \qquad \phi_2 = \left(\frac{1}{12}\right)^2\left(\frac{a}{b}-\frac{b}{a}\right)^2$$

while the remaining five Hu moments are all zero.

- Similarly, the two first Hu moments of a binary elliptic object with semi-axes a and b, are given by

$$\phi_1 = \frac{1}{4\pi}\left(\frac{a}{b}+\frac{b}{a}\right), \qquad \phi_2 = \left(\frac{1}{4\pi}\right)^2\left(\frac{a}{b}-\frac{b}{a}\right)^2$$

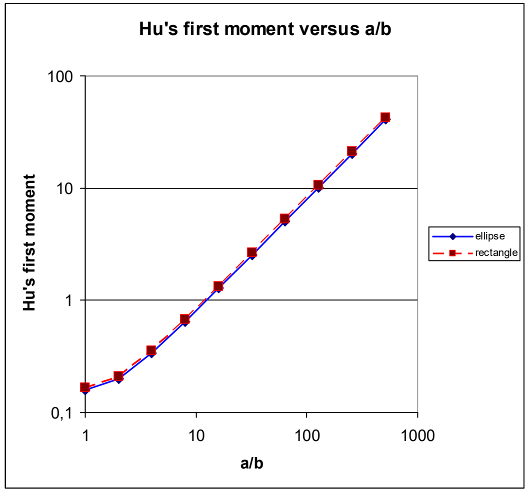
while the remaining five Hu moments are all zero.

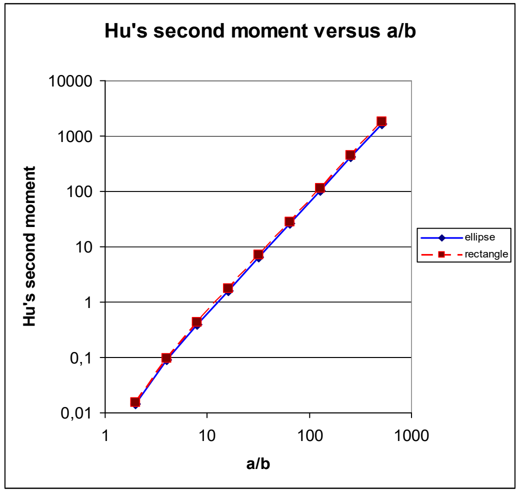
(See definitions of a, b, c and d and table of symmetry)

	η_{11}	η_{20}	η_{02}	η_{21}	η_{12}	η_{30}	η_{03}
'M'	0	+	+	-	0	0	-
'C'	0	+	+	0	+	+	0
'O'	0	+	+	0	0	0	0

Φ_1 and φ_2 versus a/b

- Only (φ1, φ2) are useful for these simple objects.
- Notice that even in the continuous case it may be hard to distinguish between an ellipse and its bounding rectangle using these two moments.
- Relative difference in φ_1 of ellipse and its object oriented bounding rectangle is constant, 4.5%.
- Relative difference in φ_2 of ellipse and its object oriented bounding rectangle is constant, 8.8%.
- Relative differences given above are also true when comparing an ellipse with a same-area rectangle having the same a/b ratio, regardless of the size and eccentricity of the ellipse.





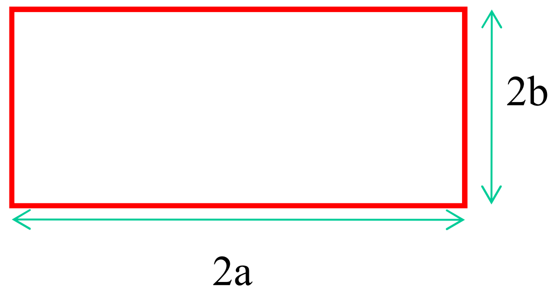
Moments as shape features

- The central moments are seldom used directly
as shape descriptors.
- Major and minor axis, radius of gyration, and eccentricity
are useful shape descriptors.
- Object orientation is normally not used directly,
but to estimate rotation.
- The set of 7 Hu moments can be used as shape features.
(Start with the first four,
as the last half are often zero for simple objects).

Moments of inertia for simple shapes

- Rectangular object (2a×2b):

  $I_{20} = 4a^3b/3 \quad , \quad I_{02} = 4ab^3/3$

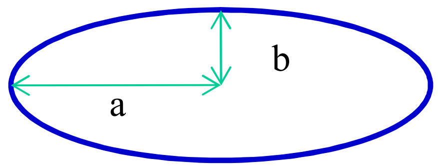


- Square (a×a):

  $I_{20} = I_{02} = a^4/12$

- Elliptical object, semi-axes (a,b):

  $I_{20} = \pi a^3b/4 \, , \quad I_{02}= \pi ab^3/4$



- Circular object, radius R:

  $I_{20} = I_{02} = \pi R^4/4$

Moments of an ellipse

- Assume that the ellipse has semimajor and semiminor axes (*a,b*), a>b. An ellipse where major axis is along x-axis is given by

$$(x/a)^2 + (y/b)^2 = 1 \quad \Rightarrow y = \pm \frac{b}{a}\sqrt{a^2 - x^2}$$

The largest second order central moment (here called I_{20}) is given by

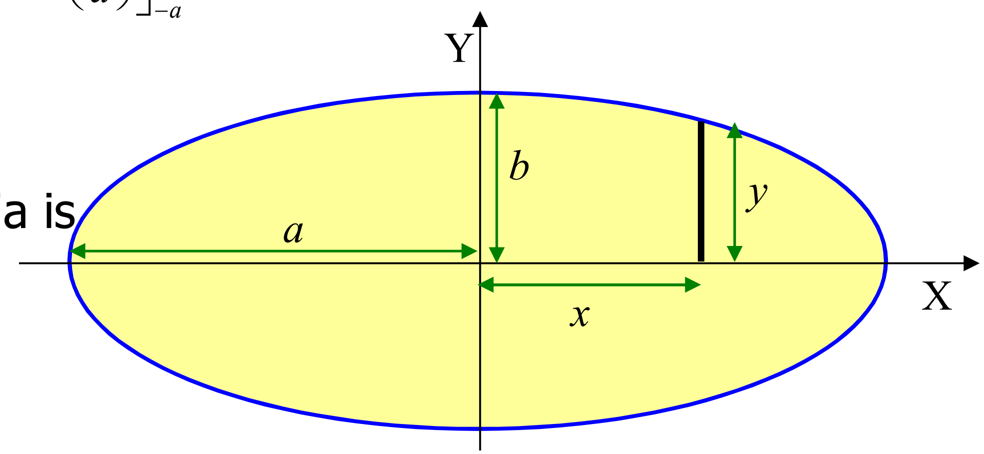
$$I_{20} = 2\int_{-a}^{a} x^2 y\, dx \;= 2\frac{b}{a}\int_{-a}^{a} x^2\sqrt{a^2 - x^2}\, dx$$

$$I_{20} = 2\frac{b}{a}\left[\frac{x}{8}\left(2x^2 - a^2\right)\sqrt{a^2 - x^2} + \frac{a^4}{8}\sin^{-1}\left(\frac{x}{a}\right)\right]_{-a}^{a}$$

$$I_{20} = 2\frac{b}{a}\left[\frac{a^4}{8}\left(\frac{\pi}{2} + \frac{\pi}{2}\right)\right] = \underline{\underline{\frac{\pi}{4}a^3 b}}$$

Similary, the smallest moment of inertia is

$$I_{\min} = \underline{\underline{\frac{\pi}{4}a\ b^3}}$$



Grayscale contrast invariants

- Abo-Zaid *et al.* have defined a normalization that cancels both scaling and contrast.
- The normalization is given by

$$\eta'_{pq} = \frac{\mu_{pq}}{\mu_{00}} \left(\frac{\mu_{00}}{\mu_{20} + \mu_{02}} \right)^{\frac{(p+q)}{2}}$$

- This normalization also reduces the dynamic range of the moment features, so that we may use higher order moments without having to resort to logarithmic representation.
- Abo-Zaid's normalization cancels the effect of changes in contrast, but not the effect of changes in intensity:

$$f'(x,y) = f(x,y) + b$$

- In practice, we often experience a combination:

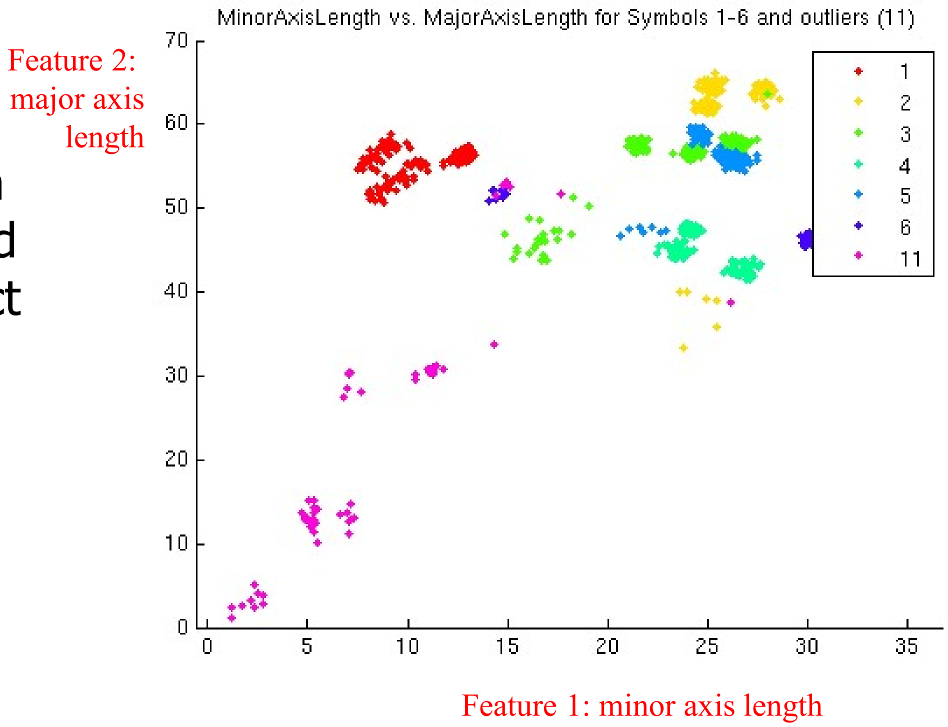
$$f'(x,y) = cf(x,y) + b$$

From features to discrimination between objects

- The following slide introduces simple tools like scatter plots to visualize how good a feature (or combination of 2-3 features) is in separating objects of different types/classes.
- To evaluate features, we use training data consisting of objects with KNOWN CLASS.

Scatter plots

- A 2D scatter plot is a plot of feature values for two different features. Each object's feature values are plotted in the position given by the features values, and with a class label telling its object class.
- Matlab: gscatter(feature1, feature2, labelvector)
- Classification is done based on more than two features,
  but this is difficult to visualize.
- Features with good class separation show clusters for each class, and different clusters should ideally be separated.



The "curse-of-dimensionality"

- Also called "peaking phenomenon".
- For a finite training sample size, the correct classification rate initially increases when adding new features, attains a maximum and then begins to decrease.
- The implication is that:
- For a high measurement complexity, we will need large amounts of training data in order to attain the best classification performance.
- => 5-10 samples
  per feature per class.

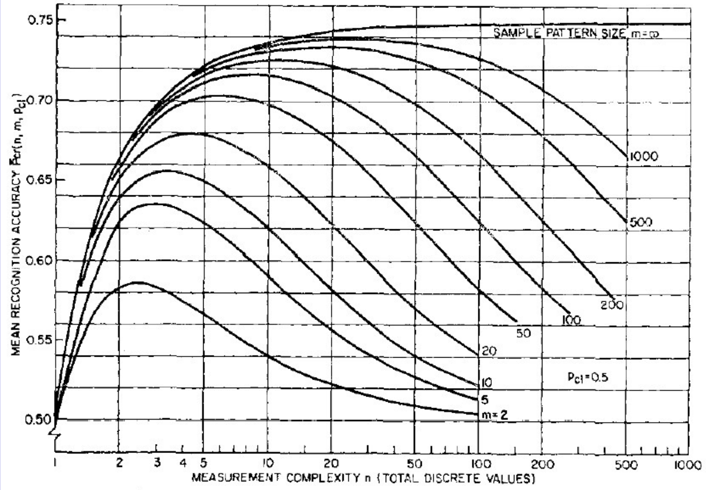


Fig. 3. Finite data set accuracy ($p_{c1} = \frac{1}{2}$).

Correct classification rate as function of feature dimensionality, for different amounts of training data. Equal prior probabilities of the two classes is assumed.

Illustration from G.F. Hughes (1968).

Finding best feature subset

- The goal: to find the subset of observed features which
  - best characterizes the differences between groups
  - is similar within the groups
  - Maximize the ratio of between-class and within-class variance.
- If we want to perform an exhaustive search through D features for the optimal subset of the $d \leq m$ "best features", the number of combinations to test is

$$n = \sum_{d=1}^{m} \frac{D!}{(D-d)!\,d!}$$

- Impractical even for a moderate number of features!
  $d \leq 5$, $D = 100$ => $n = 79.374.995$
- There exist several sub-optimal schemes to search for the best sub-set.

A simulation study design

- *H*. Schulerud and F.Albregtsen: *"Many are called, but few are chosen. Feature selection and error estimation in high dimensional spaces"*, Computer Methods and Programs in Biomedicine, 73, 91—99, 2004.
- Monte Carlo study, averaging 100 simulations per setting
- 2 classes, normally distributed, common covariance
- Up to 500 feature candidates
- Only 5 features are different between the classes

  For these 5, squared difference of class means = $\delta^2/\sqrt{5}$; $\delta^2 = 0, 1, 4$

  the rest of the continuous distributions are EQUAL!
- Stepwise forward-backward feature selection
- 20 - 1000 training samples
- 20 - 1000 test samples

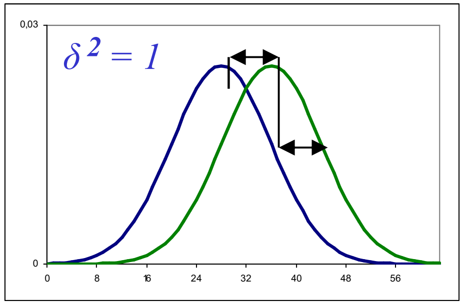
Probabilistic distance measures

- If the class-conditional probability distributions are Gaussian:

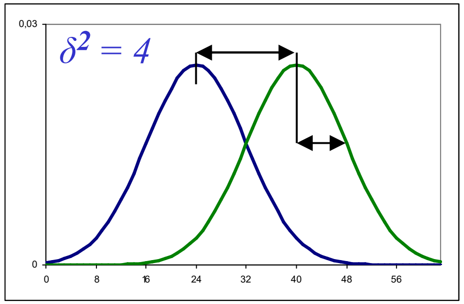
$$p(\xi \mid \omega_i) = \left[(2\pi)^d |\Sigma_i|\right]^{-1/2} \exp\left\{-\frac{1}{2}(\xi - \mu_i)^T \Sigma_i^{-1} (\xi - \mu_i)\right\}$$

where μ_i and Σ_i are the mean vector and the covariance matrix of the *i*-th class distribution; the **Mahalanobis** distance is

$$\delta^2 = (\mu_2 - \mu_1)^T \Sigma^{-1} (\mu_2 - \mu_1), \; if \; \Sigma_1 = \Sigma_2 = \Sigma$$

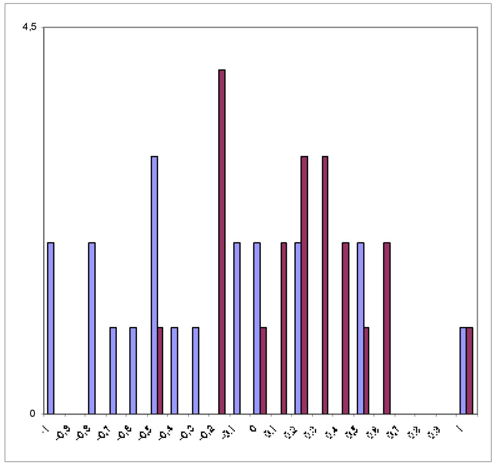


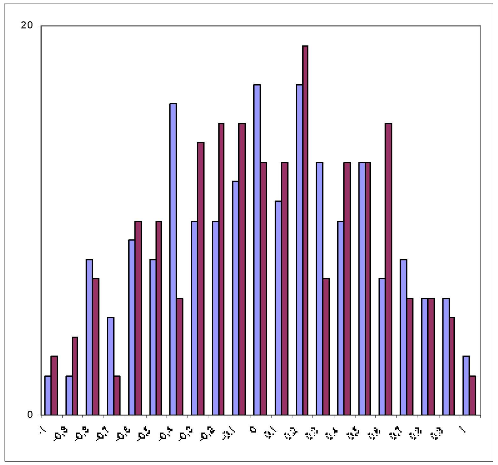
- The Bhattacharyya distance may also be useful:



$$J_B = \frac{1}{4}(\mu_2 - \mu_1)^T \left[\Sigma_1 - \Sigma_2\right]^{-1} (\mu_2 - \mu_1) + \frac{1}{2} \ln \left[\frac{\left|\frac{1}{2}(\Sigma_1 + \Sigma_2)\right|}{\sqrt{|\Sigma_1||\Sigma_2|}} \right]$$

Samples from distributions





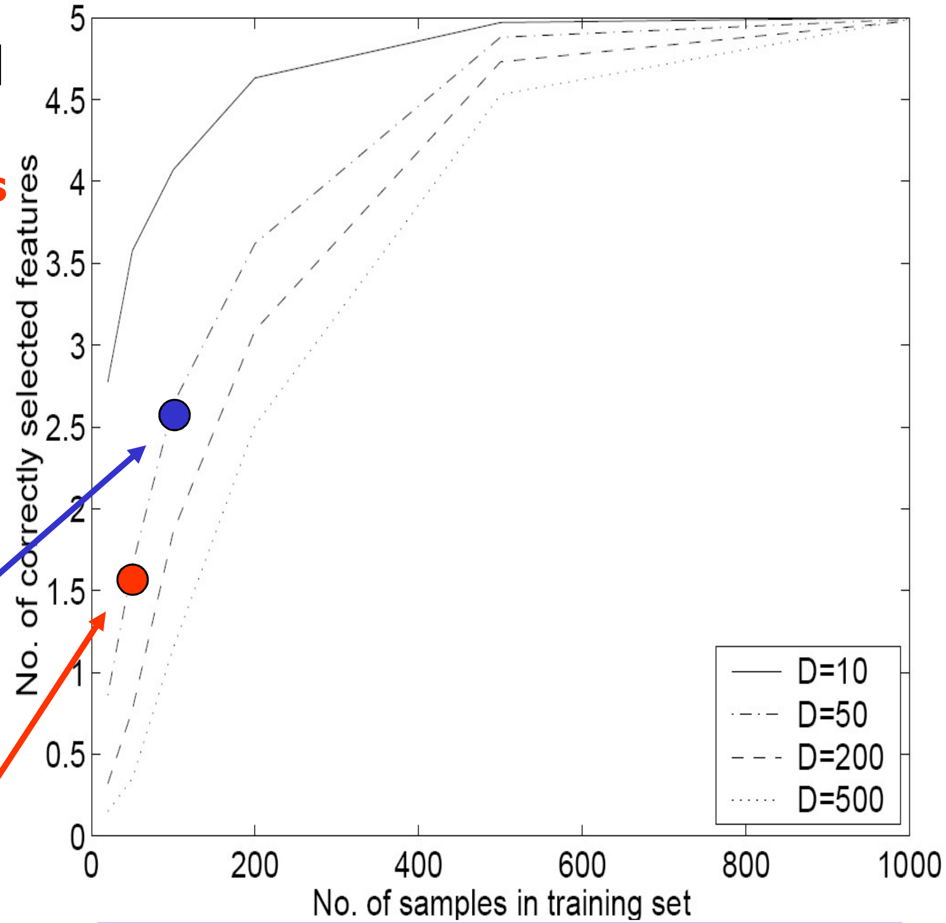
Distribution of 2 independent sets of ***20*** *samples from standardized normal distributions, $\delta^2 = 0$.*

Distribution of 2 independent sets of ***200*** *samples from standardized normal distributions, $\delta^2 = 0$.*

- For small sample sets and small class distances, observations may indicate a separation of classes, while no real difference exists !!!

Simulation results - Feature selection II

- The number of correctly selected features increases with
  - **increasing # of training samples**
  - **decreasing # of candidates**
  - **(increasing class distance)**
- For small sample sizes the number of *candidates* features is of great importance :
  - For D = 50 and $\delta^2 = 1$, half of the 5 selected features will be noise if $n_{Tr} = 100$.
  - For D = 50, $\delta^2 = 1$, $n_{Tr} = 50$, **60% of the selected features will be noise!**
  - **So, Be Careful !!!**



The average number of correctly selected features, when the class Mahalanobis distance $\delta^2 = 1$.

Caveats of Cross Validation

- A simple simulation may demonstrate the effect of performing feature selection before a cross validation to estimate classification performance on the same data.

- If the classes are overlapping, the number of training samples is small, and the number of feature candidates are high, the common approach of performing feature selection before leave-one-out error estimation on the same data results in a highly biased error estimate of the true error.

- Performing feature selection and leave-one-out error estimation in one process gives an unbiased error estimate, but with high variance.

- See Figure 7 of

  H Schulerud and F Albregtsen, *"Many are called, but few are chosen. Feature selection and error estimation in high dimensional spaces"*, Computer Methods and Programs in Biomedicine 73, 91—99, 2004.

Learning goals – object description

- Invariant topological features
- Projections and signatures – use and limitations
- Geometric features
  - Area, perimeter and circularity/compactness
  - Bounding boxes

- Moments, binary and grayscale
    - Ordinary moments and central moments
  - Moments of objects, object orientation, and best fitting ellipse
    - Focus on first- and second-order moments.
  - Invariance may be important

- Inspection of feature scatter plots
- Select your feature set with great care ! Validate correctly!