IN5520/9520 Exam 2018, Exercise 1: Texture analysis

Assume that you are given a gray level image of size MxN pixels with b bits per pixel.

a) Describe how a normalized Gray Level Coccurrence Matrix is computed, and which parameters this involves.

Answer:
Textbook stuff!
Eventually re-quantize gray level image from $G = 2^b - 1$ to L gray levels.
4 elements should be mentioned:

1. *Initialize matrix of GxG (or LxL) with 0's.*
2. *Go through all MxN pixels where pixel pair of gray levels i and j a distance d pixels apart in direction θ are inside image, and add 1 at position (i,j) in GLCM.*
3. *Finally, normalize by integer sum of matrix entries.*
4. *Parameters: G (or eventually L),d, θ (or $\Delta x, \Delta y$).*

b) For a given inter-pixel distance and direction, how do we make the normalized GLCM symmetrical about the matrix diagonal without double counting?

Answer:
The GLCM in the opposite direction of θ is the transpose of the GLCM in the direction θ.
So we have $P(d,\theta+\pi) = [P(d,\theta)]^T$ - see slide 38 of lecture 2.
Thus, double counting is avoided by adding the transpose of the GLCM to the GLCM before normalizing.
If both have been normalized, simply divide the sum by 2.

c) Say that we have accumulated a normalized symmetrical GLCM for a given inter-pixel distance and direction.
How can we find what fraction of the pixel pairs in the image that have a difference of D gray levels? Please illustrate with a small sketch!

Answer: Matrix elements on the diagonal will all represent pixel pairs with no gray level difference.
Matrix elements that are one cell away from the diagonal (along I or j-direction) represent pixel pairs with a difference of only one gray level.
So, the sum of all elements D cells away from the diagonal (along i and j-axis) will represent the fraction of pixel pairs having a difference of D gray levels. To get the differences both ways, it is important to sum both above and below the main diagonal!

	J = 0	J = 1	J = 2	J = 3
I = 0	I – j = 0	| I – j | = 1	| I – j | = 2	| I – j | = 3
I = 1	| I – j | = 1	I – j = 0	| I – j | = 1	| I – j | = 2
I = 2	| I – j | = 2	| I – j | = 1	I – j = 0	| I – j | = 1
I = 3	| I – j | = 3	| I – j | = 2	| I – j | = 1	I – j = 0

IN5520/9520 Exam 2018, Exercise 3: Cartesian geometric moments

a) Give an expression for an ordinary geometric moment of order p+q of an object in an image, and explain the terms of the expression.

Answer: The expression for an ordinary moment is

$$m_{pq} = \sum_x \sum_y x^p y^q f(x,y)$$

where f(x,y) is the pixel value at the pixel coordinates (x,y).
The summation is performed over the object.

b) Describe, in words or using mathematical expressions, how to get from an ordinary moment of order p+q to a central moment.

Answer: Shift the origin to the centre of mass of the object, and compute the moment as before. Or: Find the coordinates of the center of mass of the object, and use the expression below.

$$\mu_{p,q} = \sum_x \sum_y (x - \bar{x})^p (y - \bar{y})^q f(x,y)$$

a) What kind of invariance is obtained by such central moments, as compared to an ordinary moment? Please explain!

Answer: Central moments are invariant to the position of the object in the image, since it is essentially an ordinary moment computed with the origin in its center of mass.

b) A third kind of moment is obtained when we normalize a central moment by

$$\eta_{pq} = \frac{\mu_{pq}}{(\mu_{00})^{\gamma}}, \quad \gamma = \frac{p+q}{2} + 1, \quad p+q \geq 2$$

What kind of invariance is obtained, and what is the condition for the expression above?

Answer:
This is a scale invariant central moment (and thus also invariant to position), and the condition is that the scaling is the same in the x- and y direction.

1st question for the PhD-students only:

e) The seven Hu's moment combinations are position, scale, and rotation invariant, and may be useful to obtain features that will discriminate between different objects.

We have seen that for rectangles and ellipses, only the two first Hu moment combinations are different from 0.
The figure below gives a logarithmic plot of the first two Hu moments for binary rectangular objects of size 2a by 2b, and ellipses with semi-axes a and b, for a/b = [1,2,4,8,16].

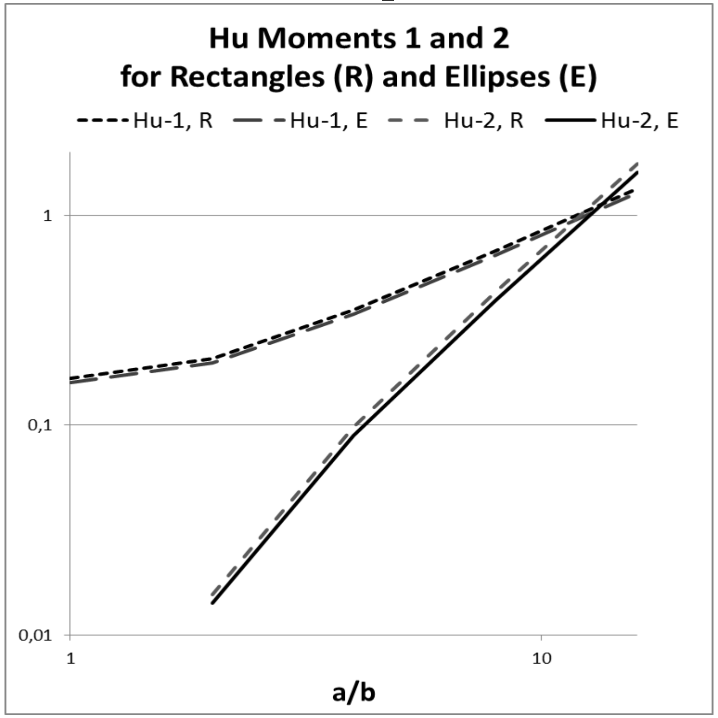
In the continuous case, the first two Hu moments of rectangles are given by

$$\phi_1 = \frac{1}{12}\left(\frac{a}{b}+\frac{b}{a}\right), \qquad \phi_2 = \left(\frac{1}{12}\right)^2\left(\frac{a}{b}-\frac{b}{a}\right)^2$$

and for ellipses they are given by

$$\phi_1 = \frac{1}{4\pi}\left(\frac{a}{b}+\frac{b}{a}\right), \qquad \phi_2 = \left(\frac{1}{4\pi}\right)^2\left(\frac{a}{b}-\frac{b}{a}\right)^2$$

What implication do you draw from the equations above and from this plot?



Answer: The relative difference between a rectangle and a same a/b ellipse feature is very small, only 4.5% for Hu-1 and 8.8% for Hu-2, regardless of eccentricity. Relative diference = [Hu(R) – Hu(E)]/Hu(R). Thus, this will be useful only for large objects & little noise. (For a circle and its bounding square, $\varphi_2 = 0$.)

Besides, features for a given rectangle (a/b) will be the same as for a slightly more eccentric ellipse.

2nd question for the PhD-students only:

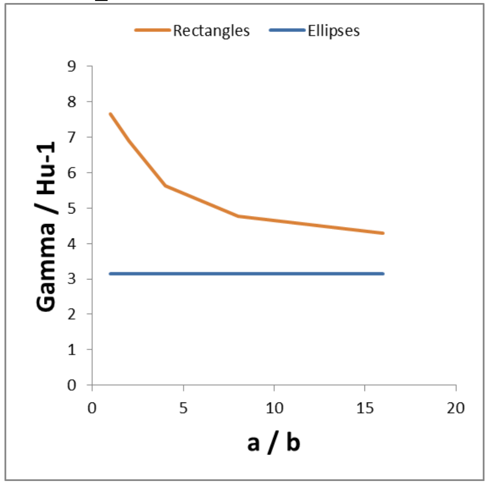
f) We define object compactness $\gamma = P^2/(4\pi A)$, where P is the perimeter length and A is the area. For a disc, γ is minimum and equals 1, while γ attains a high value for both complex objects, and for very elongated simple objects, like rectangles and ellipses where the a/b ratio is high.

For ellipses and rectangles, the compactness measure in the continuous case is:

$$\gamma_{ellipse} = \frac{1}{4}\left(\frac{a}{b} + \frac{b}{a}\right), \qquad \gamma_{rectangle} = \frac{1}{\pi}\left(\frac{a}{b} + \frac{b}{a} + 2\right)$$

In an object classification setting, would you use both the given compactness measure and the first Hu moment to characterize ellipses or rectangles? Please explain!

Answer: We notice that for ellipses, the first Hu moment is a simple linear function of its compactness measure, given by $\varphi_1 = \gamma/\pi$. So, do not use both γ and φ_1 on ellipses of different eccentricity!.
For rectangles the relation is: $\varphi_1 = (\pi\gamma + 2)/12$. So for a square the ratio $\gamma/\varphi_1 = 24/\pi$, while the ratio converges to $12/\pi$ as illustrated in the plot to the right. So, using both the compactness measure and the first Hu moment to distinguish between rectangles of different a/b ratios may make sense. [But not ellipses and rectangles together!]



IN5520/9520 Exam 2018, Exercise 5: Alternative approaches to object size estimation

The graylevel image to the right is from one of the chapters of the textbook used throughout this course.

It depicts a number of bright wood dowels of different brightness on a dark background.

There may be visible structures both in each object (dowel) and in the background.

a) Please detail maximum two different approaches to obtain an estimate of the distribution of object sizes in the image.

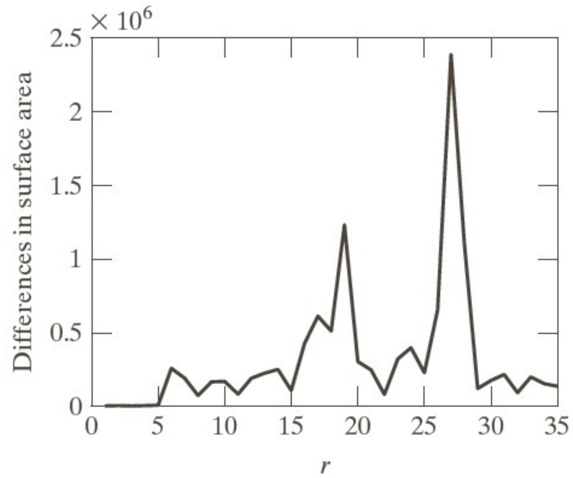
Answer:

1) At least describe in detail the pattern matching approach from Chapter 9 of GW.

- This is termed "Granulometry": determining the size distribution of particles in an image.

$$f \circ S = (f \ominus S) \oplus S$$
$$= \max(\min(f))$$

- Principle: perform a series of morphological *openings* (erosion followed by dilation), i.e., first creating an image containing the running local **minimum** gray level in the region defined by the structuring element, and based on that output image creating a second output image containing the running local **maximum** gray level in the region defined by the (reflected) structuring element.
- Let pixel (x,y) in the outimage have this value.
- Compute the sum of all pixel values after the opening.
- Repeat this with increasing radius r of structuring element.
- Compute the difference in this sum between radius r and r-1, and plot this as a function of radius, see figure.



2) An obvious alternative is to do edge detection (gradient detection), followed by Hough transform to estimate circle radii.
Since there are irrelevant structures both in the objects and in the background, Canny's method may seem to be relevant, and that should be described.
Sobel/Prewitt filtering should be mentioned anyway, since it occurs in Canny's method and since it gives the gradient direction that is very useful in the HT.