IN5520/9520 Exam 2019, Exercise 1: Texture Analysis

Assume that you are given a gray level image of size MxN pixels with b bits per pixel.

a) Describe how a normalized Gray Level Coccurrence Matrix (GLCM) is computed, and which parameters this involves.

Answer: Textbook stuff!
Eventual re-quantization of the input gray level image from $G = 2^b - 1$ to L gray levels should be motivated.
4 elements should be mentioned:

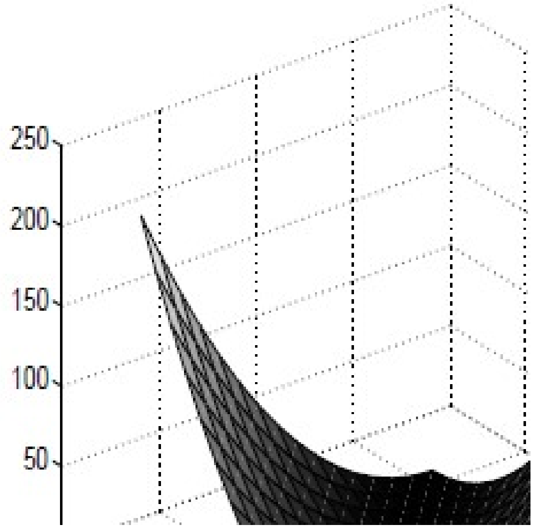
1. *Initialize matrix of GxG (or LxL) with 0's.*
2. *Go through all MxN pixels where pixel pair of gray levels i and j a distance d pixels apart in direction θ are inside image, and add 1 at position (i,j) in GLCM.*
3. *Finally, normalize by integer sum of matrix entries. A plus if you give an expression of what this sum is.*
4. *Parameters: G (or eventually L),d, θ (or Δx,Δy).*

b) For a given inter-pixel distance and direction, how do we make the normalized GLCM symmetrical about the matrix diagonal without double counting?

Answer: The GLCM in the opposite direction of θ is the transpose of the GLCM in the direction θ.
So we have $P(d,\theta+\pi) = [P(d,\theta)]^T$ - see slide 38 of lecture 2.
Thus, double counting is avoided by adding the transpose of the GLCM to the GLCM before normalizing.
If both have been normalized, simply divide the sum by 2.
Both options should be mentioned.

c) Assume that we have accumulated a normalized symmetrical GLCM for a given inter-pixel distance and direction.
Give the expression for a GLCM feature that has a weighting function equal to zero along the diagonal (i = j), and increases quadratically away from the diagonal, as illustrated for G=15.

What will the effect of this weight function be, and what kind of images will get a high feature value?

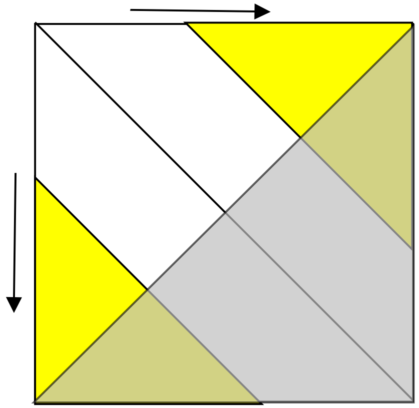


Answer: The expression is:

$$\text{Inertia} = \sum_{i=1}^{G} \sum_{j=1}^{G} (i$$

This expression will favor contributions from P(i, j) away from the diagonal (i ≠ j), i.e., give higher values for images with high local contrast.

d) How can we find what fraction of pixel pairs at the given inter-pixel distance and direction in the image that have an absolute difference $| i-j | \geq D$ gray levels, while the sum (i+j) is in the upper half of the possible range? Please illustrate!

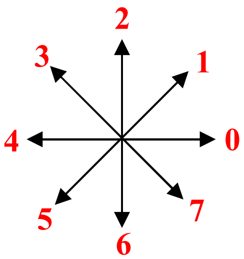


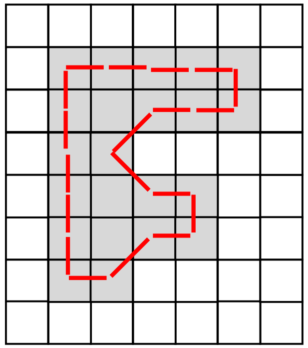
Answer: Matrix elements on the main diagonal represent pixel pairs with same gray level. The sum of all elements from D cells away from the diagonal (along i and j-axis) into the corners of the matrix will represent the fraction of pixel pairs having a difference of D gray levels or more, illustrated in yellow for a given value of D.
Restricted to upper half of the possible range
=> The transparent gray triangle in the lower right half of the matrix.
So the answer is the sum of the normalized matrix elements in the intersection of yellow and gray.

It is also entirely possible to think of decomposing the 2D GLCM into sum and difference histograms. ***Then the answer is the intersection of those pixel pairs in the low and high end of the difference histogram, and at the same time in the upper half of the sum histogram (which has twice the range G).***

IN5520/9520 Exam 2019, Exercise 2: Chain Codes

You are given the 8-directional chain code and the binary object below.





a) Find the <u>absolute</u> chain code of the boundary of the object clockwise from the upper left pixel.

Answer: The absolute code starting at the upper left point and moving clockwise is 19 digits long:

0000644570645422222

b) Which technique, based on the 8-directional absolute chain code, can be used to make a description of the object that is independent of the start point? Demonstrate this by starting at the lower right pixel of the object, instead of the upper left.

Answer: **A minimum circular shift of the clockwise absolute chain code gives start point invariance.**
Demonstration: Absolute chain code starting at the lower right point gives:
4222220000644570645.
The minimum circular shift of this is: 0000644570645422222,
which is the same code as when we started at the upper left in a).

c) Which technique, based on the clockwise **relative** chain code, will give you the same description of the object, independent of the start point? Demonstrate this by starting at the upper left and the lower right object pixel.

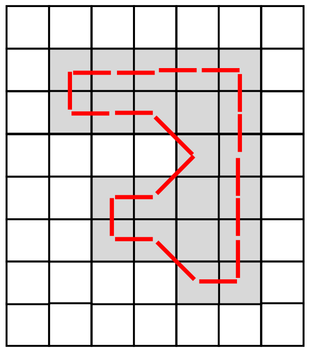
Answer: A minimum circular shift of the clockwise relative chain code gives a normalization for start point.

Demonstration: The clockwise relative code, starting at the upper left pixel of the object, is the 19 digits: 0222002343003102222.

The minimum circular shift of this is: 0023430031022220222.

With start point at the lower right pixel we get 1022220222002343003 which has a minimum circular shift 0023430031022220222, which is the same as above.

d) Rotation invariance is inherent in relative chain codes. But what if the object has been flipped horizontally. How can you then determine if it is the same object?



Answer: The simplest way is to flip it horizontally, select a starting point, and do as in the exercise above.

Alternatively, clockwise relative code from upper left pixel: 0222022220130034320; reversed (back-to front) followed by a minimum circular shift you get 0023430031022220222. So it is the same object!

Another possibility also exists: Anti-clockwise relative code using a flipped code, starting upper right, is 0222002343003102222, which has a minimum circular shift; 0023430031022220222. So it is the same object.

And as a vertical flip is equivalent to a rotation by 180 degrees followed by a horizontal flip, these solutions will work for vertical flips too! Instructive to check flips and rotations!

IN5520/9520 Exam 2019, Exercise 3: Geometric Moments and Hough Transform

Assume that you have thresholded a gray level image into a binary image ***b(x,y)*** containing a solid object (pixel value = 1) and a background (pixel value 0).

a) Describe a moment-based approach to find the center of mass of the object.

Answer: We use first order moments to find the center of mass

$$m_{10} = \sum_x \sum_y x\, b(x,y) = \bar{x} m_{00} \quad \Rightarrow \bar{x} = \frac{m_{10}}{m_{00}}$$

$$m_{01} = \sum_x \sum_y y\, b(x,y) = \bar{y} m_{00} \quad \Rightarrow \bar{y} = \frac{m_{01}}{m_{00}}$$

b) Assume that the object is an equilateral triangle, located somewhere in the image. Give a definition of the object orientation, and describe a moment-based approach to estimate the orientation of the object. In this case, is the result unique?

Answer: We either use central moments defined by

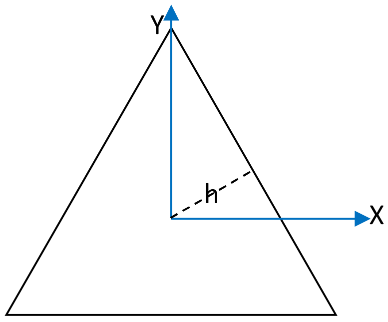
$$\mu_{pq} = \sum_x \sum_y (x - \bar{x})^p \ (y - \bar{y})^q \, b(x, y)$$

And use the three second order central moments μ_{11}, μ_{20}, and μ_{02} to estimate the orientation of the object by

$$\theta = \frac{1}{2} \tan^{-1} \left[\frac{2\mu_{11}}{(\mu_{20} - \mu_{02})} \right]$$

Or we shift the origin to the center of mass, and use ordinary moments to find orientation. Since the orientation is defined as "the angle, relative to the X-axis, of an axis through the centre of mass that gives the lowest moment of inertia", we have three possible orientations of this triangle.

c) Assume that we translate the origin to the center of mass of the triangle, then apply a gradient detector to the binary image, and use the "normal representation" Hough Transform. Assume that one side of the triangle is parallel to the x-axis, describe the contents of the Hough space.



Answer:

- *There will be three TH peaks, π/3 apart.*
- *The peaks will have the same height, since the three line segments have the same length, s.*
- *The normal onto the three sides are of the same length, => all peaks occur at ρ=h.*

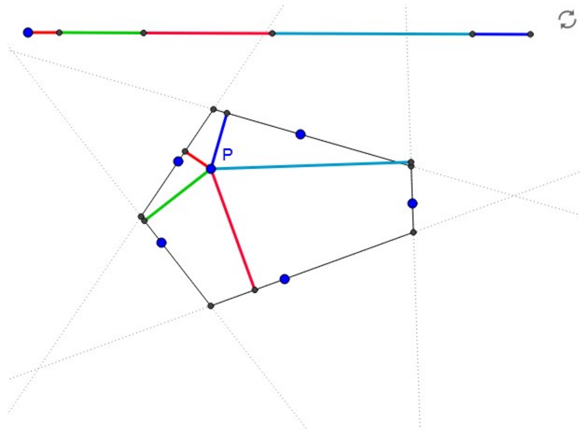
d) What happens in the Hough space if the origin is moved inside the triangle, so that it does not coincide with the center of mass?

Answer:

- *The three TH peaks will still be π/3 apart.*
- *The peaks will still have the same height, but the normals onto the three sides are different.*

The following two questions are intended for the PhD-students:

e) What will happen in the Hough domain, if an equi-angular polygon is rotated anti-clockwise around an off-centered origin, P, inside the polygon?



Answer:

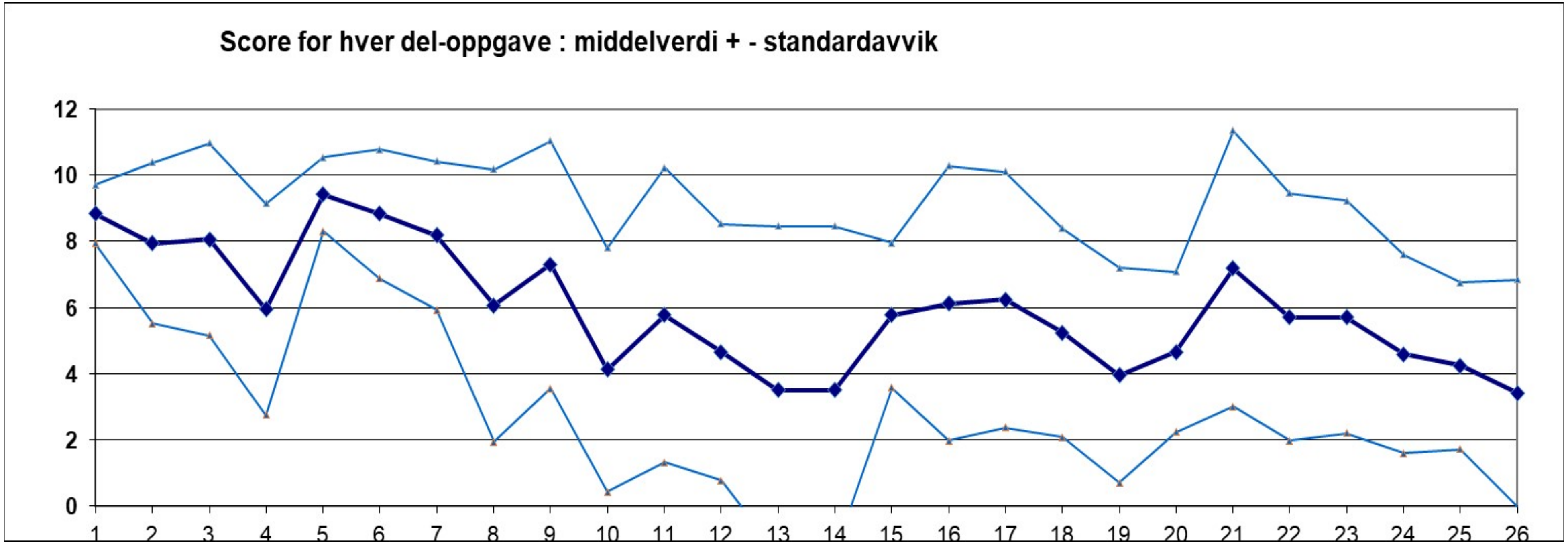
- *The N peaks in the Hough space will be 180*(1-2/N)° apart on the θ-axis.*
- *The normals onto the N sides are in general of different length, so the peaks occur at different ρ-values.*
- *The sides of the polygon are in general different length, so the Hough space peaks have different heights.*

- *The N peaks in the (θ,ρ)-domain will slide in the positive θ-direction, keeping the distance between the maxima and the ρ-values, sliding out of the $[-\pi/2,\pi/2]$-domain at $\pi/2$ and reappearing at $-\pi/2$.*

f) Which polygon feature in the HT domain is invariant to the location of the origin inside an equiangular polygon, and may be useful when working with noisy images?

Answer: "The sum of distances from an interior point to the sides of an equiangular polygon ($\sum\rho$) does not depend on the location of the point, and is that polygon's invariant." This is known as "Viviani's theorem".

So an intentional hint is that the length of the bar above the sketched polygon is constant when P moves, while the length of the colored normals onto the N sides will vary.



Thank You for Your Attention so far!