IN 5520 Weekly exercises on Support Vector Machines.

Exercise 1.

Show that the criterion

$$y_i(w^T x_i + w_0) \geq 1, \quad i = 1,2,\ldots N$$

corresponds to correct classification for all N samples in a binary classification problem with classes -1 and 1.

Exercise 2.

Given a binary data set:

$$\text{Class} -1: \begin{bmatrix} 1 & 9 \\ 5 & 5 \\ 1 & 1 \end{bmatrix} \quad \text{Class} +1: \begin{bmatrix} 8 & 5 \\ 13 & 1 \\ 13 & 9 \end{bmatrix}$$

Plot the points in a plot. Sketch the support vectors and the decision boundary for a linear SVM classifier with maximum margin for this data set.

Exercise 3.

Given the binary classification problem:

$$\text{Class -1: } \begin{bmatrix} 2 & 2 \\ 3 & 3 \\ 4 & 4 \\ 5 & 5 \\ 4 & 6 \\ 3 & 7 \\ 4 & 8 \\ 5 & 9 \\ 6 & 10 \end{bmatrix} \quad \text{Class +1: } \begin{bmatrix} 6 & 2 \\ 7 & 3 \\ 8 & 4 \\ 9 & 5 \\ 8 & 6 \\ 7 & 7 \\ 7 & 8 \\ 7 & 9 \\ 8 & 10 \end{bmatrix}$$

a) Sketch the point in a scatterplot.
b) In the plot, sketch the mean values and the decision boundary you would get with a Gaussian classifier with $\Sigma=\sigma I$.
c) What is the error rate of the Gaussian classifier on the training data set?
d) Sketch on the plot the decision boundary you would get using a SVM with linear kernel and a high cost of misclassifying training data. Indicate the support vectors and the decision boundary on the plot.
e) What is the error rate of the linear SVM on the training data set?

Exercise 4.

Download the two datasets mynormaldistdataset.mat and mybananadataset.mat from undervisningsmateriale/week9.

You can use a library for SVM e.g. svmtrain and svmclassify in Matlab

Familiarize you with the data sets by studying scatterplots.

Load mynormaldistdataset.mat. Stick with the linear SVM, but change the C-parameter ('BoxConstraint' in svmtrain).
Rerun the experiments a couple of times, and visualize the data using 'ShowPlot'. How does the support vectors and the boundary change with the parameter?
Try to remove some of the non-support-vectors and rerun – does the solution change?

Load mybananadataset.mat. Try various values values of the C-parameter with a linear SVM. Can the linear SVM classifier make a good separation of the feature space?

Change kernel to a RBF (radial basis function), and rerun. Try changing the sigma-parameter ('rbf_sigma' in svmtrain). Make sure you know why we now get a non-linear decision boundaries.

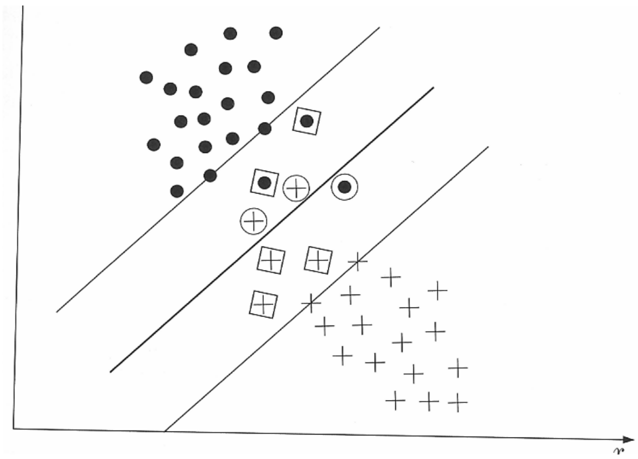
Implement a grid search of the C- and sigma-parameters based on 10-fold crossvalidation of the training data (the A-dataset). Find the best values of C and sigma, retrain on the entire A-data set, and then test on the B-data set. Does the average 10-fold crossvalidation estimate of the overall classification error match the result we get when testing on the independent 'B'-dataset?
Use the parameter range listed in the lecture foils.

Exercise 5: Support vector machine classifiers (from 2017 exam)

a) Consider a linear SVM with decision boundary $g(x) = w^T x + w_0$.
In SVM classification, explain why it is useful to assign class labels -1 and 1 for a binary classification problem.

b) The basic SVM optimization problem is to minimize $J = \frac{1}{2} ||w||^2$
What are the additional constraints for this optimization problem?
Ideally, you should answer both by math and explain what this expression means.

c) Explain how slack variables ξ_i are used to solve a non-separable case like the one below:



d) Discuss how likely a Gaussian classifier and an SVM classifier are to overfit to the training data.

e) Explain how an SVM can be used for a classification problem with M classes.

f) Explain briefly how SVM parameters should be determined

Exercise 6 (from 2018 Exam) : Support vector machines

a) The basic optimization problem for a support vector machine classifier is:

$$\text{minimize} \quad J(w) = \frac{1}{2}\|w\|^2$$

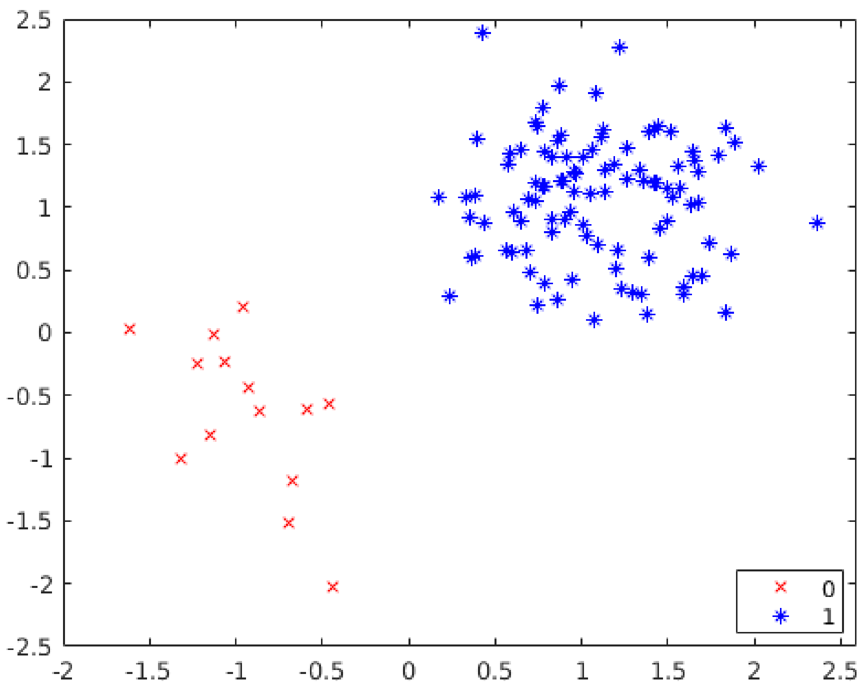
$$\text{subject to} \quad y_i(w^T x_i + w_0) \geq 1, \quad i = 1,2,\ldots N$$

What is the total margin for this problem?

b) Support vector machines are fundamentally different from Gaussian classifiers in terms of how the decision boundary is found – explain why.

c) Support vector machine classifiers can also be explained based on convex hulls. Explain the relationship between the convex hull of two regions and the hyperplane with maximum margin.

d) Given below is a scatter plot of a binary classification problem. Sketch the convex hulls on the figure and use this to find an approximate hyperplane.



e) In the general case the optimization problem is given as:

$$\max_{\lambda} \left(\sum_{i=1}^{N} \lambda_i - \frac{1}{2} \sum_{i,j} \lambda_i \lambda_j y_i y_j x_i^T x_j \right)$$

$$\text{subject to} \quad \sum_{i=1}^{N} \lambda_i y_i = 0 \quad \text{and} \quad 0 \le \lambda_i \le C \quad \forall i$$

Explain briefly which terms in the equation that can be computed using kernels in a high-dimensional space, and also explain what the kernels measure.