INF-GEO 4310 Imaging

Lecture 09.09.2010

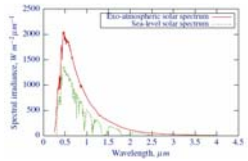
Fritz Albregtsen

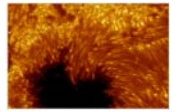
- ***Themes today:***
- Introduction to imaging
- Geometrical optics - Reflection
- Geometrical optics – Refraction

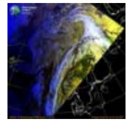
- ***Literature:***
- F. Albregtsen: ”1. Imaging – an introduction” (8 pages)
- F. Albregtsen: ”2. Reflection, refraction, diffraction, and scattering” (the first 36 of 80 pages)

Introduction to imaging

- Contents of ”Introduction”
  - Passive and active imaging
  - Spectral distribution of radiation
    - Planck’s equation
    - Wien’s distribution law
    - spectral emission and spectral irradiance
    - Atmospheric absorption
  - The color of images
    - Color perception
    - Pseudo-color
    - False-color images







Imaging

- Imaging is a process that produces an image of some part of our surroundings.

- Rendering a mathematical function is not considered to be imaging, but visualization.

- The image that is produced may be projected onto a screen for viewing, or it may be captured on film or some digital detector matrix for storage and processing.

Image not created by imaging

Image dimensions

- Image may be a 2-D matrix of intensity values.
- 2-D images at several wavelengths => 3-D image.
- Time-instances of 3-D image => 4-D image.
- 3-D medical image (US, CT, MR, PET) or seismic
  - 3-D image at several frequencies => 4-D image
  - 4-D image at several instances in time => 5-D image.

Images or coefficients?

- Stored data is not necessarily an image.
- May be a matrix of transform coefficients
  - may later be transformed into an image.
- Imaging may occur
  - at wavelengths
  - using physical principles

  that are not part of our own visual system
  - the eye and the brain,

  giving images unobtainable by the naked eye.

Passive imaging

- Utilizes energy sources external to imaging system – usually naturally available sources.
- Either using image sources present in the scene, or letting source light up objects within the scene.
- Examples:
  - infrared (IR) imaging of
    - heat sources
    - leaks in a construction.
  - Astronomical images of
    - Visual, gamma, X-ray, UV, IR, MW and radio sources.

High resolution passive image

Passive imaging of reflected radiation

- We image radiation reflected at a given wavelength, or absorbed and reemitted at a different wavelength.
- Different objects or different parts of the same object may have different absorption / reflection properties.
  - Highly absorbing objects look dark
  - spectral absorption determine color of object.
- Reflection properties determine whether objects are
  - Mirror-like (specular reflection)
  - matte surfaces (diffuse reflection).
- Orientation, shape and fine structure of surface, influence how the object is imaged.

Qualities of energy source

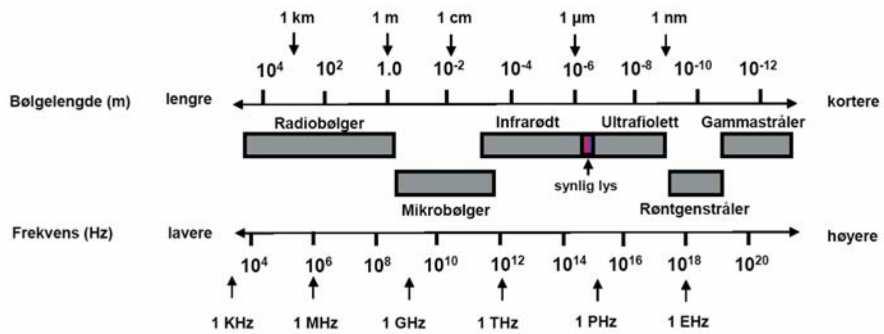
- Passive imaging depends on energy source.
- The most frequently used source is sunlight.
- Amount and color of light depends on
  - atmospheric conditions
  - elevation
  - local topography
  - time of day
  - time of year
  - position on Earth

Active imaging

- We have to provide the energy, and then image radiation
  - reflected from the object
  - absorbed and re-emitted at other wavelengths
  - passing through the object
- Pros:
  - not hampered by unpredictable variations in natural light source
  - able to obtain images regardless of time of day or time of year.
- Examples:
  - radar, sonar and seismic
  - medical applications – microscopy (EM, laser, confocal, flurecent)
  - medical applications - US, X-ray, CT, MR, PET

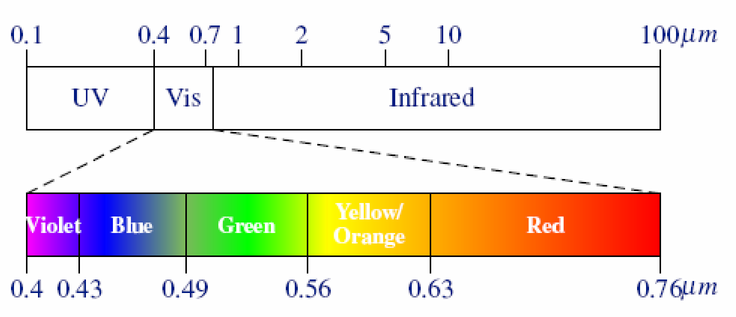
Wavelength and frequency

The electromagnetic spectrum:



The visual part of the spectrum

Human eye responds to radiation
in 0.4 – 0.7 μm range



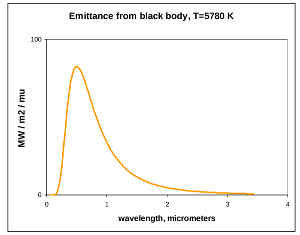
Planck's equation

- Energy emitted versus wavelength depends on temperature of source.
- Emitted energy varies according to Planck's equation

$$M(\lambda, T) = \frac{2hc^2}{\lambda^5} \cdot \frac{\pi}{e^{\frac{hc}{\lambda kT}} - 1}$$

where
- λ is the wavelength (m)
- T is the temperature (K)
- h is Planck's constant = $6.6260693 \bullet 10^{-34}$ Js,
- c is the speed of light = $2.99792458 \bullet 10^{8}$ m/s,
- k is Boltzmann's constant = $1.38065 \bullet 10^{-23}$ J/K
- M is the spectral emittance, given in W per unit area (m^2) per wavelength (m).

- The Sun behaves like a "black body" radiator with T ≈ 5780 K, peaking in visual part of spectrum.



Wien's displacement law

- Gives wavelength of maximum emission by a "black body"

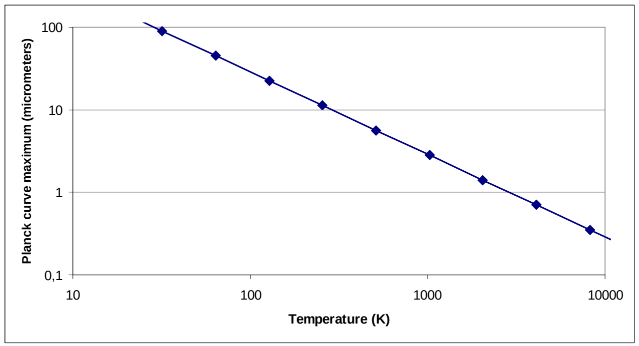
$$\lambda_{max} = \frac{2897}{T}$$

λ_{max} is wavelength of maximum emittance, given in μm.

- The Sun's average surface temperature T ≈ 5780 K implies λ_{max} = 2897/5780 ≈ 0.5 μm = 500 nm.

- Average temperature of the Earth's surface is about 287 K, emission spectrum peaks around 10 μm.

Log-log plot of Wien's law

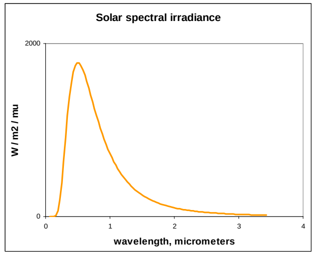


Exo-atmospheric irradiance

- Earth receives only a fraction of emitted solar radiation.
  - Energy from surface of Sun is spread out over sphere
  - radius equal to the distance of the Earth from the Sun.
  - Exo-atmospheric solar spectral irradiance, is given by

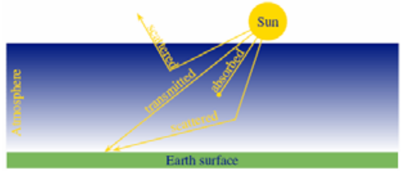
$$\boxed{E_0(\lambda,T) = \frac{2hc^2}{\lambda^5} \cdot \frac{\pi}{e^{\frac{hc}{\lambda kT}} - 1} \left(\frac{r}{d}\right)^2}$$

  - where
  - r is the radius of the Sun
    - r = 6.36 •10⁸ m
  - d is the mean distance Earth – Sun
    - d = 1.5 •10¹¹ m.
  - Irradiance curve is a scaled Planck curve

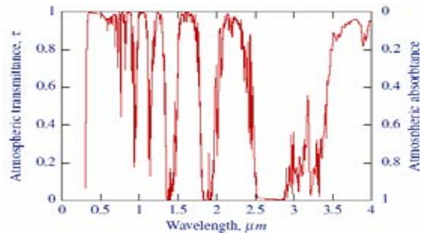


Atmospheric transmittance

- Only a part of the total exo-atmospheric solar irradiance reaches the Earth's surface.
  - Some is absorbed by gases and particles in the atmosphere
  - Some is scattered back out to space by aerosols and clouds



- Transmittance of the standard atmosphere when light travels vertically down through the atmosphere to sea level.
- Note :
  - strong UV absorption
  - IR absorption bands



Path length through atmosphere

- Direct solar radiation incident on a horizontal surface

$$I_{direct} = E_0 \tau^m \cos(\psi)$$

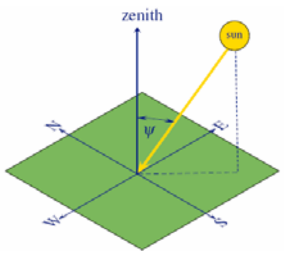
  - where τ is the atmospheric transmittance, *m* is the air mass and *ψ* is the solar zenith angle
- The air mass at sea level, *m*, is given by

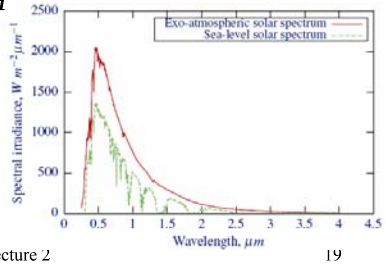
$$m = \cos^{-1}(\psi)$$

- The solar zenith angle, *ψ*, is given by

$$\cos\psi = \sin\phi \sin\delta + \cos\phi \cos\delta \cos H$$

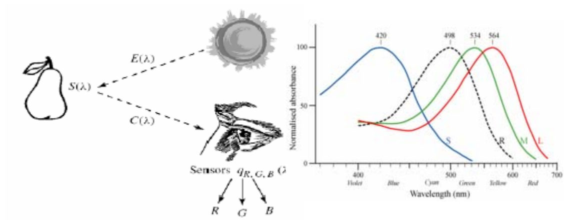
  - where *φ* is the geographical latitude of the site
  - *δ* is the solar declination angle
    δ = -23.4° cos(360(*D* +10)/365))
    where *D* is the day of year.
  - *H* is the hour angle of the Sun,
    given by H ≈ 15(12-h),
    where h is the local solar time in hours.





Color of images

- Light from the Sun, having a spectral distribution E(λ) falls on object.
- Surface of object has a spectral reflection function S(λ).
- Light entering the eye is detected by the three cone types,
  - each having a spectral sensitivity function, q(λ).
- CIE-defined RGB primaries:
  - Blue=436 nm, Green=546 nm, Red=700.0 nm,
  - and standard light sensitivity curves for the three color components.
- Three analog signals expressing a three-channel image by integrals:



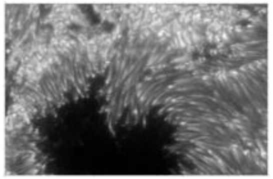
$$R = \int E(\lambda)\ S(\lambda)\, q_R(\lambda)\, d\lambda$$

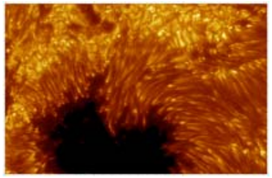
$$G = \int E(\lambda)\ S(\lambda)\ q_G\ (\lambda)\, d\lambda$$

$$B = \int E(\lambda)\ S(\lambda)\ q_B\ (\lambda)\, d\lambda$$

Pseudo-color images

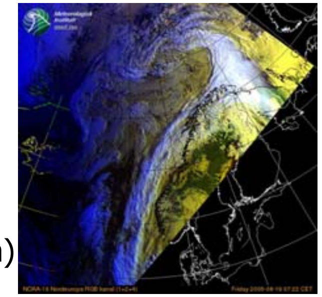
- Are single-channel (graylevel) images where a color has been assigned to each graylevel.
- Example: Sunspot image.
  - Monochromatic filter gives graylevel image.
  - Lookup-table maps each graylevel to an RGB-value.
- Used extensively in many imaging modalities!



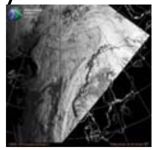


False-color images

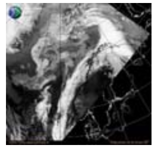
- We have graylevel images from three wavelengths.
- Assign each of them to the primary colors in (RGB), even though wavelengths do not correspond to (RGB).
- The result is an image of the observed scene using colors that are different from those actually observed.
- Ex.: NOAA AVHRR
- Three images:
  - 580-680 nm
  - 725 – 1 000 nm
  - 1 030 – 1 130 nm
- displayed as RGB (435, 546, 700 nm)



Channel 1+2+4 as RGB



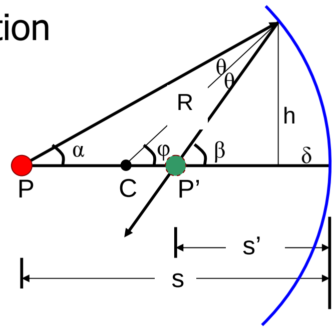
Channel 2



Channel 4

Geometrical optics: Reflection

- Reflectance
- Diffuse and specular reflection
- Imaging by planar mirrors and retro-reflectors
- Sign rules for image formation
- Imaging by curved mirrors
  - Spherical mirrors
  - Optical aberrations
- Optical and non-optical telescopes



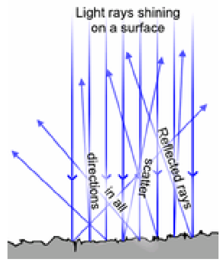
Reflection

- Reflection occurs when a wave hits the interface between two dissimilar media,
  - at least part of the wave returns into the medium of origin.
- Common examples:
  - reflection of rays of light
  - reflection of surface waves
    - Surface waves in a pool of water
    - Sound waves reflected as echo from a wall
- Reflection may be
  - Specular: occurs on a blank mirroring surface that retains the geometry of the beams of light.
  - Diffuse: occurs on a rougher surface, not retaining the imaging geometry, only the energy.

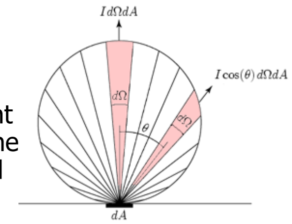
Reflectance

- The ratio of reflected power to incident power, generally expressed in dB or %.
- Reflectance varies with the angle of incidence.
- Reflectance varies with wavelength.
- Surface reflectance is often divided into
  - diffuse reflectance
  - specular reflectance
- In climatology, reflectance is called albedo.
- Also important in computer graphics.

Diffuse reflection

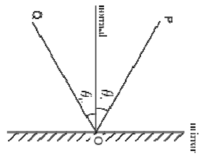


- A rough surface does not form an image.
- **Lambert's cosine law** of diffuse reflection:
  - The total radiant power observed from a "Lambertian" surface is directly proportional to the cosine of the angle θ between the observer's line of sight and the surface normal.
- When an area element is viewed from any angle, it has the same radiance.
- Although the emitted power from an area element is reduced by the cosine of the emission angle, the observed size of the area element is also reduced by that same amount.



Specular reflection

- In specular reflection, the angle of incidence θ_i equals the angle of reflection θ_r.

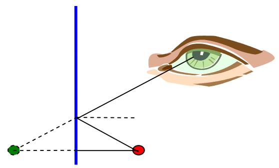


- Reflection occurs when light travels from a medium of a given refractive index into a medium with a different refractive index.
  - A fraction is reflected from the interface
  - The remainder is refracted into the transparent medium.
- Common examples of specular light reflection:
  - Mirror (a glass sheet in front of a metallic coating)
  - Surface of transparent media, such as water or glass.

"Perfect" mirrors

- A **perfect mirror** reflects light (and electromagnetic radiation in general) perfectly, and doesn't transmit it.
- Domestic mirrors are not perfect mirrors.
- Dielectric mirrors are substrates on which one or more layers of dielectric material are deposited.
- A very complex dielectric mirror can reflect up to 99.999%, for a narrow range of wavelengths and angles.
- A simpler mirror may reflect 99.9% of the light, but may cover a broader range of wavelengths.

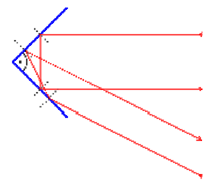
Image formation by plane mirror



- A plane mirror will form a virtual image located exactly opposite the real image, as far behind the mirror as the object is from the front of the mirror.
- A "full size" wardrobe mirror should be half your height, no matter where on your body your eyes are placed, and no matter how far away from the mirror you are standing.
- Image of extended object is exactly the same size as the object, but perceived to be at a negative image distance behind mirror.
- Object size = image size => lateral magnification = 1.

Retro reflectors

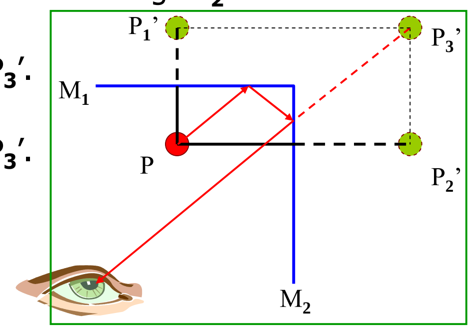
- Retro reflectors exist in both specular and diffused modes.
- A corner reflector consists of three mutually perpendicular planes.
- Image produced is inverse of one produced by a single mirror.



- Some applications:
  - simple radar reflectors for ships
  - Laser Range Finders
  - *Retroreflective or partially retroreflective surfaces*
    - Made by depositing a layer of tiny refractive spheres or by creating small pyramid like structures (cube corner reflection).
    - Applications:
      - traffic signs
      - automobile license plates
    - Here, a completely perfect retro reflection is not desired!

Imaging by retro reflectors

- An image formed by one surface can serve as the object for a second imaging surface.
  - Mirror M_1 forms a virtual image P_1' of an object point P.
  - Mirror M_2 forms another virtual image P_2'.
  - P_1' is an object for M_2, forming virtual image P_3'.
  - P_2' is an object for M_1, forming virtual image P_3'.
- This principle will be used extensively.

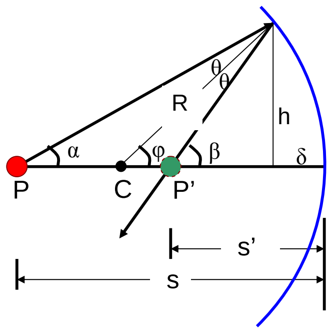


Sign rules for image formation

- Some general sign rules that will be applicable to all imaging situations that we will encounter later, when both real and virtual images are formed in front of or behind curved surfaces:
- **The object distance**: When the object is on the same side of the reflecting or refracting surface as the incoming light, the object distance s is positive; otherwise, it is negative.
- **The image distance**: When the image is on the same side of the reflecting or refracting surface as the outgoing light, the image distance s' is positive; otherwise, it is negative.
- **Radius of curvature**: When the centre of curvature C is on the same side as the outgoing light, the radius of curvature is positive; otherwise, it is negative.

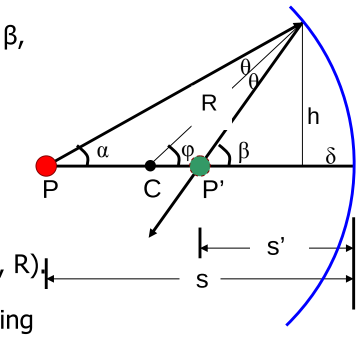
Reflection by spherical mirror - I

- The concave side of a spherical mirror with radius of curvature R is facing the incident light originating from an object point P on the optical axis.
- A ray from P at an angle α to the axis is reflected by the mirror.
- The angle of incidence and reflection are both θ, and the reflected ray crosses the optical axis at an angle β.
- All rays from P will intersect the axis at the same point P', provided that the angle α is small.



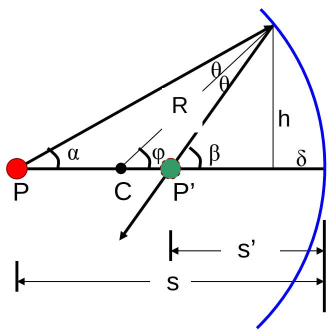
Reflection by spherical mirror - II

- We have the following relations:
  $\varphi = \alpha + \theta$ and $\beta = \varphi + \theta$,
  which implies that $\alpha + \beta = 2\varphi$.
- The expressions for the tangents of α, β, and φ are simply
  $tg(\alpha) = h/(s-\delta)$, $tg(\beta) = h/(s'-\delta)$,
  $tg(\varphi) = h/(R-\delta)$.
- If the angle α is small, so are β and φ.
- Under ***the paraxial approximation***, δ may be neglected compared to (s, s', R).
- So for small angles we have the following approximations:
  $\alpha = h/s$, $\beta = h/s'$, $\varphi = h/R$.



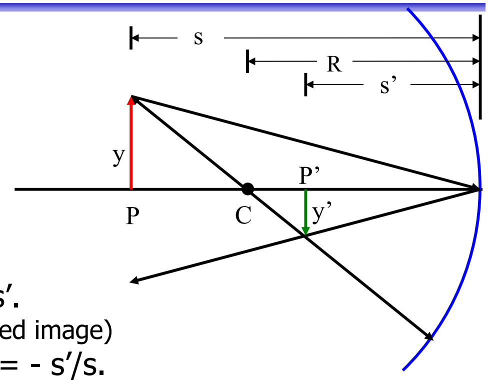
Reflection by spherical mirror - III

- Substituting this into **$\alpha + \beta = 2\varphi$** we get a general object-image relation for a spherical mirror: $\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}}$
- If the radius becomes infinite ($R = \infty$), the mirror becomes plane, and the relation above reduces to $s = -s'$ for a plane reflecting surface.
- If object is far from mirror ($s \approx \infty$), the incoming rays are parallel, and the image will be formed at a distance $s' = R/2$ from the mirror.
  - This is the focal length, f.



Imaging by spherical mirrors - I

- Given
  (y, s, R)
- What is
  - the size y'
  - position s'
  - of the real image ?



- We see that $y/s = -y'/s'$.
  - (negative sign => inverted image)
- Magnification: $m = y'/y = -s'/s$.
- Size of image: $|y'| = ys'/s$.
- Substituting s' from "object-image-relation":
  $$|y'| = yf/(s-f)$$

Imaging by spherical mirrors - II

- When the object is far from the mirror, the image of the object is smaller than the object, inverted, and real.
- If the object is infinitely far away, the image is formed in the focal plane.
- As the object is moved closer to the mirror, the image moves farther from the mirror and increases in size.
- When the object is in the focal plane, the image is at infinity.
- If the object is inside the focal point, the image becomes larger than the object, erect, and virtual.

Optical aberrations, curved mirrors

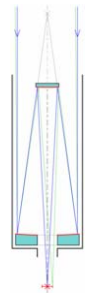
- ***Spherical aberration***
  - Light striking nearer the periphery focuses closer to the mirror, while light striking near the center focuses farther away. Eliminated by paraboloid mirrors.
- ***Coma***
  - Light entering at an angle to the optical axis, are distorted into tiny "tear drops".
- ***Astigmatism***
  - Rays of light in different planes do not focus at the same distance from the mirror.
- ***Curvature of field***
  - Different parts of a sharp image are formed at different distances from the mirror.
- ***Distortion***
  - Straight lines in the object plane are imaged as curved lines.
- ***Vignetting***
  - A darkening of the image towards the corners of the field of view.

Astronomical telescopes

- Three main types of optical astronomical telescopes:
  - refracting (dioptric) telescopes use an arrangement of lenses.
  - reflecting (catoptric) telescopes use an arrangement of mirrors.
  - catadioptric telescopes use a combination of mirrors and lenses.
- Telescopes increase the apparent angular size of objects, as well as their apparent brightness.
- An eyepiece may be used to view the image.
- In a prime focus design, the image detector sits at the focal point.
- Radio telescopes also often have a prime focus design.

Optical mirror telescopes - designs

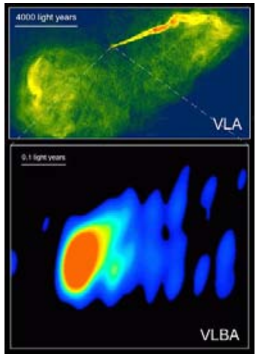
- **Newtonian:** A paraboloid (spherical) primary, and a flat diagonal secondary reflects the light to a focal plane at the side of the top of the telescope tube.
- **Cassegrain:** A parabolic primary, a convex hyperbolic secondary that reflects the light back through a hole in the primary.
- **Ritchey-Chrétien:** two hyperbolic mirrors (instead of a parabolic primary).
- **Dall-Kirkham:** A concave elliptical primary mirror and a convex spherical secondary.
- **Schiefspiegler:** Tilted mirrors to avoid the secondary casting a shadow on the primary.
- **Maksutov:** A full aperture corrector lens, a spherical primary, and a spherical secondary as an integral part of the corrector lens.
- **Gregorian:** A concave secondary mirror and in this way achieves an upright image.
- **Schmidt:** A spherical primary mirror, and an aspherical correcting lens.
- **Nasmyth:** Similar to Cassegrain, a third mirror reflects the light to the side and out of the tube.
- **Coudé:** Adding further optics to a Nasmyth design deliver the light to a fixed focus point.



Radio telescopes

- Single dish

- Interferometry



Non-optical telescopes

- Single-dish radio telescopes are often made of a conductive wire mesh whose openings are smaller than the reflected wavelength. The shape is often parabolic, and the detector is either placed in prime focus, or a secondary reflector is used.
- Multi-element radio telescopes are constructed from pairs or groups of antennae to synthesize apertures that are similar in size to the separation between the telescopes.
- Large baselines are achieved by utilizing space-based Very Long Baseline Interferometry (VLBI) telescopes such as the Japanese HALCA (Highly Advanced Laboratory for Communications and Astronomy) VSOP (VLBI Space Observatory Program) satellite.
- The VLBI technique is also using radio telescopes on different continents simultaneously.

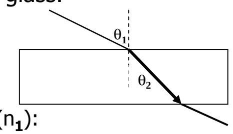
Geometrical Optics: Refraction

- Snell's law
- The refractive index
  - Sellmeier's equation,
  - Fresnel's equation
  - Critical angle, total internal reflection
- Non-imaging refraction (briefly)
  - Plane-parallel slab
  - Prism
  - Circular dics
    - Two refractions and one reflection – primary rainbow
    - Two refractions and two reflections – secondary rainbow
  - Atmospheric refraction

Snell's law

- Light travels at different velocities in different media.
- This is described by the refraction index of glass, equivalent to ratio of the speed of light in air and glass.
- The result is that the light rays are bent at media interfaces.
- A ray of light passes from air into a plane parallel slab of glass.
- Then Snell's law gives the relation between
  - angle of incidence (θ_1),
  - angle of refraction (θ_2),
  - refraction index of glass (n_2) and the surrounding medium (n_1):

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 .$$



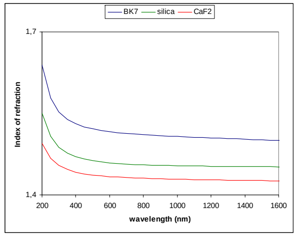
- If the index of refraction is given relative to the surrounding medium

$$\mathbf{\sin \alpha / \sin \beta = n}$$

where α is the angle of incidence and β is the angle of refraction.

Refractive index

- The refractive index depends on λ and T.
- Given for 589 nm at 25° C.
  - (CaII D-line)



- Also weaker dependencies on
  - Pressure
  - Stress
  - Material compositions (dopants, impurities)

- These variations are usually < 1%.

The Sellmeier equation

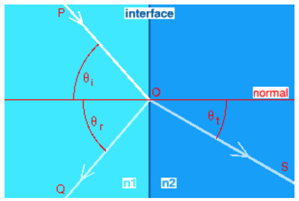
- An empirical relationship between
  the refractive index n
  and the wavelength λ
  for a particular transparent medium.
- The usual form of the Sellmeier equation for glasses is:

$$n(\lambda) = \left[1 + \sum_i \frac{B_i \lambda^2}{\lambda^2 - C_i}\right]^2$$

- Coefficients are usually quoted for λ in micrometers.
- λ is the vacuum wavelength;
  not that in the material itself, which is λ/n(λ).

Reflection and refraction

- When light moves from a medium with refractive index n_1 into a second medium with refractive index n_2, both reflection and refraction of the light may occur.



- The reflection angle is equal to the incidence angle ($\theta_r = \theta_i$).

- The angle of refraction, θ_t, is given by Snell's law.

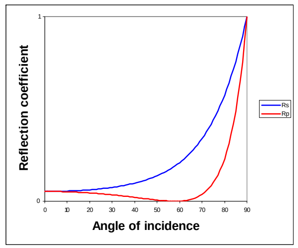
Fresnel's equation – non-magnetic materials

- May be used to calculate the fraction R of the intensity of incident light that is reflected from the interface.
- If the light is s-polarized:

$$R_S = \left[\frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}\right]^2 = \left[\frac{n_1 \cos(\theta_i) - n_2 \cos(\theta_t)}{n_1 \cos(\theta_i) + n_2 \cos(\theta_t)}\right]^2$$

- If the incident light is p-polarized:

$$R_P = \left[\frac{tg(\theta_t - \theta_i)}{tg(\theta_t + \theta_i)}\right]^2$$



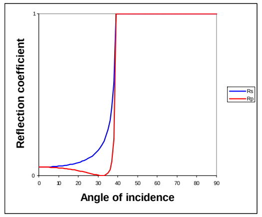
- The transmission coefficients are:
  - $T_s = 1 - R_s$ and $T_p = 1 - R_p$.
- Unpolarized
  - $R = (R_s + R_p)/2$.
- At one particular angle, a p-polarized incident ray is purely refracted.
  - Brewster's angle, ≈ 56° for a glass medium in air or vacuum.

Critical angle, total internal reflection

- When $n_1 > n_2$, all light is reflected above an incidence angle known as the *critical angle*, and $R_s = R_p = 1$.
- This phenomenon is known as *total internal reflection*.
- The critical angle is approximately 41° for glass in air.
- *Important application: Reflective optical prisms*
- When $\theta_i \approx \theta_t \approx 0$, we have:

$$R = R_S = R_P = \left[\frac{n_1 - n_2}{n_1 + n_2}\right]^2$$

$$T = T_S = T_P = 1 - R = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

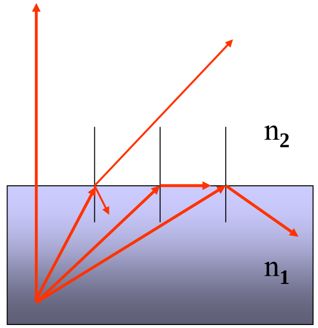


Reflection coefficients from glass to air.

Grazing refraction

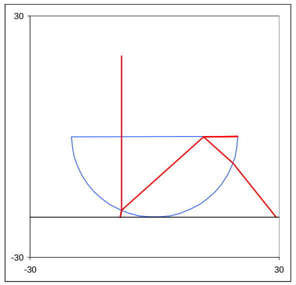
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

- When light travels from a high to a low refractive index medium, the ray is bent away from the normal.
- At the critical angle of incidence, given by
  $\theta_{crit} = \arcsin(n_2/n_1)$,
  the refracted ray will be grazing the surface.
- If $\theta < \theta_{crit}$, the ray will split.
  Part of the ray will reflect off the boundary, and some will refract as it passes through.
- If $\theta > \theta_{crit}$, all of the ray reflects from the boundary. None passes through.



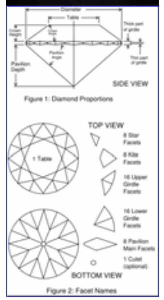
A water bowl example

- Given a semi-circular bowl of water.
- A light-ray from a 632.8 nm laser enters perpendicular to the surface 4/10 of the radius from the centre of the bowl.
- We want to obtain grazing refraction and total internal reflection.
- At 25°C, how much do you have to raise the refractive index of the water by increasing the salinity?
  - » (See http://www.luxpop.com/)



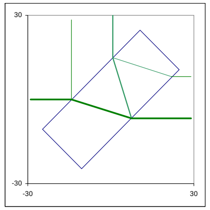
Optical fibers and diamonds

- In large core diameter optical fibers, confinement is based on total internal reflection.
- Diamond has an extremely high refractive index
  - low critical angle - about 24.4°
  - light is much more likely to be internally reflected within a diamond than in glass
    - critical angle about 41.5°.
- The "brilliant cut" is designed to achieve high total reflection of light entering the diamond, and high dispersion of the reflected light.



Refraction in plane-parallel slab

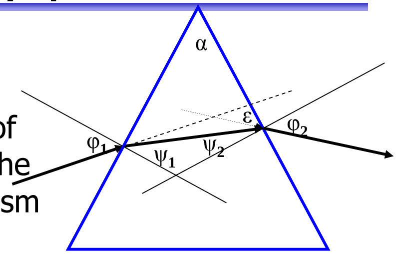
- When the beam exits the glass slab, the angle of incidence equals β, the relative index of refraction is 1/n, and the exit angle φ is given by
  n sin β= sin φ.
- The net effect is that the plane parallel glass slab has caused a parallel displacement of the light beam.
- The displacement, d, given relative to the thickness of the slab, is



$$d = \sin\alpha \left(1 - \frac{\cos\alpha}{\sqrt{n^2 - \sin^2\alpha}}\right)$$

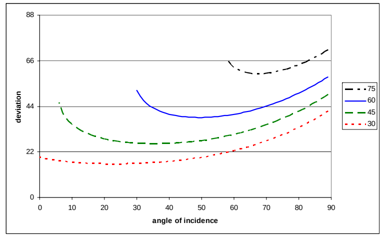
Refraction by prism

- Let α be the top angle of a symmetric triangular prism.
- A minimum value of the angle of deviation, ε, is obtained when the passage of light through the prism is *symmetrical*, and is given by



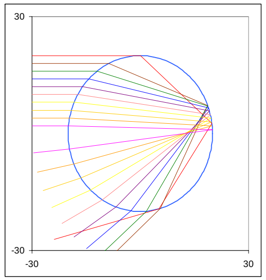
$$n = \frac{\sin\phi_1}{\sin\psi_1} = \frac{\sin\left[\frac{1}{2}(\varepsilon_{\min} + \alpha)\right]}{\sin\left(\frac{1}{2}\alpha\right)}$$

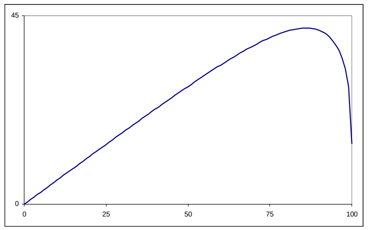
- May be used to determine n when α is known and ε is measured.



Two refractions and one reflection by a circular disc

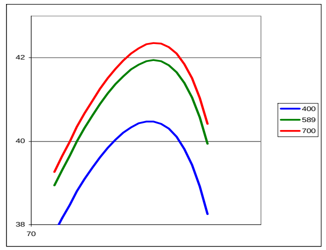
- Parallel rays of light enter a circular disc, and are refracted.
- A small fraction (4%) of the refracted light is reflected from the back of the disc.
- Refracted again upon exiting the disc.
- Exit rays "pile up" at a maximum exit angle.
- For the refraction index of water, $\varphi_M \approx 42°$.





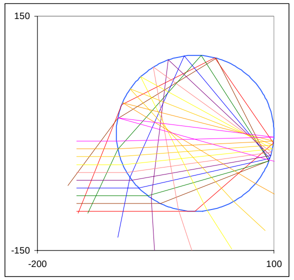
Primary rainbow

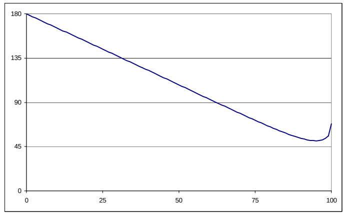
- The "piling up" of exit angles implies a bright disc having angular radius φ_M centered on the point in the sky opposite the light source.
- Disc is brightest around its rim.
- No light reaches us from $\varphi > \varphi_M$
- φ_M depends on wavelength.
- Visual color spectrum from violet to red is spread out over 40.8° – 42.5°.
- Distance does not matter
  - the rainbow is a collection of rays
  - not located at any point in space.



Two refractions and two reflections by a circular disc

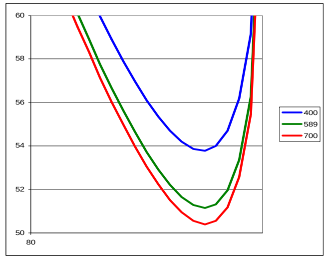
- Parallel rays of light enter a circular disc, and are refracted.
- A small fraction (0.16%) of the refracted light is reflected twice inside the disc.
- Refracted again upon exiting the disc.
- Exit rays "pile up" at a *minimum exit angle*.
- For the refraction index of water, $\varphi_{M2} \approx 52°$.





Secondary rainbow

- The "piling up" of exit angles gives a darker "hole" centered on the point in the sky opposite the light source.
- The angular radius of this "hole" is φ_{M2}, or about 52°.
- φ_{M2} depends on wavelength.
- The order of the colors is reversed.
- Secondary bow wider than primary.
- Bright "disc" of primary bow, and dark "hole" of secondary, results in dark "Alexander-band" between bows.



Atmospheric refraction

- Variation in air density as a function of altitude causes light or other electromagnetic waves to deviate from a straight line during a passage through the atmosphere.
- The atmospheric refraction causes astronomical objects to appear higher in the sky:
  - zero in the zenith
  - less than 1' at 45° altitude
  - only 5' at 10° altitude
  - 29' half a degree (one solar diameter) above horizon
  - 34' at the horizon.
- Setting / rising sun is *flattened* by about 1/6 of apparent diameter. Thermal inversion layers may create additional phenomena.
- Values above are for 10 °C and 1003 mbar.
  - Add 1% to the refraction for every 3° C colder, subtract if hotter.
  - Add 1% for every 9 mbar higher pressure, subtract if lower.
- Atmospheric refraction near the ground can produce *mirages*.
- Light passing through layers and bubbles of air having different densities, will continuously alter direction, causes stars to appear as *twinkling*.

Next: Refraction, Diffraction, Scattering

- Imaging by refraction
  - Thin lenses; "The lensmakers equation"
  - The camera; "Depth-of-field"
  - The eye
  - The magnifier
  - The eyepiece
  - Microscopes
  - Telescopes
- Fraunhofer diffraction pattern
  - Single slit, twin slit, multiple slits
  - Diffraction grating, spectrograph, spectroheliograph, slitless spectrograph
- Diffraction profile of circular aperture
  - Airy disc and Rayleigh criterion
- The smallest visible detail
- Depth of focus
- Convolving PSF and sampling aperture
- What is scattering?
- Some effects of scattering
  - Atmospheric blurring and straylight in images
  - Turbidity in liquids
  - Subsurface scattering in non-metallic materials and in tissues
- Doppler-shifted straylight