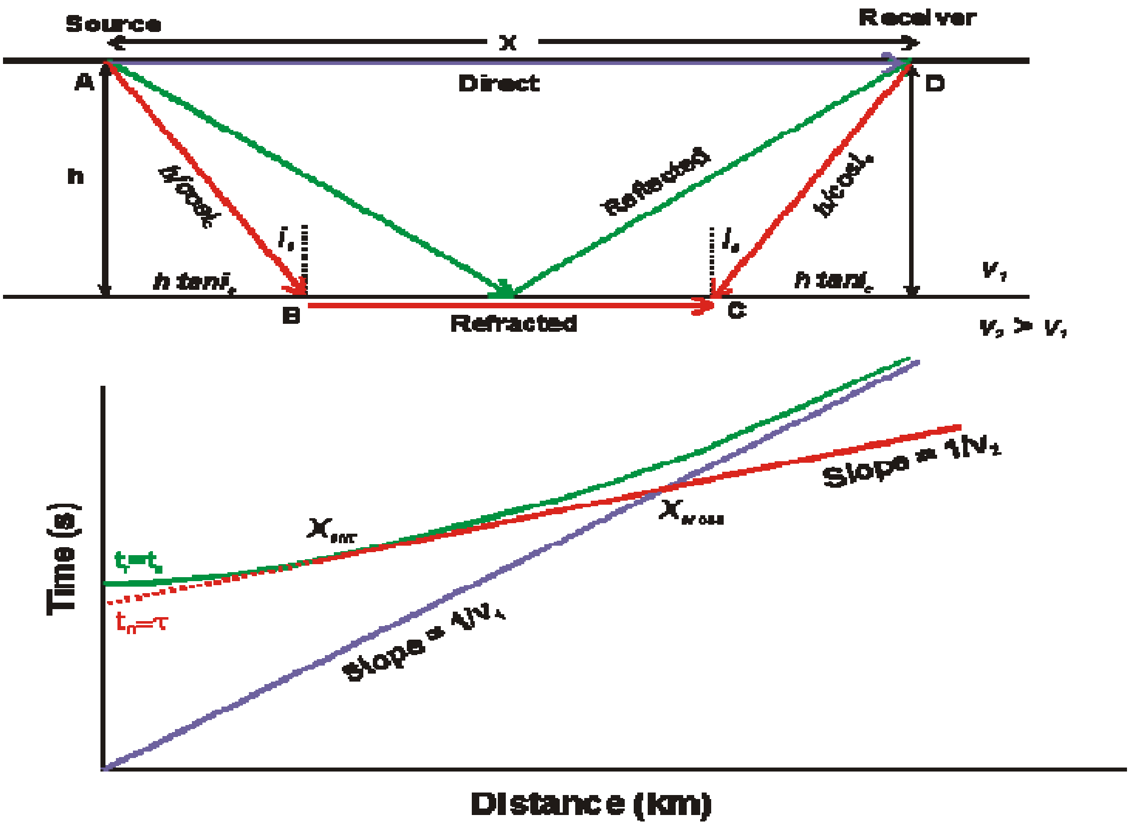
Exercise



We consider a simple two-layer model as shown in the figure above, where the P-wave velocity of layer *1* and *2* is v_1 and v_2 , respectively (note $v_2 > v_1$). In general, a receiver will measure three different wave contributions: direct wave, reflected wave and refracted wave.

(a) Find an expression for the traveltime to an arbitrary receiver for the *direct* wave in the first layer as a function of offset x (i.e. source-receiver distance).

(b) The *refracted* wave can only be received after the critical distance x_{crit}. Find an explicit expression for this distance.

(c) The cross-over distance x_{cross} is the distance where the *refracted* wave starts to take over to be the first arrive at a point. Show that this distance can be written as:

$$x_{cross} = 2h\sqrt{\frac{v_1 + v_2}{v_2 - v_1}}$$

(d) Find an expression for the traveltime to an arbitrary receiver for the *reflected* wave for this 2-layer model.

SOLUTION

(a) The traveltime for the direct wave:

$t_{direct} = x/v_1$

(b) The critical angle is defined by:

$\sin i_c = v_1/v_2$

from the figure it follows

$$\tan i_c = \frac{\sin i_c}{\cos i_c} = \frac{x_{crit}/2}{h} \quad \Rightarrow \quad x_{crit} = 2h\frac{\sin i_c}{\sqrt{1-\sin^2 i_c}} = \frac{2h}{\sqrt{\left(\frac{v_2}{v_1}\right)^2 - 1}}$$

(c) First find traveltime of refracted wave:

$$t_{refract} = \frac{2\sqrt{h^2 + (x_{crit}/2)^2}}{v_1} + \frac{(x - x_{crit})}{v_2} = \frac{2h}{v_1}\frac{\sqrt{v_2^2 - v_1^2}}{v_2} + \frac{x}{v_2}$$

Next, the cross-over distance is defined by :

$$t_{direct} = t_{refract} \quad \Rightarrow \quad \frac{x_{cross}}{v_1} = \frac{2h}{v_1}\frac{\sqrt{v_2^2 - v_1^2}}{v_2} + \frac{x_{cross}}{v_2} \quad \Rightarrow$$

$$x_{cross} = 2h\frac{\sqrt{v_2^2 - v_1^2}}{(v_2 - v_1)} = 2h\sqrt{\frac{(v_2 + v_1)}{(v_2 - v_1)}}$$

(d) Start with ray path and knowledge of Snell's law

$$\left(v_1\frac{t_{refl}}{2}\right)^2 = \left(\frac{x}{2}\right)^2 + h^2 \quad \Rightarrow \quad t_{refl} = \frac{\sqrt{x^2 + 4h^2}}{v_1}$$

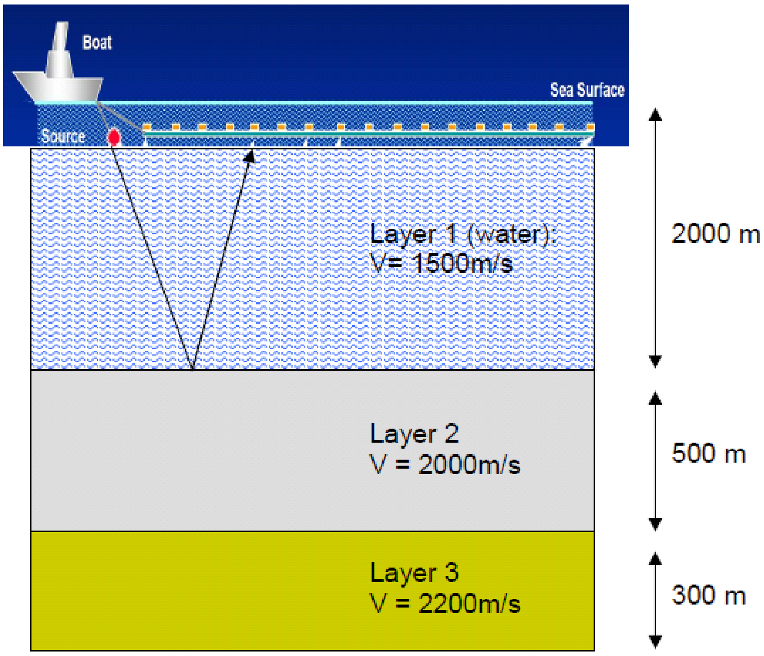
This expression can be reformatted as

$$\frac{v_1^2 t_{refl}^2}{4h^2} - \frac{x^2}{4h^2} = 1$$

which is the equation for a hyperbola curve

Exercise 2

Consider a horizontally 3-layered earth model as shown in the figure and assume that 2D marine seismic data are acquired at the surface employing a streamer with 120 hydrophone groups (group interval Δg=25 m). Distance between shot and first hydrophone group is 100 m.



a) **Compute the zero-offset reflection coefficient at the seafloor if the density of layer 2 is set to 2.2 g/cm^3. Compute also the maximum *normal moveout* (i.e. the difference between maximum traveltime recorded and a zero-offset case) for the reflection events from the seafloor.**

- Zero-offset reflection coefficient:

$$R = \frac{2000 \cdot 2.2 - 1500 \cdot 1}{2000 \cdot 2.2 + 1500 \cdot 1} \cong 0.49$$

- Maximum normal moveout:

$$\Delta t_1 = \sqrt{t_0{}^2 + \left(\frac{x}{V}\right)^2} - t_0, \qquad t_0 = \frac{2Z}{V}$$

$$\Delta t_1 = \sqrt{\left(\frac{2Z_1}{V_1}\right)^2 + \left(\frac{100\mathbf{m} + 25\mathbf{m}\cdot(120-1)}{V_1}\right)^2} - \frac{2Z_1}{V_1}$$

$$\Delta t_1 = \sqrt{\left(\frac{2\cdot 2000\mathbf{m}}{1500\,\mathbf{m/s}}\right)^2 + \left(\frac{3075\mathbf{m}}{1500\,\mathbf{m/s}}\right)^2} - \frac{2\cdot 2000\mathbf{m}}{1500\,\mathbf{m/s}}$$

$$\Delta t_1 \cong \underline{\underline{0.70\mathrm{s}}}.$$

b) Determine the maximum incidence angle for reflections at the interface between layers 2 and 3.

Maximum angle: $\sin\theta = \frac{V_{p2}}{V_{p3}} = \frac{2000}{2200} \Rightarrow \theta = \underline{\underline{65.4°}}.$