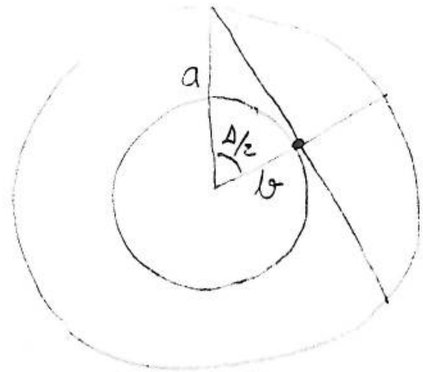
3) Maximum distance for PcP:

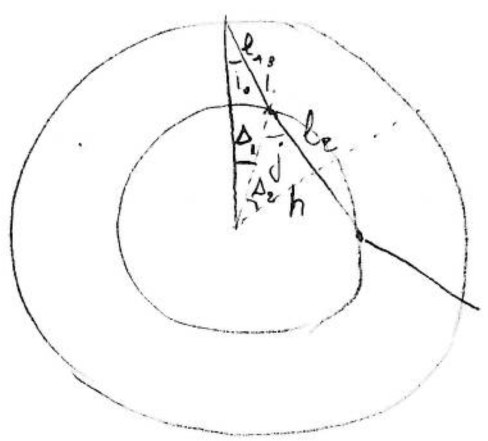


$$a \cos\frac{\Delta}{2} = b$$

$$\hookrightarrow \Delta = 2 \arccos\left(\frac{b}{a}\right)$$

$$= 113.8°$$

4) Distances for PKP and SKS.



- $\frac{a \sin i_0}{v_p} = \frac{b \sin i}{v_p} = \frac{b \sin j}{v_c} = \frac{h}{v_c}$

  with v_s instead of v_p for SKS.
- $i_0 + (\pi - i) + \Delta_1 = \pi$

  $\hookrightarrow \Delta_1 = i - i_0$
- $j + \frac{\pi}{2} + \Delta_2 = \pi \rightarrow \Delta_2 = \frac{\pi}{2} - j$
- $\Delta = 2(\Delta_1 + \Delta_2)$

  $= 2\left(i - i_0 + \frac{\pi}{2} - j\right)$

  $\underline{\Delta = \pi + 2(i - i_0 - j)}$

- $i_0 = 0 \Rightarrow i = 0,\ j = 0 \Rightarrow \underline{\Delta = \pi}$ for PKP and SKS.
- $i_0 = 10, 20, 30$ for PKP and SKS

  $\hookrightarrow i = \arcsin\left(\frac{a}{b} \sin i_0\right) = 18.5°,\ 38.8°,\ 66.3°$

PKP: $j = \arcsin\left(\frac{a}{b} \cdot \frac{v_c}{v_p} \sin i_0\right) = 13.8°,\ 28.0°,\ 43.3°$

SKS: $j = \arcsin\left(\frac{a}{b} \cdot \frac{v_c}{v_s} \sin i_0\right) = 28.5°,\ 70.0°,$ not an angle (no refracted wave)

$$t = 2\left(\frac{l_1}{v_p} + \frac{l_2}{v_c}\right)$$ with v_s instead of v_p for SKS

with $l_1 = \sqrt{a^2 \sin^2\Delta_1 + (a\cos\Delta_1 - b)^2}$

and

$l_2 = b \sin \Delta_2$

$\hookrightarrow \Delta_1 = i - i_0 = 8.5°, 18.5°, 36.3°$

$\Delta_2 = \frac{\pi}{2} - j = 76.2°, 62°, 46.7°$ for PKP

$= 61.5°, 20°, -$ for SKS

$\hookrightarrow$ $\boxed{\Delta = 2(\Delta_1 + \Delta_2) = 169.4°, 161°, 166° \text{ for PKP} \quad = 140°, 77°, \text{ for SKS}}$

$t =$ 444.7 s, 536.9 s, 626.3 s, 710.2 s for PKP

$=$ 889. s, 1074. s, 1253 s for SKS

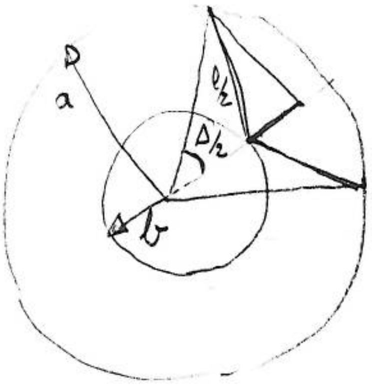
for $i =$ 0 10 20 30°

You don't get any PKP wave but you get SKS waves at the furthest stations.

The arrival time is 12:10:45.1, that is 645.1 after 12:0[illegible]

The origin time using this wave is therefore $\underline{\underline{12:00:00.2}}$

2 – Radius of the core



$$\left(\frac{l}{2}\right)^2 = \left(a \cdot \sin\frac{\Delta}{2}\right)^2 + \left(a \cdot \cos\frac{\Delta}{2} - b\right)^2$$

$$= a^2 \sin^2\frac{\Delta}{2} + a^2\cos^2\frac{\Delta}{2} + b^2 - 2ab\cos\frac{\Delta}{2}$$

$$\left(\frac{l}{2}\right)^2 = a^2 + b^2 - 2ab\cos\frac{\Delta}{2}$$

$$t_{PcP} = \frac{l}{v_P}$$

$$\hookrightarrow\ b^2 - 2ab\cos\frac{\Delta}{2} + a^2 - \left(\frac{v_P \cdot t_{PcP}}{2}\right)^2 = 0$$

$$\Delta' = \left(a\cos\frac{\Delta}{2}\right)^2 - a^2 + \left(\frac{v_P\, t_{PcP}}{2}\right)^2$$

$$b = +a\cos\frac{\Delta}{2} \pm \sqrt{\Delta'}$$

$$= a\cos\frac{\Delta}{2} \pm \sqrt{\left(a\cos\frac{\Delta}{2}\right)^2 - a^2 + \left(\frac{v_P\, t_{PcP}}{2}\right)^2}$$

$$= a\left[\cos\frac{\Delta}{2} \pm \sqrt{\cos^2\frac{\Delta}{2} - 1 + \left(\frac{v_P\, t_{PcP}}{a\ 2}\right)^2}\right]$$

$$\frac{v_P \cdot t_{PcP}}{2a} = \frac{12 \times 696.8}{2 \times 6371} = 0.6562$$

$$\cos\frac{\Delta}{2} = \cos\left(\frac{74.8}{2}\right) = 0.7944$$

$$\hookrightarrow\ b = 6371\left[0.7944 \pm \underbrace{\sqrt{0.7944^2 - 1 + 0.6562^2}}_{0.2483}\right]$$

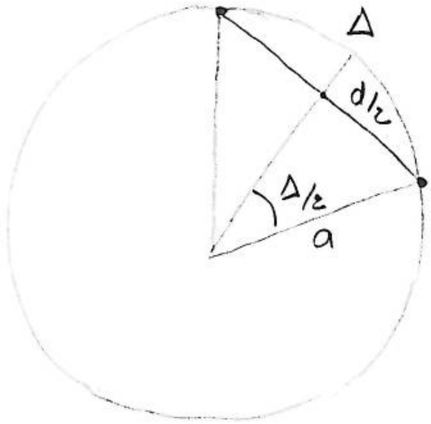
$\underline{b = 3479.\ \text{km}}$

Solution

1) We have first to calculate the difference in time between P and S waves, and then use it to calculate the epicentral distances to the 5 stations.

$\Delta t = 645.1\,s,\ 621.2\,s,\ 486.0\,s,\ 417.6\,s,\ 712.5\,s$

Relation between traveltime and distance.



$$\frac{d}{2} = a \cdot \sin\frac{\Delta}{2} \Rightarrow \Delta = 2 \arcsin\left(\frac{d}{2a}\right)$$

$$t_p = t_0 + \frac{d}{v_p}$$

$$t_s = t_0 + \frac{d}{v_s}$$

$$\hookrightarrow t_s - t_p = d\left(\frac{1}{v_s} - \frac{1}{v_p}\right)$$

$$\hookrightarrow \Delta = 2 \arcsin\left(\frac{t_s - t_p}{\left(\frac{1}{v_s} - \frac{1}{v_p}\right)} \cdot \frac{1}{2a}\right)$$

$$\simeq 2 \arcsin\left(\frac{\Delta t}{1062}\right)$$

$\Delta = 74.8°,\ 71.6°,\ 54.5°,\ 46.3°,\ 84.3°$

Locate the earthquake graphically.

The true location is <u>35°N, 135°E.</u>

Origin time : travel time for P at first station is

$$\frac{d}{v_p} = \frac{2a}{v_p} \sin\left(\frac{\Delta}{2}\right) = \frac{2 \times 6371}{12} \sin\left(\frac{74.8}{2}\right) = 644.9\,s$$