INF2080 Challenge I: Sample Solution

In principle, it would be possible to create a GNFA with one state that accepts the language $\{a^{120k} \mid k \geq 0\}$: a single start/accept state with a loop to itself, labeled with a^{120}. Since, however, the challenge only allows for DFAs, NFAs, or all-NFAs, we are going to need some more states.

We can use the prime factorization of 120 to our advantage when constructing an all-NFA: Since an all-NFA only accepts if *all* computational branches accept, we will use the prime factorization to create various cycles (each containing one accepting state), such that the computations halt at all accepting states iff the number of a's is divisible by 120.

The prime factorization of 120 is as follows:

$$120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 = 8 \cdot 3 \cdot 5$$

Thus, an 8-cycle, a 3-cycle, and a 5-cycle will simultaneously complete their cycles precisely at multiples of 120. Using this, we are able to construct an all-NFA consisting of these three cycles and a starting node, so $8 + 3 + 5 + 1 = 17$ states, as depicted in the figure on the next page.

Note that using three cycles of, say, lengths 4, 6, and 5 is *not* possible. These cycles are simultaneously completed at multiples of 120 as well. However, they also reach their accepting states at multiples of 60. It is vitally important for the different cycle lengths to share no common divisors. For those who are interested, a proof of this is based on an algebraic argument: the cyclic group $\mathbb{Z}_{m \cdot n}$ is isomorphic to $\mathbb{Z}_m \times \mathbb{Z}_n$ iff m and n share no common divisors. In our situation, each cycle corresponds to one factor in the Cartesian product, and we want $\mathbb{Z}_{120}$ to be isomorphic to said Cartesian product. There are multiple options, e.g., $\mathbb{Z}_{24} \times \mathbb{Z}_5$ or $\mathbb{Z}_8 \times \mathbb{Z}_{15}$, but the one with the fewest number of "states" (algebraically, the sum over the orders of each factor in the Cartesian product) is $\mathbb{Z}_8 \times \mathbb{Z}_3 \times \mathbb{Z}_5$.

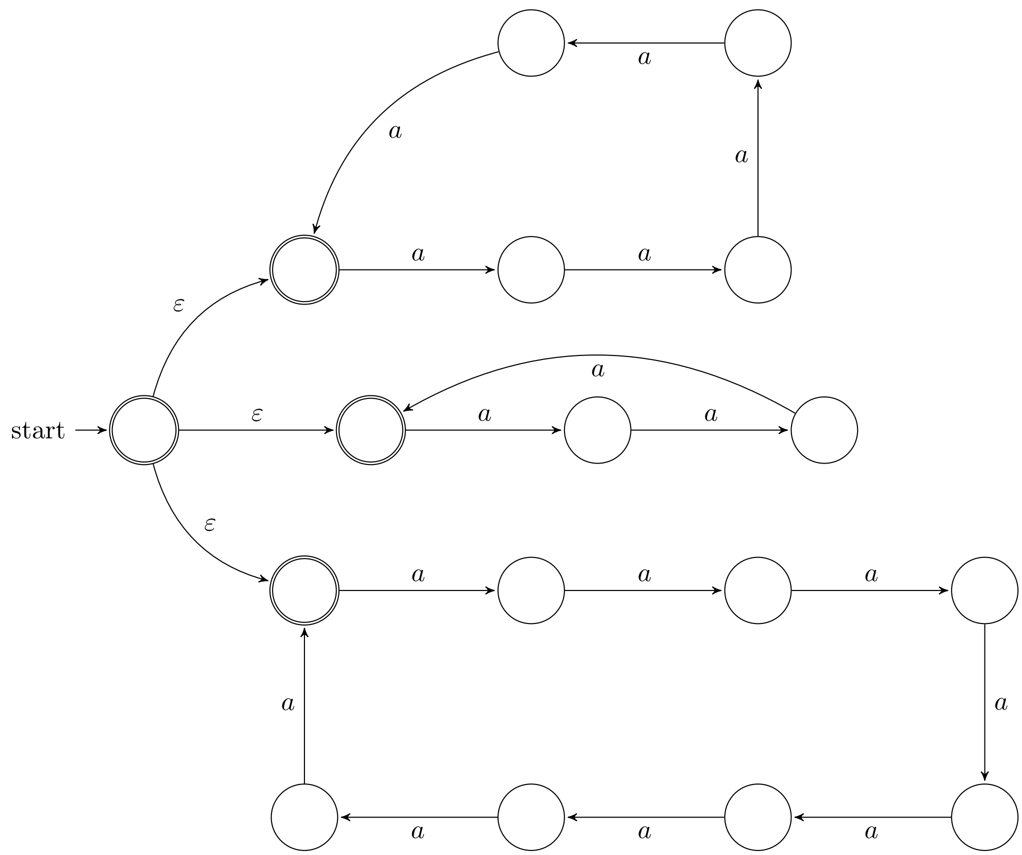

Figure 1: State diagram.