INF2080

2. Regular Expressions and Nonregular languages

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Department of Informatics

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Last week

- Deterministic finite automata (DFA)

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- Regular languages are those languages accepted by DFA's

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- If water reaches any of the coffee filters (accept states), the water can finally drip through!
- you get a cup of coffee $\leftrightarrow$ your input has been accepted!

Regular Expressions

Definition (Regular Expression)

Given an alphabet Σ, a *regular expression* is

- a for some $a \in \Sigma$,
- ε,
- $\emptyset$,
- $(R_1 \cup R_2)$ for regular expressions R_1, R_2,
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$\rightarrow$ Regular expressions represent languages!

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- $(1(0 \cup 1)^*1) \cup (0(0 \cup 1)^*0) \cup 0 \cup 1$

What is the connection between RE and DFA/NFA?

Regular Expressions - Automata

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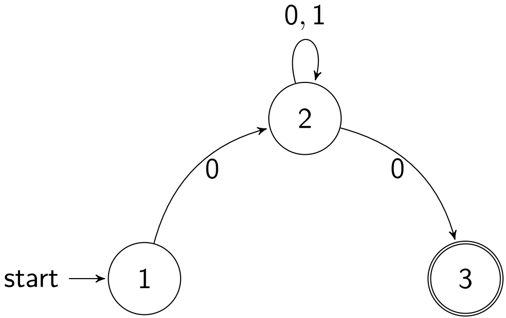

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Yes!

Regular Expressions and Automata

Proposition

Every language described by an RE is regular.

Proof based on inductive definition of RE!

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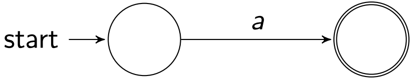

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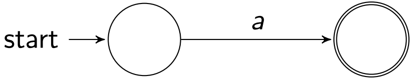

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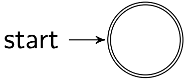

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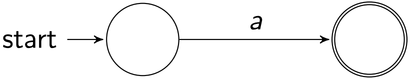

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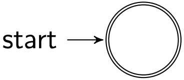

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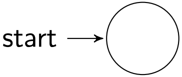

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The rest is union, concatanation and Kleene star of regular languages, as discussed last week!

Regular Expressions and Automata

So we've just proven

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Next:

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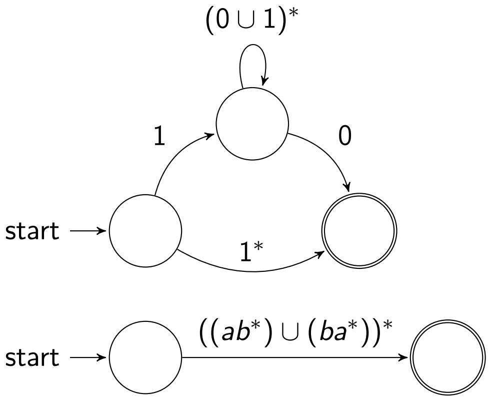

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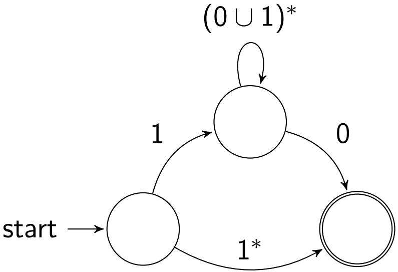

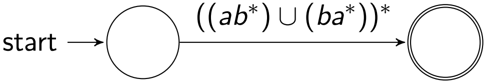

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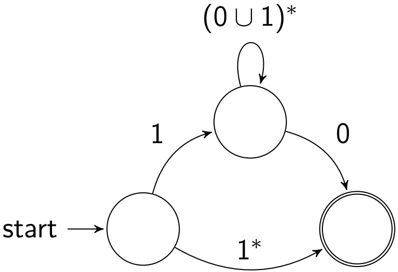

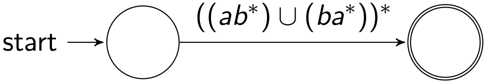

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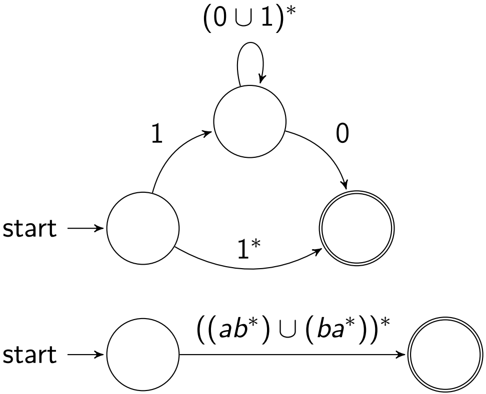

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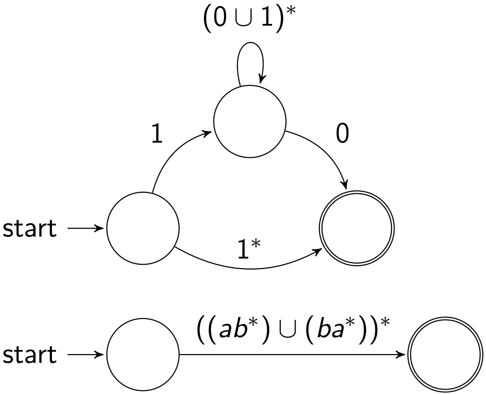

Generalized Nondeterministic Finite Automata

Definition

A generalized nondeterministic finite automaton (GNFA) is a 5-tuple $(Q, \Sigma, \delta, q_{start}, q_{accept})$ where

1. Q is a finite set of states
2. Σ is the input alphabet
3. $\delta : (Q \setminus \{q_{accept}\}) \times (Q \setminus \{q_{start}\}) \to \mathcal{R}$ is the transition function, where $\mathcal{R}$ is the set of all RE's over Σ,
4. q_{start} is the start state, and
5. q_{accept} is the accept state.

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Proof idea: take DFA and transform into a GNFA that accepts the same language. Iteratively remove (non-starting and non-final) states so that the same language is accepted, until only the starting and accepting state remain. Then the RE along the transition between the two states describes the regular language.

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Proof:

- Given a DFA M, we construct an equivalent GNFA G by adding a new start state q_{start} with an ε transition to the old start state q_0, as well as a new accepting state q_{acept}, with ε transitions from all old accept states.
- add $\emptyset$ transitions for all state pairs that do not have a transition in M.

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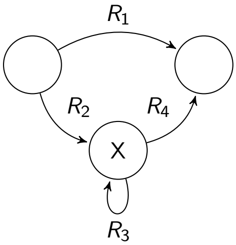

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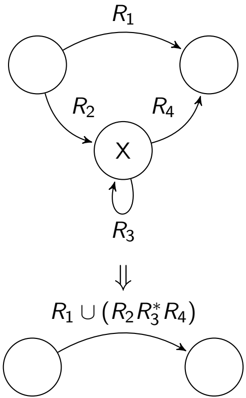

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$$\delta'(q_i, q_j) = R_1 \cup R_2 R_3^* R_4$$

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4. correctness still remains to be shown! See book for details!

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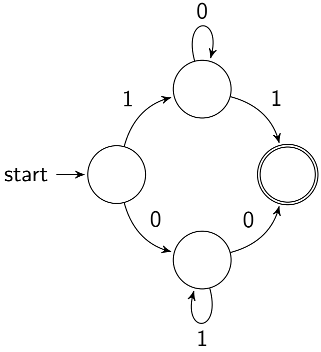

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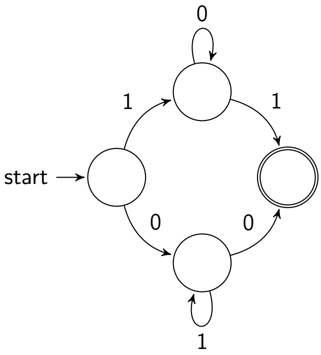

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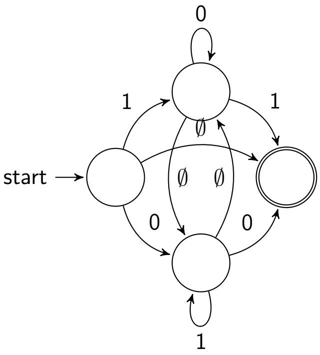

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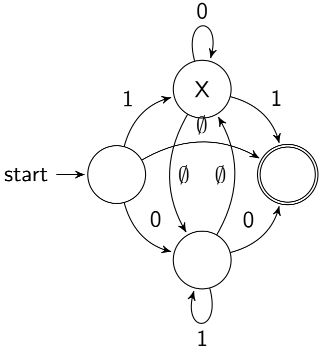

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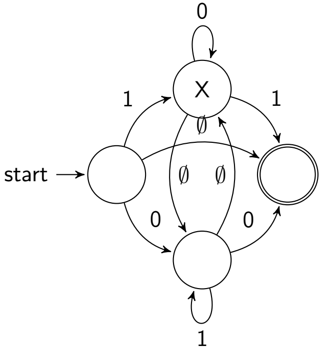

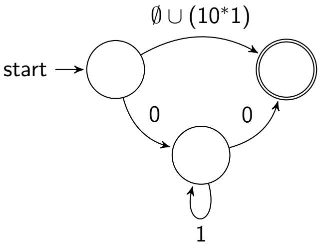

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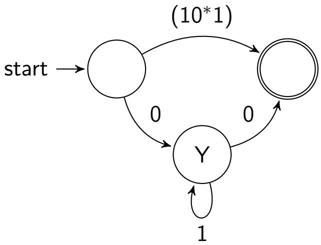

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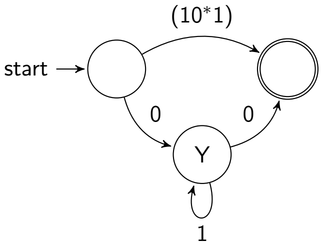

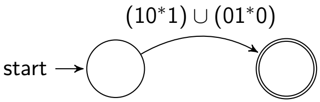

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$\Rightarrow$ Pumping Lemma!

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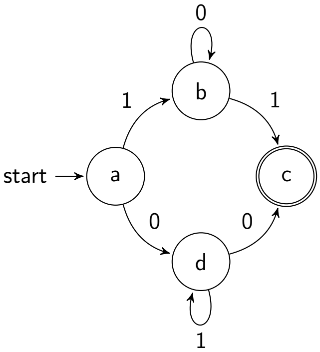
- Pumping lemma gives a *pumping length*: if a string is longer than the pumping length, it can be *pumped*, i.e., there is a substring that can be repeated arbitrarily often such that the string remains in the language
- If a DFA has p states, and a string has length $\geq p$, then the accepting path in the DFA must visit at least $p + 1$ states. In other words, at least one state appears twice. $\Rightarrow$ loop!
- This loop can be repeated while staying in the language.

Pumping Lemma - Example



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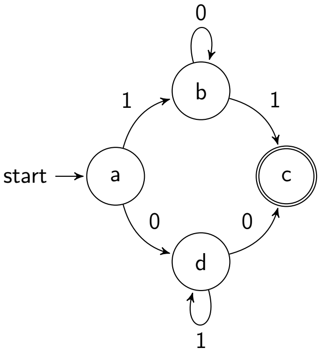

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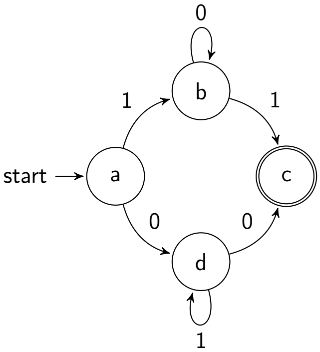

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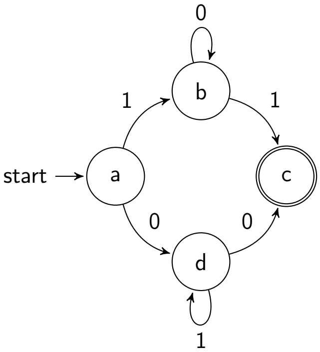

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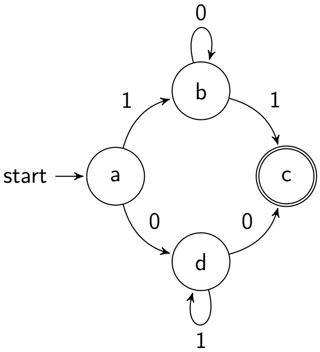

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- $\Rightarrow$ path must contain a loop (in this case, at node b)

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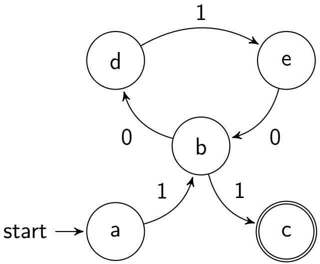

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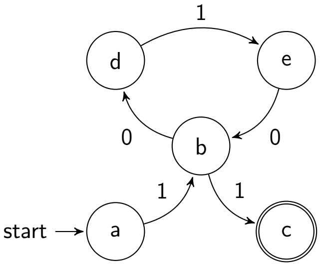
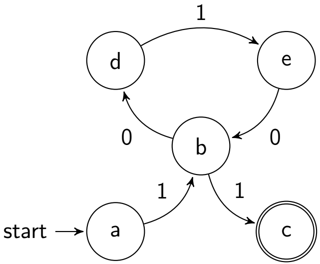


- Language $1(010)^*1$

Pumping Lemma - Example



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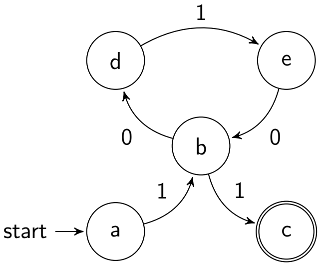

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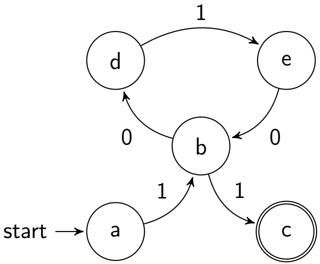

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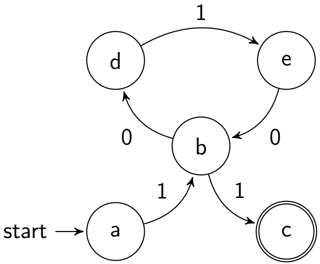

- Language $1(010)^{*}1$
- DFA has 5 states
- consider string 10101, length 5
- $\Rightarrow$ path must contain a loop (in this case, at nodes b,d,e)
- $\Rightarrow$ 10100101 is also a word!

Pumping Lemma

Lemma (Pumping Lemma)

If A is a regular language, then there is a number p, called the pumping length, where if s is a word in A of length $\geq p$ then s can be divided into three parts, $s = xyz$, such that

1. $xy^iz \in A$ *for every* $i \geq 0$,
2. $|y| > 0$,
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- $\rightarrow$ not an if and only if statement!

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- Is A regular?
- If it is, then the pumping lemma gives us a pumping length p.
- Let $s = 0^p1^p$.

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- $\Rightarrow$ then xy^iz for $i \geq 2$ has more 0s than 1s. Contradiction! $\Rightarrow$ A is nonregular.

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- $A = \{0^n 1^n \mid n \geq 0\}$.
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- and $A = B \cap 0^*1^*$
- if B is regular and since 0^*1^* is regular, then A must be as well, contradiction!
- another way of saying this is: if a language contains an nonregular language, it must be nonregular as well!

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- every regular language has a pumping length
- useful for determining if a language is *nonregular*