INF2080

1. Introduction and Regular Languages

Daniel Lupp

Universitetet i Oslo

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Department of Informatics

University of Oslo

Details on the Course

- consists of two parts: computability theory (first half of semester) and complexity theory (second half, held by Evgenij Thorstensen)

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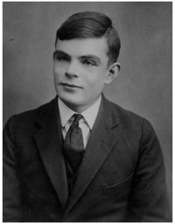
- as always: lectures are useful, but doing exercises yourself is the most important! $\rightarrow$ group exercises

Setup for Computability Theory

For the first half of the course:

- Thursday lecture: new theory and material
- Friday lecture: new theory and material, but mostly reserved for in-depth discussion, examples, open Q&A

So what's it all about?



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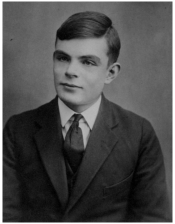

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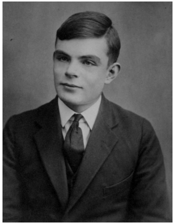

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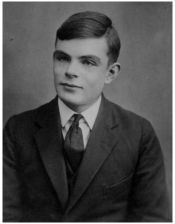

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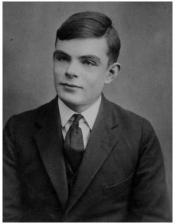

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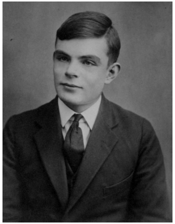

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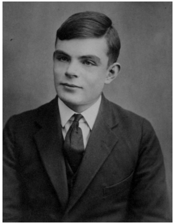

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- Noam Chomsky (1928-)
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- classification of formal languages

→ Chomsky hierarchy

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- Does a "solver" for a given problem always terminate?
- If yes, how expensive is it? ($\rightarrow$ complexity)

The Basics

- **Set**: an unordered collection of distinct objects called *elements*
- $\{a, b\} = \{a, a, b\} = \{b, a\}$
- **Set union**: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- **Set intersection**: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- **Set complement**: $\bar{A} = \{x \mid x \notin A\}$
- **de Morgan's laws**: $\overline{A \cup B} = \bar{A} \cap \bar{B}$ and $\overline{A \cap B} = \bar{A} \cup \bar{B}$
- **Power Set**: $\mathcal{P}(A) = \{S \mid S \subseteq A\}$

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- **Power Set**: $\mathcal{P}(A) = \{S \mid S \subseteq A\}$, example: $\mathcal{P}(\{0, 1\}) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$.

The Basics

- **Tuple**: ordered collection of objects
- $(a, a, b) \neq (a, b)$
- **Cartesian product**: $A \times B = \{(a, b) \mid a \in A, b \in B\}$
- **Function**: $f : A \to B$. Assigns to each element $a \in A$ a unique element $f(a) \in B$.

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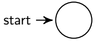
- always exactly one start node: 

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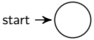
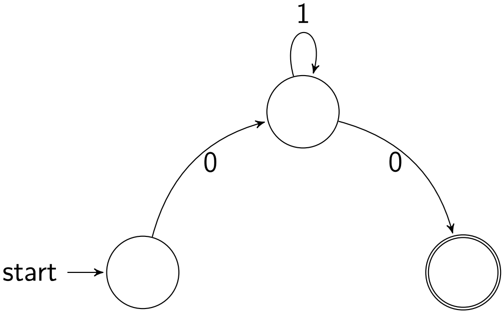
- always exactly one start node: 

- as well as some accept states: 

Finite Automata

Example:



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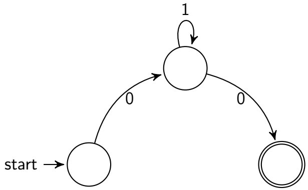

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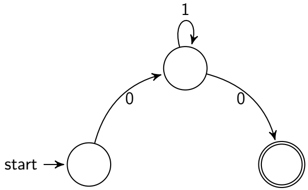

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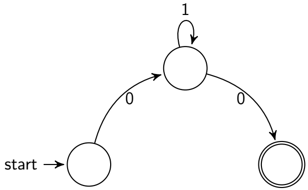

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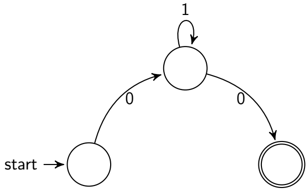

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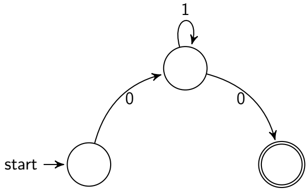

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- the example automaton accepts all inputs, *words*, that start and end with 0, with only 1's in between.

Deterministic Finite Automata

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A deterministic finite automaton (DFA) is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

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5. $F \subseteq Q$ the set of accept states.

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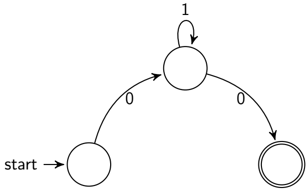

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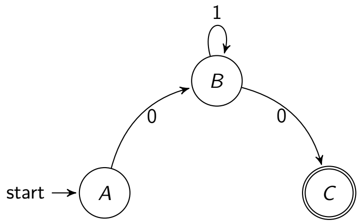

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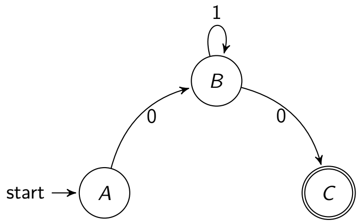

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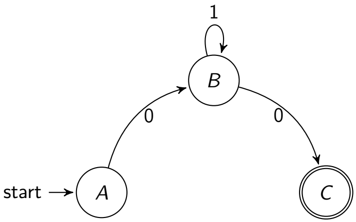

describes

$$L_1 = \{w \mid w \text{ starts and ends with } 0, \text{ with only 1's in between}\}.$$

$\rightarrow L_1$ is regular

Regular languages

Since languages are sets, we can apply various operations on them:

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- Kleene star: $L_1^* = \{x_1 x_2 \cdots x_k \mid k \geq 0, \text{ each } x_i \in L_1\}$

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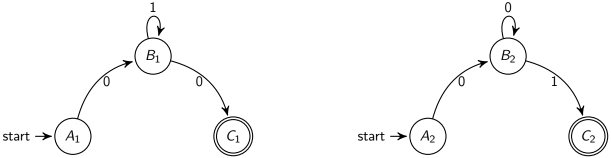
Union of regular languages: Example



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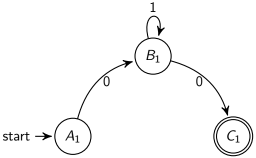

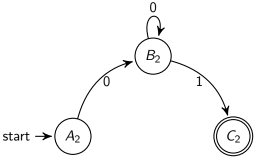

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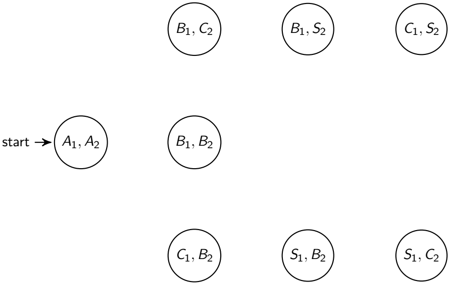

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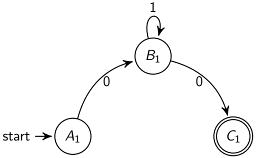

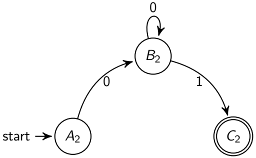

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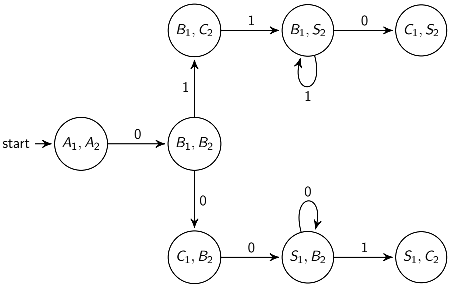

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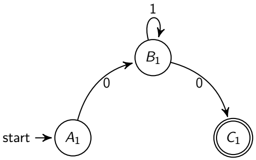

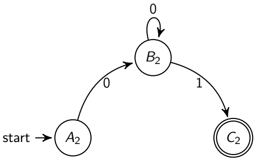

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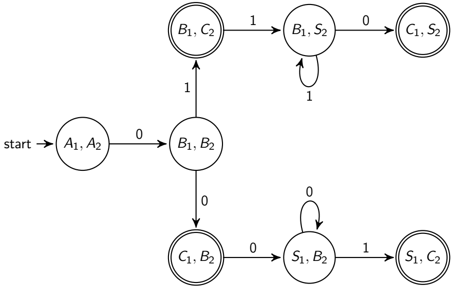

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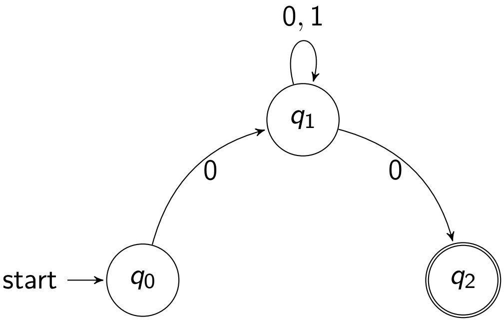
Now we allow for multiple possible "next" states.$\rightarrow$ nondeterminism

NFA - An example



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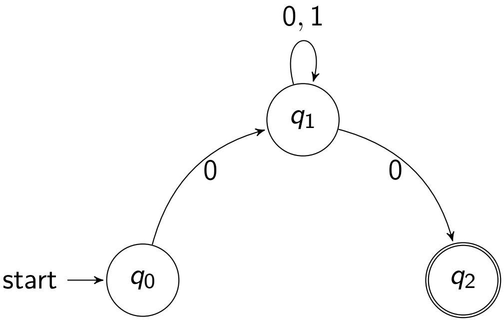
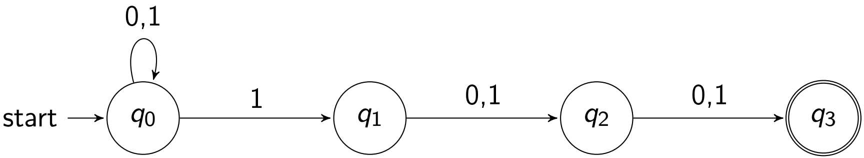


Language consists of all 0,1 sequences starting and ending with 0.

NFA - Another example



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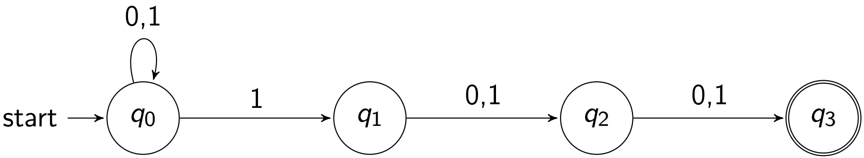
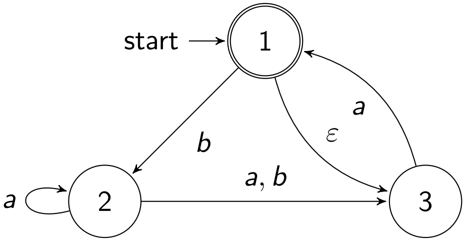


Language consists of all 0,1 sequences with a 1 in the third position from the end.

NFA - an example with empty transitions



Nondeterministic Finite Automata

Definition

A nondeterministic finite automaton (NFA) is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where

1. Q is a finite set of states,
2. Σ is a finite alphabet,
3. $\delta : Q \times \Sigma_\varepsilon \to \mathcal{P}(Q)$ is the transition function, and
4. $F \subseteq Q$ is the set of accepting states.

NFA

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- DFA's accept regular languages, but what languages do NFA's accept?
- As it turns out: regular languages! In other words, in a sense, DFA=NFA.

DFA=NFA

Theorem

Every NFA $N = (Q, \Sigma, \delta, q_0, F)$ *has an equivalent DFA* $M = (Q', \Sigma, \delta', q_0', F')$.

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Proof:

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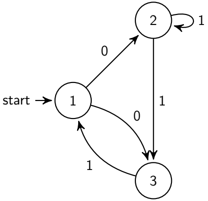
- Let us first assume there are no ε transitions.

DFA=NFA: Example



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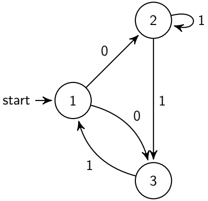

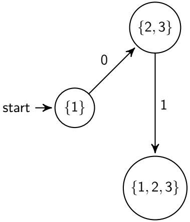

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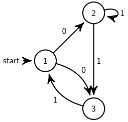

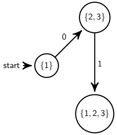

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In other words, we have just proven:

Theorem

A language is regular iff (if and only if) there exists an NFA that accepts it.

Concatenation of regular languages

- So what about concatanation of languages? Given two regular languages L_1, L_2, is $L_1 L_2 = \{w_1 w_2 \mid w_1 \in L_1, w_2 \in L_2\}$ a regular language?

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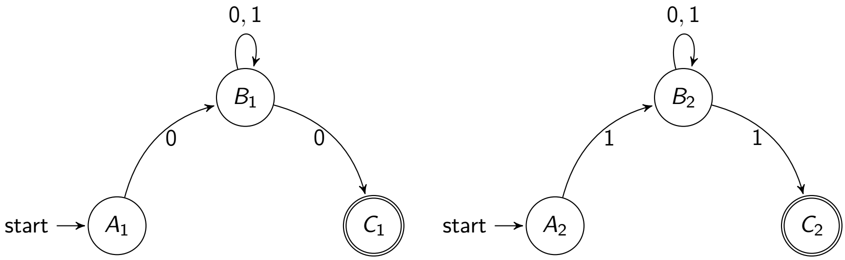
- for concatenation: after having M_1 parse a part of the input that led to an accept state, nondeterministically "hop" into the second automaton M_2.

Concatanation of regular languages



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