INF2080
Oblig 3

Deadline: Friday April 20th 23:59

Hand-in and deadline

Hand in a single PDF file with your answers. You can scan written answers and compile the scans into a PDF file, but make sure the pages are correctly oriented, and that they are readable. Your answer may be in English or a Norwegian-like language.

Hand in your answers using Devilry. Deadline is **April 20th, at 23:59**.

Definitions

A *literal* is a formula on the form X or $\overline{X}$, where X is a variable (i.e., atomic formula). A formula ϕ is on *Conjunctive Normal Form* (CNF) if

$$\phi = (l_1^1 \vee \ldots \vee l_{k_1}^1) \wedge \ldots \wedge (l_1^n \vee \ldots \vee l_{k_n}^n),$$

where l_i^j is the i-th literal of the j-th clause, and k_m is the number of literals in the m-th clause. A formula ϕ is on *Disjunctive Normal Form* (DNF) if

$$\phi = (l_1^1 \wedge \ldots \wedge l_{k_1}^1) \vee \ldots \vee (l_1^n \wedge \ldots \wedge l_{k_n}^n).$$

We define the following languages:

$$\begin{aligned}
CNFSAT &= \{\phi \mid \phi \text{ is on CNF, and } \phi \text{ is satisfiable}\} \\
DNFSAT &= \{\phi \mid \phi \text{ is on DNF, and } \phi \text{ is satisfiable}\} \\
CNFUNSAT &= \{\phi \mid \phi \text{ is on CNF, and } \phi \text{ is unsatisfiable}\} \\
DNFUNSAT &= \{\phi \mid \phi \text{ is on DNF, and } \phi \text{ is unsatisfiable}\} \\
CNFTAUT &= \{\phi \mid \phi \text{ is on CNF, and } \phi \text{ is a tautology}\} \\
DNFTAUT &= \{\phi \mid \phi \text{ is on DNF, and } \phi \text{ is a tautology}\}
\end{aligned}$$

We define the complexity class $coNP$ as the class of languages that are the complements of languages in NP. Formally, let A be a language. Then $A \in NP$ if and only if $\overline{A} \in coNP$.

A language B is $coNP$-complete if (i) B is in $coNP$, and (ii) every language A in $coNP$ is polynomial time reducible to B (i.e., $A \leq_P B$).

When answering the following problems, assume that

$$P \neq NP \quad \text{and} \quad NP \neq coNP.$$

Problem 1

At least one of the above languages is in P. Identify them, and prove that they are in P.

Problem 2

At least one of the above languages is NP-complete. Identify them, and prove that they are NP-complete.

Problem 3

At least one of the above languages is $coNP$-complete. Identify them, and prove that they are $coNP$-complete.