INF2080
Oblig 4

Deadline: Friday May 11th 23:59

Hand-in and deadline

Hand in a single PDF file with your answers. You can scan written answers and compile the scans into a PDF file, but make sure the pages are correctly oriented, and that they are readable. Your answer may be in English or a Norwegian-like language.

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Problem 1 (Sipser 8.14)

An undirected graph is bipartite if its nodes may be divided into two sets so that all edges go from a node in one set to a node in the other set. A graph is bipartite if and only if it does not contain a cycle that has an odd number of nodes. Using this fact, show that the language $BIP = \{G \mid \text{G is a bipartite graph}\}$ is in NL.

Problem 2 (Sipser 9.20)

Describe the error in the following fallacious "proof" that $\mathsf{P} \neq \mathsf{NP}$. Assume for contradiction that $\mathsf{P} = \mathsf{NP}$. Then $SAT \in \mathsf{P}$ and so for some k, $SAT \in \mathsf{TIME}(n^k)$. Because every language in NP is polynomial time reducible to SAT, we have that $\mathsf{NP} \subseteq \mathsf{TIME}(n^k)$. Therefore, $\mathsf{P} \subseteq \mathsf{TIME}(n^k)$. But by the time hierarchy theorem, $\mathsf{TIME}(n^{k+1})$ contains a language that isn't in $\mathsf{TIME}(n^k)$, which contradicts $\mathsf{P} \subseteq \mathsf{TIME}(n^k)$. Therefore, $\mathsf{P} \neq \mathsf{NP}$.

Problem 3

Show that $\mathsf{P} \neq \mathsf{SPACE}(n)$. Hint: Assume that they are equal, and look for a contradiction involving the space hierarchy theorem.