INF2270 — Spring 2010

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Lecture 1: Digital Representation

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Digital Electronics

- Digital electronics is everywhere: PCs, cell phones, DVD and MP3 players, cameras, cars, fridges ...
- ’Digital’ comes from latin ’digitus’ meaning ’finger’ which we often use for discrete counting
- Digital representation: discontinuous/discrete representation (discrete numbers/integers/$\mathbb{Z}$) as opposed to analog/continuous

A Short History of Computers

- The Analytical Engine by Charles Babbage in 1837 (mechanical)
- G. Stibitz' Model-K (1937) and K. Zuse's Z3 (1941) (electro mechanical, relays)
- ENIAC, 1946 (vacuum tubes)
- Computer revolution with transistors, first used to build a computer in 1953 (ca. 600 transistors)

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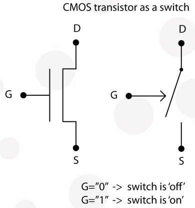
Transistors

- Transistors are electronic switches that are in turn controlled by an electronic signal, i.e. three terminal electronic devices



  - State of the art transistors can be extremely small, down to 32nm (0.000000032m) long, and hundreds of millions integrated on a single computer chip. The Intel Core^{tm} i7 Quad Extreme (Bloomfield) has 731 millioner transistorer in a 45nm-technology (2008)
  - Transistors need in the order of only nano-seconds to switch

Binary Representation

- Switches obviously have two states: 'on' and 'off'
- Thus they are suited to represent digital numbers in the binary system (rather than the decimal that we are used to)
- The two states can also be used to represent the two states in predicate logic/Boolean algebra: 'true' and 'false', respectively '1' and '0'

Binary Numbers

'10010' as a binary number means:
$1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 16 + 2 = 18$

just as '18' as a decimal number means:
$1 \times 10^1 + 8 \times 10^0$

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Boolean Algebra

- Boolean algebra/logic is a set of operations defined on variables that have only two values: '0' and '1'
- There are three basic opeartoions: NOT, AND, OR
- NOT a can be written as $\neg a$, $\bar{a}$ or a'
- a AND b can be written as $a \wedge a$ or $a \times b$ (not to be mixed up with normal multiplication)
- a OR b can be written as $a \vee a$ or $a + b$ (not to be mixed up with normal addition)

Truth Tables

a	$\bar{a}$
0	1
1	0

a	b	$a \wedge b$
0	0	0
0	1	0
1	0	0
1	1	1

a	b	$a \vee b$
0	0	0
0	1	1
1	0	1
1	1	1

Derived Boolean Functions

$$a \text{ XOR } b = (a \wedge \bar{b}) \vee (\bar{a} \wedge b)$$

$$a \text{ XNOR } b = (a \wedge b) \vee (\bar{a} \wedge \bar{b})$$

$$a \text{ NAND } b = \overline{a \wedge b}$$

$$a \text{ NOR } b = \overline{a \vee b}$$

Rules that Apply

$\bar{\bar{a}}=a$		
$a\wedge b\vee c = (a\wedge b)\vee c$	$a\vee b\wedge c = a\vee(b\wedge c)$	
$a\wedge\bar{a}=0$	$a\vee\bar{a}=1$	
$a\wedge a=a$	$a\vee a=a$	
$a\wedge 1=a$	$a\vee 0=a$	
$a\wedge 0=0$	$a\vee 1=1$	
$a\wedge b = b\wedge a$	$a\vee b = b\vee a$	(commutative)
$(a\wedge b)\wedge c=a\wedge(b\wedge c)$	$(a\vee b)\vee c=a\vee(b\vee c)$	(associative)
$a\wedge(b\vee c)=(a\wedge b)\vee(a\wedge c)$	$a\vee(b\wedge c)= (a\vee b)\wedge(a\vee c)$	(distributive)
$\overline{a\vee b} = \bar{a}\wedge\bar{b}$	$\overline{a\wedge b} = \bar{a}\vee\bar{b}$	(deMorgan)

Checking deMorgan's Theorem

for NOR:

a	b	$\overline{a \vee b}$

a	b	$\bar{a} \wedge \bar{b}$

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Examples: simplify

$a \wedge b \vee a \wedge \bar{b} = ?$

$a \wedge b \wedge c \vee \bar{a} \wedge b \wedge c \vee \bar{a} \wedge b \wedge \bar{c} \wedge (a \vee c) = ?$

Solution 1

$$
\begin{aligned}
& a \wedge b \vee a \wedge \bar{b} \\
= \quad & a \wedge (b \vee \bar{b}) \\
= \quad & a \wedge 1 \\
= \quad & a
\end{aligned}
$$

Solution 2

$$
\begin{aligned}
& a \wedge b \wedge c \vee \bar{a} \wedge b \wedge c \quad && \vee \quad && \bar{a} \wedge b \wedge \bar{c} \wedge (a \vee c) \\
= \quad & (a \vee \bar{a}) \wedge b \wedge c && \vee && \bar{a} \wedge b \wedge \bar{c} \wedge a \vee \bar{a} \wedge b \wedge \bar{c} \wedge c \\
= \quad & 1 \wedge b \wedge c && \vee && 0 \vee 0 \\
= \quad & b \wedge c
\end{aligned}
$$

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Examples: representation with truth tables (1/2)

$F = a \wedge b \vee \bar{a} \wedge \bar{b}$

a	b	F

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Examples: representation with truth tables (2/2)

$G = a \wedge b \wedge c \vee \bar{a} \wedge c \vee a \wedge \bar{b}$

a	b	c	G

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Representing Boolean Functions

1. expression with variables and operators
2. truth table
3. graphically with logic gates