COMPRESSION AND CODING II

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INF2310 - Digital Image Processing

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After original slides by Andreas Kleppe

TODAY'S LECTURE

- Difference transform
- Run-length transform
- LZW-compression
- JPEG compression
- Lossless predictive coding
- Sections from the compendium:
  - 18.4 *Noen transformer som brukes i kompresjon*
  - 18.7.3 *Lempel-Ziv-Welch (LZW) algoritmen*
  - 18.8.0 *Koding med informasjonstap*
  - 18.8.1 *JPEG*

INTRODUCTION AND REPETITION

REPETITION: COMPRESSION

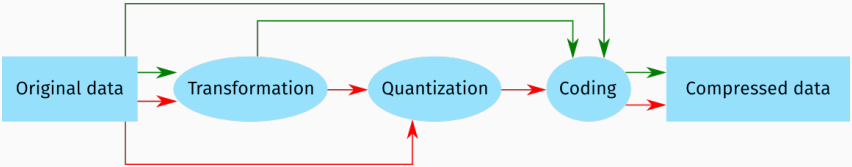


Figure 1: Three steps of compression. Green arrows: lossless, red arrows: lossy

- We can group compression in to three steps:
  - *Transformation:* A more compact image representation.
  - *Qunatization:* Representation approximation.
  - *Coding:* Transformation from one set of symbols to another.
- Compression can either be *lossless* or *lossy*.
  - *Lossless:* We are able to perfectly reconstruct the original image.
  - *Lossy:* We can only reconstruct the original image to a certain degree (but not perfect).
- There exists a number of methods for both types.

REPETITION: DIFFERENT TYPES OF REDUNDANCIES

- *Psychovisual redundancy*
  - Information that we cannot percieve.
  - Can be compressed by e.g. subsampling or by reducing the number of bits per pixel.
- *Inter-pixel temporal redundancy*
  - Correlation between successive images in a sequence.
  - A sequence can be compressed by only storing some frames, and then only differences for the rest of the sequence.
- *Inter-pixel spatial redundancy*
  - Correlation between neighbouring pixels within an image.
  - Can be compressed by e.g. run-length methods.
- *Coding redundancy*
  - Information is not represented optimally by the symbols in the code.
  - This is often measured as the difference between average code length and some theoretical minimum code length.

COMPRESSION METHODS AND REDUNDANCY

Types of redundance →	Psycho-visual	Inter-pixel temporal	Inter-pixel spatial	Coding
Shannon-Fano coding				✓
Huffman coding				✓
Arithmetic coding				✓
Lossless predicative coding in time		✓		
Lossless JPEG			✓	✓
Lossy JPEG	✓		✓	✓
Defference transform			✓	
Run-length transform			✓	
LZW transform			✓	✓

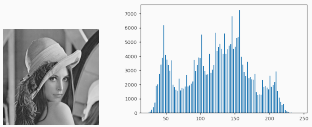
SOME TRANSFORMS

DIFFERENCE TRANSFORM

- Horizontal pixels have often quite similar intensity values.
- Transform each pixelvalue $f(x, y)$ as the difference between the pixel at (x, y) and $(x, y-1)$.
- That is, for an $m \times n$ image f, let $g[x, 0] = f[x, 0]$, and

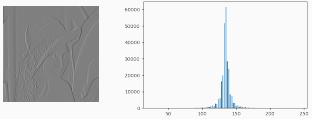
$$g[x, y] = f[x, y] - f[x, y-1], \quad y \in \{1, 2, \ldots, n-1\} \tag{1}$$

  for all rows $x \in \{0, 1, \ldots, m-1\}$.
- Note that for an image f taking values in $[0, 2^b - 1]$, values of the transformed image g take values in $[-(2^b - 1), 2^b - 1]$.
- This means that we need to use $b + 1$ bits for each $g(x, y)$ if we are going to use equal-size codeword for every value.
- Often, the differences are close to 0, which means that natural binary coding of the differences are not optimal.



(a) Original (b) Graylevel intensity histogram

Figure 2: $H \approx 7.45 \implies c_r \approx 1.1$



(a) Difference transformed (b) Graylevel intensity histogram

Figure 3: $H \approx 5.07 \implies c_r \approx 1.6$

Algorithm 1 Forward difference transform

procedure FORWARDDIFF(f) ▷ f is an image of shape $m \times n$
  $g \leftarrow 0$ ▷ Difference image with same shape as f
  for $r \in \{0, 1, \ldots, m\}$ **do**
    $g[r, 0] \leftarrow f[r, 0]$
    for $c \in \{1, 2, \ldots, n-1\}$ **do**
      $g[r, c] \leftarrow f[r, c] - f[r, c-1]$
    end for
  end for
  return g
end procedure

Algorithm 2 Backward difference transform

procedure BACKWARDDIFF(g) ▷ g is an image of shape $m \times n$
  $h \leftarrow 0$ ▷ Image with same shape as g
  for $r \in \{0, 1, \ldots, m\}$ **do**
    $h[r, 0] \leftarrow g[r, 0]$
    for $c \in \{1, 2, \ldots, n-1\}$ **do**
      $h[r, c] \leftarrow g[r, c] + h[r, c-1]$
    end for
  end for
  return h
end procedure

RUN-LENGTH TRANSFORM

- Often, images contain objects with similar intensity values.
- Run-length transform use neighbouring pixels with *the same value*.
  - Note: This requires equality, not only similarity.
  - Run-length transform is compressing more with decreasing complexity.
- The run-length transform is reversible.
- Codes sequences of values into sequences of tuples: (value, run-length).
- Example:
  - Values (24 numbers): $3, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 4, 4, 7, 7, 7, 7, 7, 7$.
  - Code (8 numbers): $(3, 6), (5, 10), (4, 2), (7, 6)$.
- The coding determines how many bits we use to store the tuples.

RUN-LENGTH TRANSFORM IN BINARY IMAGES

- In a binary image, we can ommit the *value* in coding. As long as we know what value is coded first, the rest have to be alternating values.
  - $0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1$
  - $5, 6, 2, 3, 5, 4$
- The histogram of the run-lengths is often not flat, entropy-coding should therefore be used to code the run-length sequence.

BIT SLICING

- Bit slicing is extracting the value of a bit at a certain position.
- We will look at two different ways of doing this.
- Let v be the value we want to extract bit values from, and n denote the bit position, starting from $n = 0$ at the least significant bit (LSB) to the most significant bit (MSB).
- As an example, $10_{10} = 1010_2$ has values $[1, 0, 1, 0]$ at $n = [3, 2, 1, 0]$.
- Let b be the bit value at position n in v.
- Let us use python-syntax for bitwise operators:
  - &: Bitwise and: $1100_2 \& 1010_2 = 1000$, therefore $12_{10} \& 10_{10} = 8$.
  - //: Integer division: $23//4 = 5$ since $23/4 = 5 + 3/4$.
  - $\ll$: Left bit-shift: $10_{10} \ll 3_{10} = 80_{10}$ since $1010_2 \ll 3_{10} = 1010000_2$.

Method 1

$$b = v//2^n \mod 2.$$

If the result of $v//2^n$ is odd, $b = 1$, if it is even, $b = 0$.

Method 2

$$b = [v\&(1 \ll n)] > 0.$$

$(1 \ll n)$ in binary is a 1 followed by n zeros. Therefore will a bitwise and operation on some number be 0 unless it has a bit value of 1 at position n.

Table 1: Example with $v = 234_{10} = 11101010_2$

	Method 1		Method 2	
n	$v//2^n$	b	$v\&(1 \ll n)$	b
7	1	1	128	1
6	3	1	64	1
5	7	1	32	1
4	14	0	0	0
3	29	1	8	1
2	58	0	0	0
1	117	1	2	1
0	234	0	0	0

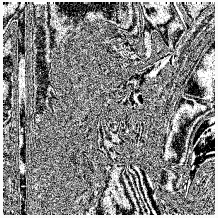
BIT SLICING IN IMAGES: EXAMPLE

(a) $n = 7$

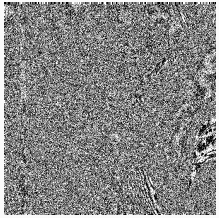
(b) $n = 6$

(c) $n = 5$

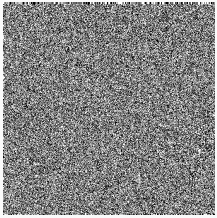
(d) $n = 4$



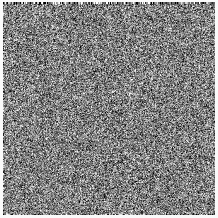
(e) $n = 3$



(f) $n = 2$



(g) $n = 1$



(h) $n = 0$

REPETITION: NATURAL BINARY CODING

- Every codeword is of equal length.
- The code of each symbol is the binary representation of the symbol's (zero indiced) index.
- Example: A 3-bits natural code has 8 possible values.

Symbol	a	b	c	d	e	f	g	h
Index	0	1	2	3	4	5	6	7
Codeword	000	001	010	011	100	101	110	111

GRAY CODE

- An alternative binary representation (or coding).
- Close numbers need not have many bit values in common, e.g. $127_{10} = 01111111_2$ and $128_{10} = 10000000$.
- Since neighbouring pixels often have similar values, this means that natural binary coded images often have high bit-plane complexity.
- Sometimes, e.g. in run-length coding, this is not desired.
- In Gray code, only one bit value is changed between adjacent integer values.
- The codewords in natural binary coding and Gray code are of equal length, the only difference is *what codeword is assigned to what value*.

Algorithm 3 Natural to Gray coding transform

procedure NATTOGRAY(n) ▷ n list of naturally coded bits
  $g \leftarrow []$ ▷ Initialize Gray list to empty
  $c \leftarrow$ `false` ▷ Boolean that decides whether to complement or not
  for $b \in n$ **do**
    if c **then**
      $g \leftarrow [1 - b]$ ▷ Append to g
    else
      $g \leftarrow [b]$
    end if
    if $b == 1$ **then**
      $c \leftarrow$ `true`
    else
      $c \leftarrow$ `false`
    end if
  end for
  return g
end procedure

Algorithm 4 Gray to Natural coding transform

```
procedure GrayToNat(g)                          ▷ g list of Gray coded bits
    n ← []                                      ▷ Initialize natural list to empty
    c ← false                                   ▷ Boolean that decides whether to complement or not
    for b ∈ g do
        if c then
            n ← [1 − b]                         ▷ Append to g
        else
            n ← [b]
        end if
        if b == 1 then
            c ← ¬c                              ▷ Switch value of c
        end if
    end for
    return n
end procedure
```

GRAY CODE EXAMPLE

Decimal	Gray code	Natural code
0	0000	0000
1	0001	0001
2	0011	0010
3	0010	0011
4	0110	0100
5	0111	0101
6	0101	0110
7	0100	0111
8	1100	1000
9	1101	1001
10	1111	1010
11	1110	1011
12	1010	1100
13	1011	1101
14	1001	1110
15	1000	1111

BIT PLANES IN NATURAL BINARY CODES AND GRAY CODES

- The figures below show bit planes from the MSB (left) to LSB (right).
- The MSB is always equal in the two representations.
- The Gray code representation has typically fewer ”noise planes”, which implies that run-length transforms can compress more.

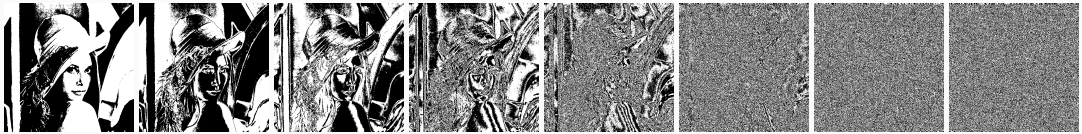


Figure 5: Natural binary representation

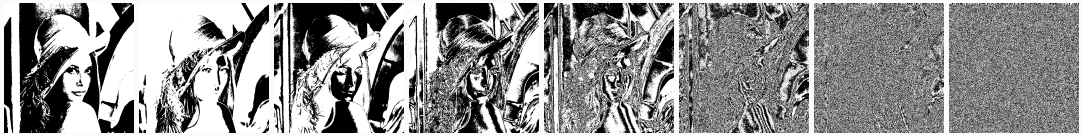


Figure 6: Gray code representation

LEMPEL-ZIV-WELCH CODING

- A member of the *LZ** family of compression schemes.
- Utilizes patterns in the message by looking at symbol occurances, and therefore mostly reduces inter sample redundancy.
- Maps one symbol sequence to one code.
- Based on a *dictionary between symbol sequence and code* that is built on the fly.
  - This is done both in encoding and decoding.
  - The dictionary is not stored or transmitted.
- The dictionary is initialized with an alphabeth of symbols of length one.

LEMPEL-ZIV-WELCH: ENCODING

Given a message W that is to be encoded, the encoding is as follows.

1. Initialize the dictionary from an alphabeth, e.g. $D = \{\# : 0, a : 1, b : 2, \ldots\}$ (where $\#$ is an end-of-message symbol).
2. Initialize the current input w, to be the first symbol in W.
3. Find the longest string $d \in D$ that matches the current input w.
4. Output (or send) the codeword of d, $D[d]$.
5. Create a new string and append it to D, this new string is created as the current string d concatenated with the next symbol in the input message W.
6. This new dictionary entry will get the next codeword not in use.
7. Set current symbol w to be the next string in W that is also in D.
8. Unless $w = \#$, go to 3.

LZW ENCODING EXAMPLE

- Message: ababcbababaaaaabab#
- Initial dictionary: { #:0, a:1, b:2, c:3 }
- New dictionary entry: current string plus next unseen symbol

Message	Current string	Codeword	New dict entry
ababcbababaaaaabab#	a	1	ab:4
abababcbababaaaaabab#	b	2	ba:5
ababcbababaaaaabab#	ab	4	abc:6
ababcbababaaaaabab#	c	3	cb:7
ababcbababaaaaabab#	ba	5	bab:8
ababcbababaaaaabab#	bab	8	baba:9
ababcbababaaaaabab#	a	1	aa:10
ababcbababaaaaabab#	aa	10	aaa:11
ababcbababaaaaabab#	aa	10	aab:12
ababcbababaaaaabab#	bab	8	bab#:13
ababcbababaaaaabab#	#	0	

- Encoded message: 1,2,4,3,5,8,1,10,10,8,0.
- Assuming original bps = 8, and coded bps = 4, we achieve a compression rate of

$$c_r = \frac{8 \cdot 19}{4 \cdot 11} \approx 3.5 \tag{2}$$

LEMPEL-ZIV-WELCH: DECODING

- Decode the encoded string codeword by codeword.
- Build the dictionary by decoding the current codeword and concatenate this encoded string with:
  - If the next codeword can be decoded (it is already in the dictionary): The first character of the *next decoded string*.
  - If the next codeword is not in the dictionary: The first character of the *current decoded string*[1].
- Processing one codeword at the time, and building the dictionary at the same time, will in the end decode the whole sequence of codewords.

[1] See next page

This explains why it makes sense to attach the first symbol of the current decoded string to the end of the new dictionary entry.

- Let the currently decoded string be X with first symbol x.
- The new dictionary entry is $X? : n$, where n is the codeword.
- We look for n in our dictionary, but see that it is not there. We know that $?$ should be the first symbol y of the decoded string Y at n, but how do we know what it is?
- The first thing to realise is that this only happens if Y was encountered immediately after the creation of $Y : n$ in the encoding.
- Therefore $X? = Y$, and therefore, $y = x$, where x was the first symbol of the string X.

LZW DECODING EXAMPLE

- Encoded message: 1,2,4,3,5,8,1,10,10,8,0
- Initial dictionary: { #:0, a:1, b:2, c:3 }
- New dictionary entry: current string plus first symbol in next string

Message	Current string	New dict entry Final	New dict entry Proposal
1,2,4,3,5,8,1,10,10,8,0	a		a?:4
1,2,4,3,5,8,1,10,10,8,0	b	ab:4	b?:5
1,2,4,3,5,8,1,10,10,8,0	ab	ba:5	ab?:6
1,2,4,3,5,8,1,10,10,8,0	c	abc:6	c?:7
1,2,4,3,5,8,1,10,10,8,0	ba	cb:7	ba?:8
1,2,4,3,5,8,1,10,10,8,0	bab	bab:8	bab?:9
1,2,4,3,5,8,1,10,10,8,0	a	baba:9	a?:10
1,2,4,3,5,8,1,10,10,8,0	aa	aa:10	aa?:11
1,2,4,3,5,8,1,10,10,8,0	aa	aaa:11	aa?:12
1,2,4,3,5,8,1,10,10,8,0	bab	aab:12	bab?:13
1,2,4,3,5,8,1,10,10,8,0	#	bab#:13	

Decoded message: ababcbababaaaaaaab#

LZW COMPRESSION, SUMMARY

- The LZW codes are normally coded with a natural binary coding.
- Typical text files are usually compressed with a factor of about 2.
- LZW coding is used a lot
  - In the Unix utility `compress` from 1984.
  - In the `GIF` image format.
  - An option in the `TIFF` and `PDF` format.
- Experienced a lot of negative attention because of (now expired) patents. The `PNG` format was created in 1995 to get around this.
- The LZW can be coded further (e.g. with Huffman codes).
- Not all created codewords are used.
- We can limit the number of generated codewords.
  - Setting a limit on the number of codewords, and deleting old or seldomly used codewords.
  - Both the encoder and decoder need to have the same rules for deleting.

LOSSY COMPRESSION

- In order to achieve high compression rates, it is often necessary with *lossy compression*.
- Note: in this case, the original signal *can not be recovered* because of loss of information.
- Some simple methods for lossy compression:
  - Requantizing to fewer graylevel intensities.
  - Resampling to lower spatial resolution.
  - Filter based methods, e.g. replacing the values in every non-overlapping $p \times q$ rectangle in an image with the mean or median value of that region.

- If we use lossy compression, we need to make sure that the result after decompression is *good enough*.
- For an $m \times n$ image f, let g be the resulting image after f has been compressed and decompressed. The error is then the difference

$$e(x, y) = f(x, y) - g(x, y).$$

- The *root mean square* (RMS) error between the images is

$$RMS = \sqrt{\frac{1}{mn} \sum_{x=1}^{m} \sum_{y=1}^{n} e^2(x, y)}$$

- If we interpret the error as noise, we can define the *mean squared signal to noise ratio* (RMS_{MS}) as

$$SNR_{MS} = \frac{\sum_{x=1}^{m} \sum_{y=1}^{n} g^2(x, y)}{\sum_{x=1}^{m} \sum_{y=1}^{n} e^2(x, y)}$$

- The RMS value of the SNR is then

$$SNR_{RMS} = \sqrt{\frac{\sum_{x=1}^{m}\sum_{y=1}^{n} g^2(x,y)}{\sum_{x=1}^{m}\sum_{y=1}^{n} e^2(x,y)}}$$

- The quality measures above considers all errors in the whole image, and treat them equally.
- Our perception does not necessary agree. E.g. small errors over the whole image will get a larger SNR_{MS} than missing or created features. But we will percieve the latter having inferior quality.
- Often, our desire is that the image quality shall mirror *our perception of the quality of the image*.
- This is especially true for image display purposes.

HOW GOOD IS THE IMAGE QUALITY, CONT.

- An image quality measure that is well aligned with our perception is typically based on several parameter
  - Each parameter should try to indicate how bad a certain compression error trait is.
  - The final image quality measure should be one value that is based on all parameters.
- Errors around edges is perceived as bad.
- Errors in the foreground are perceived worse than errors in the background.
- Missing or created structures are also bad.
- The level of compression should probably vary locally in the image.
  - Homogeneous areas should be compressed heavily. These areas carry little information, and few non-zero coefficients in the 2D DFT.
  - Edges, lines and other details should be compressed less. These carry more information, and have more non-zero 2D DFT coefficients.

JPEG

- JPEG (*Joint Photographic Expert Group*) is one of the most common compression methods.
- The JPEG-standard (originally from 1992) has both lossy and lossless variants.
- In both cases, either Huffman- or arithmetic coding is used.
- In the lossless version, *predicative* coding is used.
- In the lossy version, a *2D discrete cosinus transform (DCT)* is used.

LOSSY JPEG COMPRESSION: START

- Each image channel is partitioned into blocks of 8×8 piksels, and each block can be coded separately.
- For an image with 2^b intensity values, subtract 2^{b-1} to center the image values around 0 (if the image is originally in an unsigned format).
- Each block undergoes a 2D DCT. With this, most of the information in the 64 pixels is located in a small area in the Fourier space.

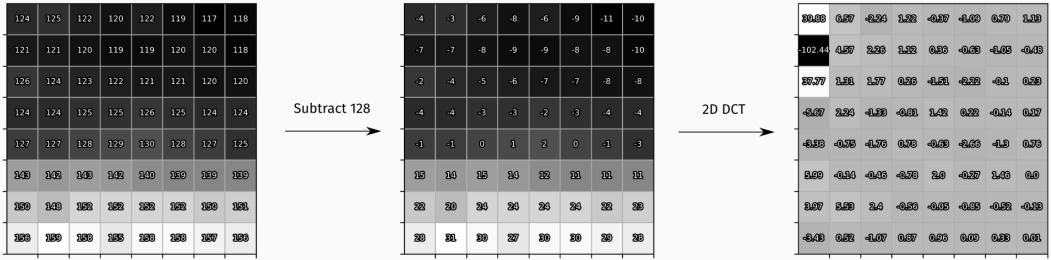


Figure 7: Example block, subtraction by 128, and 2D DCT.

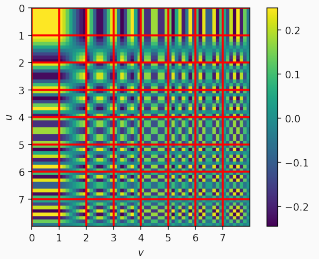
2D DISCRETE COSINUS-TRANSFORM

The main ingredient of the JPEG-compression is the 2D discrete cosinus transform (*2D DCT*). For an $m \times n$ image f, the 2D DCT is

$$F(u,v) = \frac{2}{\sqrt{mn}} c(u)c(v) \sum_{x=0}^{m} \sum_{y=0}^{n} f(x,y) \cos\left(\frac{(2x+1)u\pi}{2m}\right) \cos\left(\frac{(2y+1)v\pi}{2n}\right), \tag{3}$$

$$c(a) = \begin{cases} \frac{1}{\sqrt{2}} & \text{if } a = 0, \\ 1 & \text{otherwise.} \end{cases} \tag{4}$$

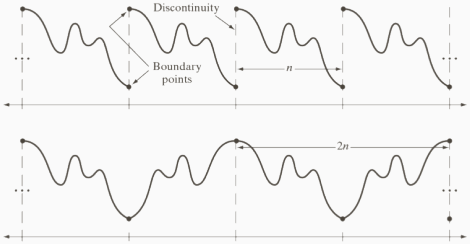
- JPEG only transforms 8×8 tiles at a time.
- Compute 8×8 (from u, v) tiles of size 8×8 (from x, y), of the cosine factor
- Compute 2D DCT coefficients by summing the dot-products of the 8×8 block in the image and every 8×8 tile in the cosine image.



WHY DCT AND NOT DFT?

For a discrete signal with n points will the implicit n-point periodicity of a DFT introduce high frequencies because of boundary-discontinuity. In JPEG, $n = 8$ and 2D, and the boundary is the boundary of the blocks, but the point still stands.

- If we remove these frequencies we introduce heavy block-artifacts.
- If we keep them, we reduce the compression rate compared to DCT, where we often don't need to keep most high frequencies.



DCT is implicitly $2n$-point periodically and symmetric about n, therefore will these hight frequencies *not be introduced*.

LOSSY JPEG COMPRESSION: LOSS OF INFORMATION

- Each of the frequency-domain blocks are then point-divided by a quantization matrix.
- The result is rounded off to the nearest integer.
- his is where we lose information, but also why we are able to achieve high compression rates.
- This result is compressed by a coding method, before it is stored or transmitted.
- The DC and AC components are treated differently.

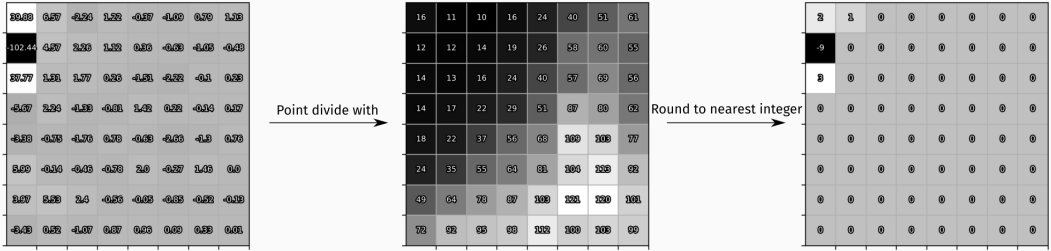


Figure 8: Divide the DCT block (left) with the quantization matrix (middle) and round to nearest integer (right)

LOSSY JPEG COMPRESSION: AC-COMPONENTS (SEQUENTIAL MODES)

1. The AC-components are zig-zag scanned:
   - The elements are ordered in a 1D sequence.
   - The absolute value of the elements will mostly decsend through the sequence.
   - Many of the elements are zero, especially at the end of the sequence.
2. A zero-based run-length transform is performed on the sequence.
3. The run-length tuples are coded by Huffman or arithmetic coding.
   - The run-length tuple is here (number of 0's, number of bits in "non-0").
   - Arithmetic coding often gives $5 - 10\%$ better compression.

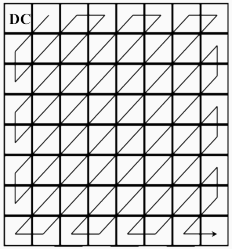


Figure 9: Zig-zag gathering of AC-components into a sequence.

LOSSY JPEG COMPRESSION: DC-COMPONENT

1. The DC-components are gathered from all the blocks in all the image channels.
2. These are correlated, and are therefore difference-transformed.
3. The differences are coded by Huffman coding or arithmetic coding.
    - More precise: The number of bits in each difference is entropy coded.

LOSSY JPEG DECOMPRESSION: RECONSTRUCTION OF FREQUENCY-DOMAIN BLOCKS

- The coding part (Huffman- and arithmetic coding) is reversible, and gives the AC run-length tuples and the DC differences.
- The run-length transform and the difference transform are also reversible, and gives the scaled and quantized 2D DCT coefficients
- The zig-zag transform is also reversible, and gives (together with the restored DC component) an integer matrix.
- This matrix is multiplied with the quantization matrix in order to restore the sparse frequency-domain block.

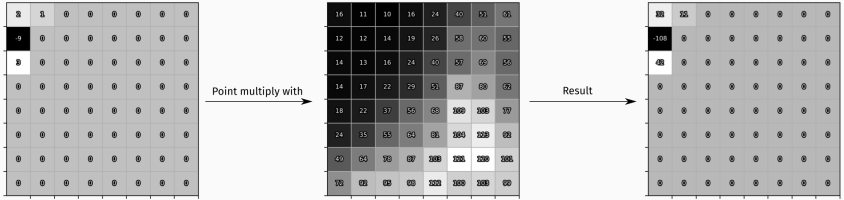


Figure 10: Multiply the quantized DCT components (left) with the quantization matrix (middle) to produce the sparse frequency-domain block (right).

LOSSY JPEG DECOMPRESSION: QUALITY OF RESTORED DCT IMAGE

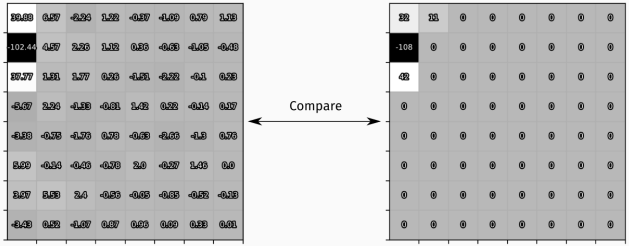


Figure 11: Comparison of the original 2D DCT components (left) and the restored (right)

- The restored DCT image is not equal to the original.
- But the major features are preserved
- Numbers with large absolute value in the top left corner.
- The components that was near zero in the original, are exactly zero in the restored version.

- We do an inverse 2D DCT on the sparse DCT component matrix.

$$f(x,y) = \frac{2}{\sqrt{mn}} \sum_{u=0}^{m} \sum_{v=0}^{n} c(u)c(v)F(u,v) \cos\left(\frac{(2x+1)u\pi}{2m}\right) \cos\left(\frac{(2y+1)v\pi}{2n}\right), \quad (5)$$

where

$$c(a) = \begin{cases} \frac{1}{\sqrt{2}} & \text{if } a = 0, \\ 1 & \text{otherwise.} \end{cases} \quad (6)$$

- We have then a restored image block which should be approximately equal to the original image block.

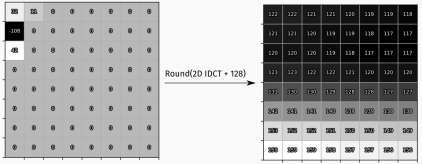


Figure 12: A 2D inverse DCT on the sparse DCT component matrix (left) produces an approximate image block (right)

LOSSY JPEG DECOMPRESSION: APPROXIMATION ERROR

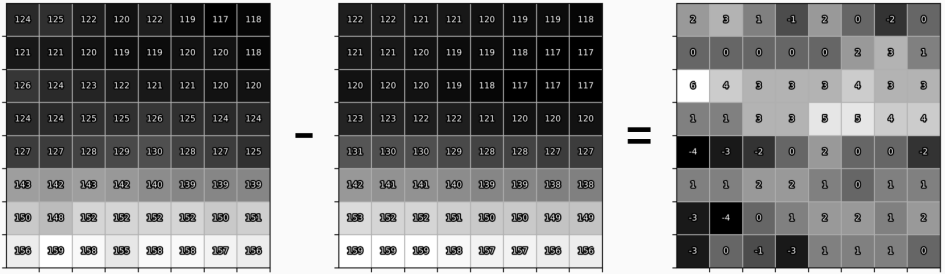


Figure 13: The difference (right) between the original block (left) and the result from the JPEG compression and decompression (middle).

- The differences between the original block and the restored are small.
- But they are, however, not zero.
- The error is different on neighbouring pixels.
- This is especially true if the neighbouring pixels belong to different blocks.
- The JPEG compression/decompression can therefore introduce *block artifacts*, which are block patterns in the reconstructed image (due to these different errors).

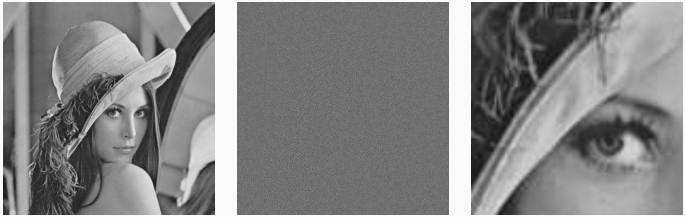
RECONSTRUCTION ERROR IN GRAYSCALE IMAGES

- JPEG compression can produce *block-artifacts*, *smoothings* and *ring-effects*.
- This is dependent on the quantization matrix, which determines how many coefficients are kept, and how precisely they are preserved.

(a) Smoothing- and ring-effects

(b) Block artifacts

BLOCK ARTIFACTS AND COMPRESSION RATE



(a) Compressed **(b)** Difference **(c)** Detail

(d) Compressed **(e)** Difference **(f)** Detail

Figure 15: Top row: compression rate = 12.5. Bottom row: compression rate = 32.7

SCALING OF QUANTIZATION MATRIX

- Lossy JPEG compression use the quantization matrix to determine what information to keep.
- The scaling factor q, of the matrix determines the compression rate c_r.

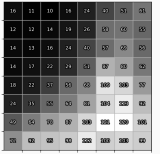
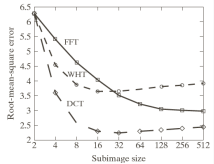


Figure 16: Quantization matrix

Figure 17: Top row, from left: $(q, c_r) : [(1, 12), (2, 19), (4, 30)]$ Bottom row, from left: $(1, c_r) : [(8, 49), (16, 85), (32, 182)]$

BLOCK SIZES

- We can vary the block size.
- The compression rate and execution time increases with increaseng block size.



- Block artifacts decreases with increasing block size,
- but the ringing-effects increases.

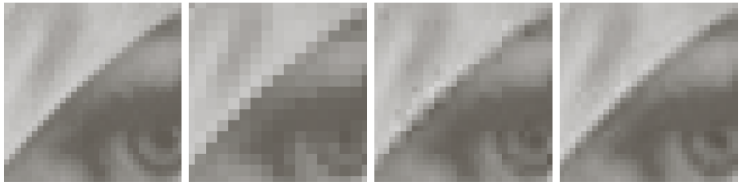
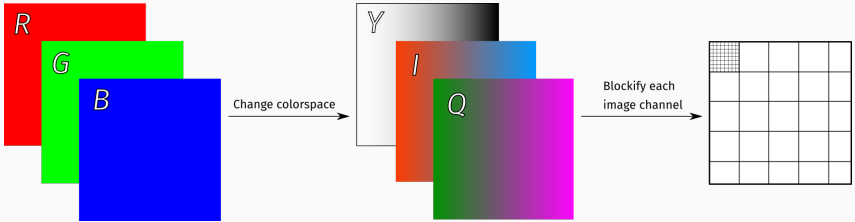


Figure 18: Original image (left). Different block sizes (left to right): $2 \times 2, 4 \times 4, 8 \times 8$.

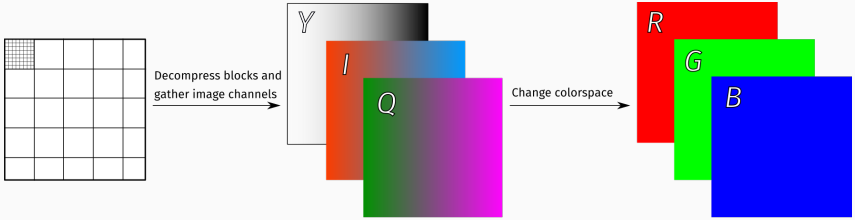
LOSSY JPEG COMPRESSION OF COLOR IMAGE

- Change color space (from RGB) in order to separate luminance from chrominance.
  - This is more aligned to how we percieve a color image.
  - Light intensity is more important to us than chromaticity.
  - Can also produce lower complexity in each channel.
- (Normally) we downsample the chromaticity-channels. Typically with a factor 2 in each channel.
- Each channel is partitioned into 8×8 blocks, and each block is coded separately as before.
- We may use different quantization matrices for the luminocity and chromaticity channels.

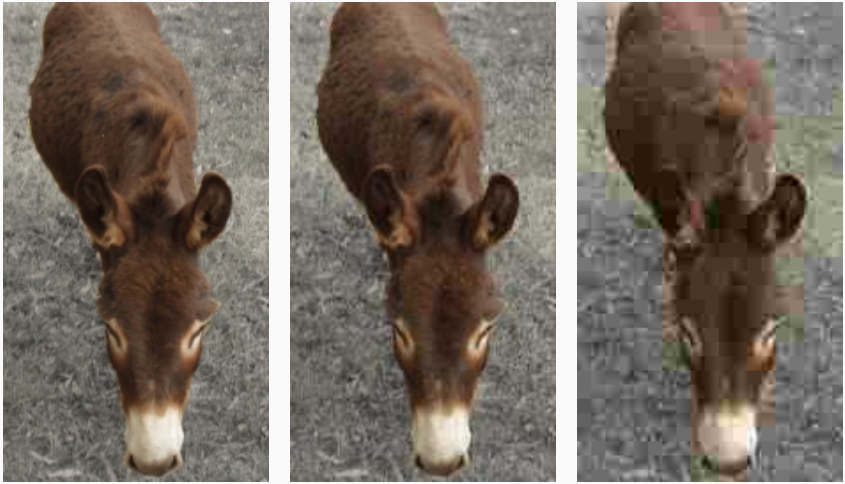


LOSSY JPEG DECOMPRESSION OF COLOR IMAGE

- Every decompressed 8×8 block in each image channel is gathered in a matrix for this channel.
- The image channels are gathered to create a color image.
- We change color space to RGB for display, or CMYK for printing.
- Even if the chromaticity channels have reduced resolution, the resolution in the RGB space is full.
  - We can get 8×8 block artifacts in intensity.
  - With 2 times downsampling in each direction in the chromaticity channels, we can get 16×16 block artifacts in chroma ("colors").



RECONSTRUCTION ERROR IN COLOR IMAGES



(a) 1.5 - 2 bbp **(b)** 0.5 - 0.75 bbp **(c)** 0.25 - 0.5 bbp

Figure 19: Compression of 2f bit color images. Compression level measured in *bits per pixel* (bbp)

JPEG2000 vs JPEG

- Original JPEG is from 1992, newer standard JPEG2000 is from 2000.
- Uses a discrete wavelet transform in stead of DCT.
- Uses more sophisticated coding algorithms.
- Higher compression and better perceptual qulity.
- No block artifacts, but ringing-effects are still present.
- More computationally demanding.
- Not widely supported, even after 17 years.

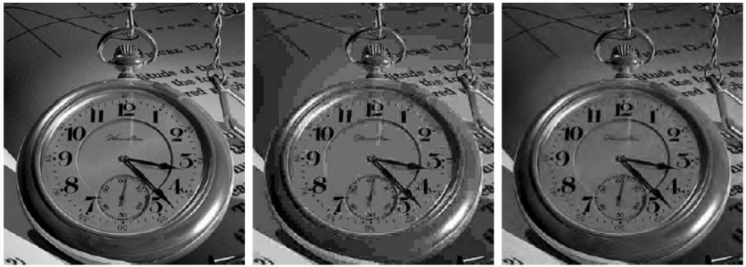


Figure 20: Original (left), JPEG (middle), and JPEG2000 (right)

LOSSLESS JPEG COMPRESSION: OVERVIEW

- The lossless JPEG variant is using *predictive coding*.
- Generally, for an image f, predictive coding codes

$$e(x, y) = f(x, y) - g(x, y),$$

  where $g(x, y)$ is predicted using neighbouring values of (x, y).
- A linear predictor of order (m, n):

$$g(x, y) = \texttt{round}\left[\sum_{i=1}^{m}\sum_{j=1}^{n}\alpha_{ij} f(x - i, y - j)\right]$$

- Equal-length coding requires an extra bit per pixel $e(x, y)$.
  - Or even more bits if the sum of the prediction-coefficients a_{ij}, exceeds 1.
  - The solution is entropy-coding.

LOSSLESS JPEG COMPRESSION: DETAIL

- In lossless JPEG compression, $f(x, y)$ is predicted using up to three previously processed elements.
  - Z is the pixel we want to predict.
  - Use some or all of the elements A, B, C.
- The prediction error is near zero, and is entropy coded with either Huffman coding or arithmetic coding.
- The compression rate is dependent on
  - Bits per pixel in the original image.
  - The entropy in the prediction error.
- For normal color images, the compression rate is about 2.
- Is mostly only used in medical applications.

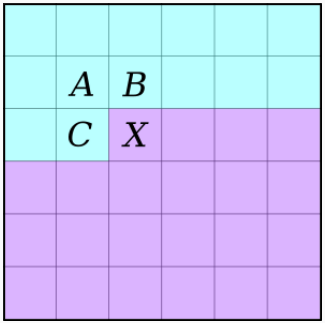


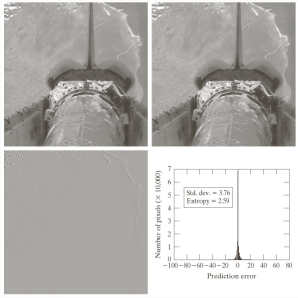
Figure 21: What elements used in predictive coding of element Z. Blue is processed, pink is not.

For a sequence of images stacked as $f(x, y, t)$, an m'th order prediction can be computed as

$$g(x, y, t) = \texttt{round}\left[\sum_{k=1}^{m} \alpha_k f(x, y, t-1)\right]$$

Motion detection and motion compensation is necessary inside so called *macro blocks* (typically of shape 16×16) to increase the compression rate.

- The difference entropy is low: $H = 2.59$.
- This gives an optimal compression rate (when single-differences is coded) of $c_r = 8/2.59 = 3$.
- Figure to the right
  - Top row: Two frames from an orbiting space shuttle video.
  - Bottom row: Prediction error image using order 1 prediction (left), and a histogram of the prediction error (right).



- Compression of digital image sequences/video is usually based on *predictive coding with motion-compensation and 2D DCT*.
- Newer standards use often prediciton based on both previous and future images in the sequence.
- With 50-60 frames per second there is a lot to gain by prediction.
- ISO/IEC standards for video compression (through the *Motion Picture Expert Group* (MPEG)): MPEG-1 (1992), MPEG-2 (1994), MPEG-4 (1998), MPEG-H (2013).
- ITU-T have also standards for video compression (throught the *Visual Coding Experts Group* (VCEG): H.120 (1984), H.26x-family (H.265 (2013) = MPEG-H part 2)

SUMMARY

- The purpose of compression is to represent ”the same” information more compactly by reducing or removing redundancy.
- Compression is based on information theory.
- The number of bits per symbol is central, and varies with the compression method and input message.
- Central algorithms:
  - Run-length transform
  - LZW transform
  - 2D DCT
  - Predictive coding
  - Difference transform
  - Huffman coding
  - Arithmetic coding

Questions?