ML lectures

- 16.09: The Algol Family and ML (Mitchell's chap. 5)
- **23.09: More on ML & Types (chap. 5 and 6)**
- 21.10: More on Types, Type Inference and Polymorphism (chap. 6)
- 28.10: Control in sequential languages, Exceptions and Continuations (chap. 8)
- Prolog I / Prolog ||

Functions and Pattern Matching

- Function declaration
  - Functions are as other values:
    - (5*6) ;

      > val it = 30 : int
    - fn x => x * 2 ;   “anonymous function”, in lambda notation $\lambda x . (x * 2)$

      > val it = fn : int -> int
    - val dbl = fn x => x * 2 ;   > val dbl = fn : int -> int
  - But we have a special syntax for defining functions:
    - fun dbl x = x * 2 ;   > val dbl = fn : int -> int
- Function declaration, general form
  - fun f (<pattern>) = <expr>
    - fun f (x,y) = x+y;   Actual par. must match pattern (x,y)
  - fn <pattern> => <expr>
    - fn (x,y) => x+y;   Anonymous function
- Multiple-clause definition
  - fun <name> <pat$_1$> = <exp$_1$> | …

    | <name> <pat$_n$> = <exp$_n$>
  - fun length (nil) = 0

    | length (x::s) = 1 + length(s);

  > val length = fn ´a list -> int

  - length [“J”, ”o”, “n”]   > val it = 3 : int

Some functions on lists

- Insert an element in an ordered list

```
fun insert (e, nil)      = [e]
 |   insert (e, x::xs) = if e>x then x :: insert(e,xs)
                                        else e::(x::xs);
- insert (3,[1,2,5]) ;
> val it = [1,2,3,5] : int list
```

- Append lists

```
fun append(nil, ys) = ys
|    append(x::xs, ys) = x :: append(xs, ys);
- append ([3,4],[1,2]) ;
>val it = [3,4,1,2] : int list
```

Data-type declarations

- Enumeration types
  - datatype color = Red | Yellow | Blue;
    - elements are: Red, Yellow, Blue <- Constructors!
- Tagged union types
  - datatype value = I of int | R of real | S of string;
    - elements are: I(9) , R(8.3) , S("hello") ...
  - datatype keyval = StrVal of string * string | IntVal of string * int ;
    - elements are: StrVal("foo","bar") , IntVal("foo",55) ...
  - datatype mylist = Nil | Cons of value * mylist
    - elements are: Nil , Cons (I(8) ,Nil) , Cons (R(1.0), Cons (I (8), Nil))
- General form

  datatype <name> = <clause> | … | <clause>
  <clause> ::= <constructor> | <constructor> of <type>

Type abbreviations

- We use *datatype* to define new types
- The keyword *type* can be used to define a type *abbreviation:*

```
- type int_pair = int * int ;
```

- The type inference will not report types as the defined abbrev.:

```
- val a = (3,5);
> val a = (3,5) : int * int
```

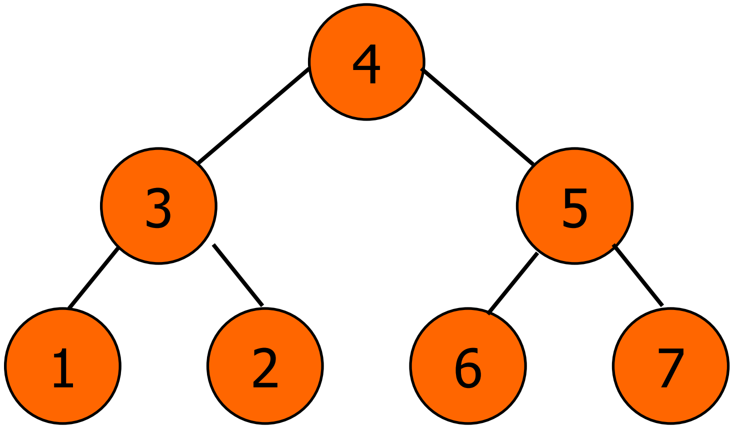
- We can force the use of type abbreviation:

```
- val a : int_pair = (3,5);
> val a = (3,5) : int_pair
```

Datatype and pattern matching

- Recursively defined data structure
  - datatype tree = Leaf of int | Node of int*tree*tree;

```
Node(4, Node(3,Leaf(1), Leaf(2)),
          Node(5,Leaf(6), Leaf(7))
      )
```



- Recursive function (sum)
  - fun sum (Leaf n) = n
    | sum (Node(n,t1,t2)) = n + sum(t1) + sum(t2);

Case expression

- Datatype

```
- datatype exp = Num of int | Var of var | Plus of exp*exp;
```

- Case expression

```
case e of Num(i)       =>  …  |
          Var(v)        => ….  |
          Plus(e1,e2) => …
- fun eval(e) = case e of  Num(i) => i
                         | Var(v) => lookUp(v)
                         | Plus(e1,e2) => eval(e1) + eval(e2)
```

- Case matching is done in order

- Use _ to catch all missing

```
- fun bintoString(i) = case x of 0 => “zero”
                          |     1 = > “one”
                          |     _ => “illegal value”;
> val bintoString = fn : int -> string
```

- Can also use _ in declarations if we don’t care about the value being matched

```
- fun hd(x::xs) = x ;
- fun hd(x::_) = x ;
```

insert: Three ”different” declarations

1.
```
fun insert (e, ls) =
    case ls of nil  => [e]
             | x::xs => if e>x then x::insert(e, xs) else
e::ls ;
```

2.
```
fun insert (e, nil)     = [e]
  | insert (e, x::xs) = if e>x then x::insert(e, xs)
                                         else e::(x::xs) ;
```

3.
```
fun insert (e: int, ls: int list) : int list =
   case ls of nil   => [e]
              | x::xs => if e>x then x::insert(e, xs) else
e::ls ;
```

Further reading

- Extra material on ML.
- See links on the course page:" Syllabus/achievement requirements "
  - Riccardo Pucella: *Notes on programming SML/NJ (Pensum/Syllabus :Secs. 1.1-1.3, 1.6, and sec. 2.)*
  - In Norwegian: Bjørn Kristoffersen: *Funksjonell programmering i standard ML; kompendium 61*, 1995.
  - **SML/NJ** http://www.smlnj.org/
  - Functions and types available at the top-level: http://www.smlnj.org/doc/basis/pages/top-level-chapter.html
- L.C. Paulson: *ML for the working programmer*

More on ML & Types

Volker Stolz
stolz@ifi.uio.no

Department of Informatics – University of Oslo

Initially by Gerardo Schneider.
Based on John C. Mitchell's slides (Stanford U.)

Outline

- ~~ML imperative constructs~~
- More recursive examples
- Higher-order functions
- Something about equality
- Something on the ML module system
- Types in programming
- Type safety

ML imperative constructs

- None of the constructs seen so far have side effects
  - An expression has a value, but evaluating it does not change the value of any other expression
- Assignment
  - Different from other programming languages:

    To separate side effects from pure expressions as much as possible

  - Restricted to *reference cells*

Variables and assignment

- General terminology: L-values and R-values
  - Assignment (pseudocode, not ML!) `y := x+3;`
    - Identifier on left refers to a *memory location*, called L-value
    - Identifier on right refers to *contents*, called R-value
- Variables
  - Most languages
    - A variable names a storage location
    - Contents of location can be read, can be changed
  - ML reference cell (L-value)
    - A reference cell has a different type than a value
    - Explicit operations to read contents or change contents
    - Separates naming (declaration of identifiers) from "variables"

ML reference cells

- Different types for location and contents

x : int	non-assignable integer value
y : int ref	location whose contents must be integer

- Operations

ref x	expression creating new cell containing value x
!y	returns the contents (value) of location y
y := x	places value x in reference cell y

- Examples

- val x = ref 0 ; create cell x with initial value 0

\> val x = ref 0 : int ref

- x := x+3; place value of x+3 in cell x; requires x:int

\> val it = () : unit (type is "unit" since it is an expression with side effects)

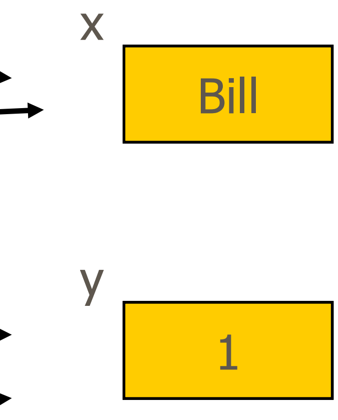
- x := !x + 3; add 3 to contents of x and store result in location x

\> val it = () : unit

- !x; > val it = 6 : int

ML examples

- Create cell and change contents
  - val x = ref "Bob";
  - x := "Bill";



- Create cell and increment
  - val y = ref 0;
  - y := !y + 1;
  - y := y + 1 Error!
- In summary:
  - x : int not assignable (like constant in other PL)
  - y : int ref assignable reference cell

Imperative programming in ML

```
val i = ref 0;
while !i < 5 do
    (i := !i +1 ;
     print("i is :"^Int.toString(!i)^"\n")
     );
```

- References
- In ML you evaluate a series of expressions
  - By evaluating $(e_1; e_2; . . . ;e_n)$, the expressions e_1 to e_n are evaluated from left to right
  - The result is the value of e_n. The other values are discarded
- While command : while e1 do e2
  - while e1 do e2 ≡ if e1 then (e2; while e1 do e2) else () ;
- print : string -> unit
  - print returns it : () but has a side effect.

Outline

- More examples on recursion
- Higher-order functions
- Something about equality
- Something on the ML module system
- Types in programming
- Type safety

More on list functions

- Writing a recursive function is not difficult, but what about efficiency?
- Example: Reverse a list
  (remember [1,2] @ [3,4] = [1,2,3,4])

```
fun  rev [] = []
   |  rev (x::xs)        = (rev xs) @ [x] ;
```

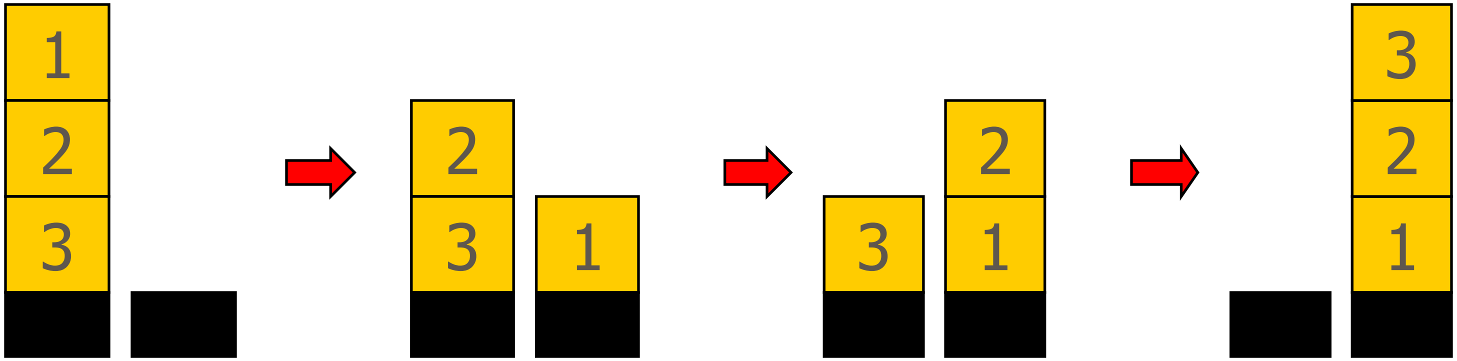
- Questions
  - How efficient is reverse?
  - Can you do this with only one pass through list?

More efficient reverse function

```
fun revAppend ([],ys) = ys
  | revAppend (x::xs,ys) = revAppend(xs,(x::ys)) ;

fun rev xs = revAppend(xs,[]);
```

Tail recursive function!



Two factorial functions

- Standard recursion

```
- fun fact n =
    if n = 0 then 1 else n * fact(n-1) ;
```

- Tail recursive (iteritative)

```
- fun facti(n,p) =
    if n = 0 then p else facti(n-1,n*p) ;
- fun fact n = facti(n,1) ;
```

- More examples in Pucella sec. 2.7

Outline

- More examples on recursion
- Higher-order functions
- Something about equality
- Something on the ML module system
- Types in programming
- Type safety

Monomorphism vs. Polymorphism

- *Monomorphic* means ”having only one form”, as opposed to *Polymorphic*
- A type system is monomorphic if each constant, variable, etc. has unique type
- Variables, expressions, functions, etc. are polymorphic if they ”allow” more than one type

Example. In ML, the *identity* function fn x => x is polymorphic: it has infinitely many types!

```
- fn x => x
> val it = fn : 'a -> 'a
```

Warning! The term ”polymorphism” is used with different specific technical meanings (more on this in ML-lecture 3)

Higher-order functions (functionals)

- In ML functions are computational values ("first-class objects")
  - can be constructed during execution
  - stored in data structures
  - passed as arguments to other functions
  - returned as values

A *functional* is a function that operates on other functions

- Programs are more concise and clear when using functionals
- Functionals on lists have been very popular in Lisp
- The use of functionals is a powerful tool for *modularisation* which is what gives FPLs one of its conceptual advantages (Hughes 1984)

Higher-order functions (functionals)

- Map: apply a function to every element in a list

```
- fun map (f, nil) = nil
   |    map (f, x::xs) = f(x) :: map (f,xs);
> val map = fn : ('a -> 'b) * 'a list -> 'b list


- fun incr x = x+1 ;
> val incr = fn : int -> int


- map (incr, [1,2,3]);        →        [2,3,4]


- map (fn x => x*x, [1,2,3]);       →        [1,4,9]
```

Higher-order functions (functionals)

- Map: apply a function to every element in a list

```
- fun map (f, nil) = nil
  |    map (f, x::xs) = f(x) :: map (f,xs);
> val map = fn : ('a -> 'b) * 'a list -> 'b list

    - fun bintoString(i) =
       case x of  0  => “zero”
                | 1  => “one”
                | _  => “illegal value”;

    > val bintoString = fn : int -> string

   - map (bintoString , [1,0,2,0]);
   > val it = ["one","zero","illegal value","zero"] : string list
```

Higher-order functions (functionals)

- filter: apply a predicate to every element of list

```
- fun filter (p, nil) = nil
     | filter (p, (x::xs))  = if p(x) then x :: (filter (p,xs))
                                  else filter (p,xs) ;

- val odd = fn : int -> bool

- val mylist = [1,2,3,4,5,6,7,8];
- filter (odd, mylist);                 > val it = [1,3,5,7] : int list

- map (fn x => x*x, (filter(odd,mylist)));

                                   > val it = [1,9,25,49] : int list

- val pairs = [(1,2),(4,3),(8,9),(0,9),(0,0),(5,1)] ;

-   filter ((op <) , pairs);

              > val it = [(1,2),(8,9),(0,9)] : (int * int) list
```

Curried functions

- A function can have only one argument
  - tuples are used for more than one argument
- Multiple arguments may be realized by giving a function as a result
  - *Currying* -> after the logician Haskell B. Curry
- A function over pairs has type

  ’a * ’b -> ’c

  while a curried function has type

  ’a -> (’b -> ’c)
- A curried function allows *partial application*: applied to its 1st argument (of type ’a), it results in a function of type ’b -> ’c

Curried functions

- Example: function to add two numbers

```
- fun  pluss (x,y) = x + y ;
> val pluss = fn : int * int -> int
- pluss (2,3) ;
➢ val it = 5 : int
```

- Curried version of the same function

```
- fun cPluss x y = x + y ;
> val cPluss = fn : int -> int -> int
- cPluss 2 3 ;
> val it = 5 : int
- val addTwo = cPluss 2 ;
> val addTwo = fn : int -> int
- addTwo 5 ;
> val it = 7 : int
```

Curried functions

- Curry and uncurry

```
- fun curry f x y = f (x,y) ;
> val curry = fn : ('a * 'b -> 'c) -> 'a -> 'b -> 'c

- fun uncurry f (x,y) = f x y  ;
> val uncurry = fn : ('a -> 'b -> 'c) -> 'a * 'b -> 'c
```

Example: the map function

- Recall that map can be defined as

```
fun map (f, nil) = nil
   | map (f, x::xs) = f(x) :: map (f,xs);
   > val map = fn : ('a -> 'b) * 'a list -> 'b list

   - map (fn x => x+1, [1,2,3]);
   > val it = [2,3,4] : int list
```

- By currying it, we can define map as

```
fun map f nil = nil
   | map f (x::xs) = (f x) :: map f xs;
 > val map = fn : ('a -> 'b) -> 'a list -> 'b list

 - map (fn x => x+1)  [1,2,3];
     > val it = [2,3,4] : int list
```

More on the map function

- We can have a function having as argument a function which has another function as an argument
- Thanks to currying, we can combine functionals to work on lists of lists

  Example:

```
- map (map (fn x => x+1))  [[1], [1,2], [1,2,3]];
→ [ map (fn x => x+1) [1], map (fn x => x+1)[1,2], map (fn x => x+1)[1,2,3]]
→ [ [2], [2,3], [2,3,4]]
```

What does it give as a result?

```
> val it = [[2],[2,3], [2,3,4]] : int list list
```

Outline

- More recursive examples
- Higher-order functions
- Something about equality
- Something on the ML module system
- Types in programming
- Type safety

Equality

- Equality in (S)ML is defined for many types but not all – E.g., it is defined for:
  - Integers
  - Booleans
  - Strings
  - Characters

- What about floating points (reals), compund types (tuples, records, lists), functions, abstract data types, etc?

Equality

◆ When are two expressions equal?
  - The so-called *Leibniz's Principle of the Identity of Indiscernables*:

”e1 and e2 are **equal** iff they cannot be distinguished by any operation in the language”

”e1 and e2 are **distinct** iff there is some way to tell them apart”

◆ What is difficult about Leibniz's Principle?

Problems with Equality

- Equality, as defined by Leibniz's principle, is undecidable

In general, there is no program which determines whether two expressions are equal in Leibniz's sense.

Also:

- Problems with reference cells (aliasing)

```
- val s = ref 1 ; val t = ref 1 ;
- s = t ;                > false
- !s = !t                > true
- val s = t ;
- s = t ;                > true
```

- Polymorphic equality complicates the compiler

Equality Types

- An equality type is a type admiting equality test
- Types admiting equality in (S)ML
  - *int, bool, char, string*
  - *tuples* and *records*, **if all their components admit equality**
  - *datatypes*, **if every constructor's parameter admits equality**
  - *lists* admit equality **if the underlying element type admits equality**
    - Two lists are equal if they have the same length and the same elements in corresponding positions

Equality Types (cont.)

- Do **not** admit equality in (S)ML
  - *reals*
  - *functions*
  - *tuples, records and datatypes* not mentioned in the previous slide
  - *abstract data types*
- Equality type variable: ”a

```
- fun equals (x,y) = if x = y then true else false ;
> stdIn:7.25 Warning: calling polyEqual
val equals = fn : ''a * ''a -> bool
```

Equality: Examples

- Equality tests on functions is not computable since

  f = g iff for all x, f(x) = g(x)

- No ”standard” notion of equality for an abstract type
  - What is supposed to be the equality on *trees*? Is it defined structurally? Is it over the list of their elements? By DFS or BFS?
- Ex:

```
fun find x nil = false
   | find x (y :: ys) =   x = y orelse find x ys ;

> = stdIn:30.31 Warning: calling polyEqual
val find = fn : "a -> "a list -> bool
```

(don’t worry, only a performance issue)

Outline

- ML imperative constructs
- More recursive examples
- Higher-order functions
- Something about equality
- Something on the ML module system
- Types in programming
- Type safety

Modularity: Basic Concepts

- Component
  - Meaningful program unit
    - Function, data structure, module, …
- Interface
  - Types and operations defined within a component that are visible outside the component
- Specification
  - Intended behavior of component, expressed as property observable through interface
- Implementation
  - Data structures and functions inside component

Example: Function Component

- Component
  - Function to compute square root
- Interface
  - function sqrt (float x) returns float
- Specification
  - If x>1, then sqrt(x)*sqrt(x) ≈ x.
- Implementation

```
float sqroot (float x){
   float y = x/2; float step=x/4; int i;
   for (i=0; i<20; i++){if ((y*y)<x) y=y+step; else y=y-step; step = step/2;}
   return y;
}
```

Something on ML Modules

- Signatures and structures are part of the standard *ML module system*
- An ML structure is a module, which is a collection of:
  - Types
  - Values
  - Structure declarations
- Signatures are module interfaces
  - Kind of ”type” for a structure

Example: Point

- Signature definition (Interface)

```
signature POINT =
 sig
   type point
   val mk_point : real * real -> point  (*constructor*)
   val x_coord : point -> real          (*selector*)
   val y_coord : point -> real          (*selector*)
   val move_p : point * real * real -> point
 end;
```

Example: Point (cont.)

- Structure definition (Implementation)

```
structure pt : POINT =
 struct
   type point = real * real
   fun mk_point(x,y) = (x,y)
   fun x_coord(x,y) = x
   fun y_coord(x,y) = y
   fun move_p((x,y):point,dx,dy) = (x+dx, y+dy)
 end;
```

- To be able to use the implementation:

```
 - open pt;
```

Example: Point (cont.)

Open the structure by writing open <structname>

```
- open pt;
 ...
```

After that you may use the struct operations

```
- val p1 = mk_point(4.3, 6.56);
> val p1 = (4.3,6.56) : point
- y_coord (p1);
> val it = 6.56 : real
- move_p (p1, 3.0, ~1.0);
> val it = (7.3,5.56) : point
```

You may use the struct without opening it by prefixing a function with the struct name.

```
- pt.mk_point(1.0,1.0);
> val it = (1.0,1.0) : point
```

E.g. we would like to use the min function to get the smallest of two ints.

```
- min(1,2);
> stdIn:1.1-1.4 Error: unbound variable or constructor: min
```

The function is defined in the Int struct so we must use Int as a prefix

```
- Int.min(1,2);
> val it = 1 : int
```

See: http://www.smlnj.org/doc/basis/pages/sml-std-basis.html for an overview of the structures and signatures in The Standard ML Basis Library. Follow the link: Top-level Environment to see which functions are available in the top level environment, i.e. which you can use without prefixes.

Outline

- More recursive examples
- Higher-order functions
- Something about equality
- Something on the ML module system
- Types in programming
- Type safety

Type

A type is a collection of computational entities sharing some common property

◆ Examples

- Integers
- [1 .. 100]
- Strings
- int → bool
- (int → int) → bool

◆ "Non-examples"

- {3, true, 5.0}
- Even integers
- {f:int → int | if x>3 then f(x) > x*(x+1)}

Distinction between types and non-types is language dependent

Uses for types

- Program organization and documentation
  - Separate types for separate concepts
    - E.g., customer and accounts (banking program)
  - Types can be checked, unlike program comments
- Identify and prevent errors
  - Compile-time or run-time checking can prevent meaningless computations such as 3 + true - “Bill”
- Support optimization
  - Short integers require fewer bits
  - Access record component by known offset

Type errors

- Hardware error
  - Function call x() (where x is not a function) may cause jump to instruction that does not contain a legal op code
    - If x = 512, executing x() will jump to location 512 and begin execute "instructions" there
- Unintended semantics
  - int_add(3, 4.5): Not a hardware error, since bit pattern of float 4.5 can be interpreted as an integer

General definition of type error

- A *type error* occurs when execution of program is not faithful to the intended semantics
- Type errors depend on the concepts defined in the language; not on ***how*** the program is executed on the underlying software
- All values are stored as sequences of bits
  - Store 4.5 in memory as a floating-point number
    - Location contains a particular bit pattern
  - To interpret bit pattern, we need to know the type
  - If we pass bit pattern to integer addition function, the pattern will be interpreted as an integer pattern
    - Type error if the pattern was intended to represent 4.5

Subtyping

- Subtyping is a relation on types allowing values of one type to be used in place of values of another
  - ***Substitutivity:*** If A is a subtype of B (A<:B), then any expression of type A may be used without type error in any context where B may be used
- In general, if f: A -> B, then f may be applied to x if x: A
  - Type checker: If f: A -> B and x: C, then C = A
- In languages with subtyping
  - Type checker: If f: A -> B and x: C, then C <: A

Remark: No subtypes in ML!

Outline

- More recursive examples
- Higher-order functions
- Something about equality
- Something on the ML module system
- Types in programming
- Type safety

Type safety

- A Prog. Lang. is *type safe* if no program can violate its type distinction
  - E.g. use an integer as a function
  - Access memory not allocated to the program.
- Examples of not type safe language features:
  - Type casts (a value of one type used as another type)
    - Use integers as functions (jump to a non-instruction or access memory not allocated to the program) (C)
  - Pointer arithmetic
    - *(p) has type A if p has type A*
    - x = *(p+i) what is the type of x?
  - Explicit deallocation and dangling pointers
    - Allocate a pointer p to an integer, deallocate the memory referenced by p, then later use the value pointed to by p

Relative type-safety of languages

- Not safe: BCPL family, including C and C++
  - Casts; pointer arithmetic
- Almost safe: Algol family, Pascal, Ada.
  - Explicit deallocation; dangling pointers
    - No language with explicit deallocation of memory is fully type-safe
- Safe: Lisp, ML, Smalltalk, Java, Haskell
  - Lisp, Smalltalk: dynamically typed
  - ML, Haskell, Java: statically typed

Compile-time vs. run-time checking

- Lisp uses run-time type checking

  (car x)   check first to make sure x is list
- ML uses compile-time type checking

  f(x)   must have f : A → B and x : A
- Basic tradeoff
  - Both prevent type errors
  - Run-time checking slows down execution (compiled ML code, up-to **4** times faster than Lisp code)
  - Compile-time checking restricts program flexibility

    Lisp list: elements can have different types

    ML list: all elements must have same type
- Combination of Compile/Run-time eg. Java
  - Static type checking to distinguish arrays and integers
  - Run-time checking to detect array bounds errors

Compile-time type checking

- ***Sound*** type checker: no program with error is considered correct
- ***Conservative*** type checker: some programs without errors are considered to have errors
- Static typing is always conservative

```
if  (possible-infinite-run-expression)
        then  (expression-with-type-error)
        else   (expression-with-type-error)
```

Cannot decide at compile time if run-time error will occur

(from the undecidability of the Turing machine's halting problem)

Remarks – Further reading

- Mitchell doesn't cover the material presented on Equality – See section 2.9 of Pucella's notes
- *signatures* and *structures* are part of ML Module system. See section 9.3.2 of Mitchell's book
- Types: Mitchell's section 6.1, 6.2
- Imperative programming in ML: See chapter 8 of Paulson's book