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Polymorphism and Type Inference

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Initially by Gerardo Schneider.
Based on John C. Mitchell's slides (Stanford U.)

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Compile-time vs. run-time checking

- Lisp uses run-time type checking
  (car x)   check first to make sure x is list
- ML uses compile-time type checking
  f(x)   must have $f : A \rightarrow B$ and $x : A$
- Basic tradeoff
  - Both prevent type errors
  - Run-time checking slows down execution (compiled ML code, up-to **4** times faster than Lisp code)
  - Compile-time checking restricts program flexibility
    Lisp list: elements can have different types
    ML list: all elements must have same type
- Combination of Compile/Run-time eg. Java
  - Static type checking to distinguish arrays and integers
  - Run-time checking to detect array bounds errors

Compile-time type checking

- ***Sound*** type checker: no program with error is considered correct
- ***Conservative*** type checker: some programs without errors are considered to have errors
- Static typing is always conservative

  if (possible-infinite-run-expression)

  then (expression-with-type-error)

  else (expression-with-type-error)

  Cannot decide at compile time if run-time error will occur

  (from the undecidability of the Turing machine's halting problem)

Outline

- **Polymorphism*s***
  - **parametric** polymorphism
  - ***ad hoc*** polymorphism
  - **subtype** polymorphism

- Type inference

- Type declaration

Polymorphism: three forms

- Parametric polymorphism
  - Single function may be given (infinitely) many types
  - The type expression involves *type variables*

Example: in ML the identity function is polymorphic

```
- fn x => x;
> val it = fn : 'a -> 'a
```

This pattern is called *type scheme*

Type variable may be replaced by *any* type

An *instance* of the type scheme may give:

int→int, bool→bool, char→char,
int*string*int→int*string*int, (int→real)→(int→real), ...

Polymorphism: three forms

- Parametric polymorphism
  - Single function may be given (infinitely) many types
  - The type expression involves *type variables*

Example: polymorphic sort

```
- sort : ('a * 'a -> bool) * 'a list -> 'a list

- sort((op<),[1,7,3]);
> val it = [1,3,7] : int list
```

Polymorphism: three forms (cont.)

- Ad-hoc polymorphism (or Overloading)
  - A single symbol has two (or more) meanings (it refers to more than one algorithm)
  - Each algorithm may have different type
  - Overloading is resolved at compile time
  - Choice of algorithm determined by type context

Example: In ML, + has 2 different associated implementations: it can have types int*int→int and real*real→real, no others

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Polymorphism: three forms (cont.)

- Subtype polymorphism
  - The subtype relation allows an expression to have many possible types
  - Polymorphism not through type parameters, but through subtyping:
    - If method ***m*** accept any argument of type ***t*** then ***m*** may also be applied to any argument from any subtype of ***t***

REMARK 1: In OO, the term “polymorphism” is usually used to denote subtype polymorphism (ex. Java, OCAML, etc)

REMARK 2: ML does not support subtype polymorphism!

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Parametric polymorphism

- Explicit: The program contains type variables
  - Often involves explicit instantiation to indicate how type variables are replaced with specific types
  - Example: C++ templates
- Implicit: Programs do not need to contain types
  - The type inference algorithm determines when a function is polymorphic and instantiate the type variables as needed
  - Example: ML polymorphism

Parametric Polymorphism: ML vs. C++

- C++ function template
  - Declaration gives type of funct. arguments and result
  - Place declaration inside a template to define type variables
  - Function application: type checker does instantiation automatically
- ML polymorphic function
  - Declaration has no type information
  - Type inference algorithm
    - Produce type expression with variables
    - Substitute for variables as needed

ML also has module system with explicit type parameters

Example: swap two values

- C++

```
void swap (int& x, int& y){
    int tmp=x;  x=y;  y=tmp;
}
```

```
template <typename T>
void swap(T& x, T& y){
    T tmp=x; x=y; y=tmp;
}
```

- Instantiations:
  - int i,j; … swap(i,j); //use swap with T replaced with int
  - float a,b;… swap(a,b); //use swap with T replaced with float
  - string s,t;… swap(s,t); //use swap with T replaced with string

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Example: swap two values

- ML

```
- fun swap(x,y) =
      let val z = !x in x := !y; y := z end;
> val swap = fn : 'a ref * 'a ref -> unit

- val a = ref 3 ; val b = ref 7 ;
> val a = ref 3 : int ref
> val b = ref 7 : int ref
- swap(a,b) ;
> val it = () : unit
- !a ;
> val it = 7 : int
```

Remark: Declarations look similar in ML and C++,
but compile code is very different!

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Parametric Polymorphism: Implementation

- C++
  - Templates are instantiated at program link time
  - Swap template may be stored in one file and the program(s) calling swap in another
  - Linker duplicates code for each type of use
- ML
  - Swap is compiled into one function (no need for different copies!)
  - Typechecker determines how function can be used

Parametric Polymorphism: Implementation

- Why the difference?
  - C++ arguments passed by reference (pointer), but local variables (e.g. tmp, of type T) are on stack
    - Compiled code for swap depends on the size of type T => Need to know the size for proper addressing
  - ML uses pointers in parameter passing (*uniform data representation*)
    - It can access all necessary data in the same way, regardless of its type; Pointers are the same size anyway
- Comparison
  - C++: more effort at link time and bigger code
  - ML: run more slowly, but give smaller code and avoids linking problems
  - Global link time errors can be more difficult to find out than local compile errors

ML overloading

- Some predefined operators are overloaded
  - + has types int*int→int and real*real→real
- User-defined functions must have unique type

```
- fun plus(x,y) = x+y;
```
(compiled to int or real function, not both)

In SML/NJ:

```
- fun plus(x,y) = x+y;
> val plus = fn : int * int -> int
```

If you want to have plus = fn : real * real -> real you must provide the type:

```
- fun plus(x:real,y:real) = x+y;
```

ML overloading (cont.)

- Why is a unique type needed?
  - Need to compile code implies need to know which **+** (different algorithm for distinct types)
  - Overloading is *resolved* at compile time
    - The compiler must choose one algorithm among all the possible ones
    - Automatic conversion is possible (**not** in ML!)
    - But in e.g. Java : consider the expression (1 + “foo”) ;
  - Efficiency of type inference – overloading complicates type checking
  - Overloading of user-defined functions is not allowed in ML!
  - User-defined overloaded function can be incorporated in a fully-typed setting using *type classes* (Haskell)

Parametric polymorphism vs. overloading

- Parametric polymorphism
  - One algorithm for arguments of many different types
- Overloading
  - Different algorithms for each type of argument

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Outline

- Polymorphism*s*
- **Type inference**
- Type declaration

Type checking and type inference

- Type checking: The process of checking whether the types declared by the programmer "agrees" with the language constraints/ requirement
- Type inference: The process of determining the type of an expression based on information given by (some of) its symbols/sub-expressions
  - Provides a flexible form of compile-time/static type checking
- Type inference naturally leads to polymorphism, since the inference uses type variables and some of these might not be resolved in the end

ML is designed to make type inference tractable

(one of the reason for not having subtypes in ML!)

Type checking and type inference

- Standard type checking

```
int f(int x) { return x+1; };
int g(int y) { return f(y+1)*2;};
```

  - Look at body of each function and use declared types of identifies to check agreement

- Type inference

  ~~int~~ f(~~int~~ x) { return x+1; };

  ~~int~~ g(~~int~~ y) { return f(y+1)*2;};

  - Look at code without type information and figure out what types could have been declared

Type inference algorithm: Some history

- Usually known as Milner-Hindley algorithm
- **1958:** Type inference algorithm given by H.B. Curry and Robert Feys for the *typed lambda calculus*
- **1969:** Roger Hindley extended the algorithm and proved that it gives the most general type
- **1978:** Robin Milner -independently of Hindley- provided an equivalent algorithm (for ML)
- **1985:** Luis Damas proved its completeness and extended it with polymorphism

ML Type Inference

- Example

  - fun f(x) = 2+x;

  > val f = fn : int → int

- How does this work?
  - + has two types: int*int → int, real*real→real
  - 2 : int, has only one type
  - This implies + : int*int → int
  - From context, need x: int
  - Therefore f(x:int) = 2+x has type int → int

Overloaded + is unusual - Most ML symbols have unique type

In many cases, unique type may be polymorphic

ML Type Inference

- Example
  - fun f(g,h) = g(h(0));

- How does this work?
  - h must have the type: $int \rightarrow$ ´a, since 0 is of type int
  - this implies that g must have the type: ´a $\rightarrow$ ´b
  - Then f must have the type:

    (´a $\rightarrow$ ´b) * (int $\rightarrow$ ´a) $\rightarrow$ ´b

Information from type inference

- An interesting function on lists

```
- fun reverse (nil) = nil
  |     reverse (x::lst) = reverse(lst);
```

- Most general type

  > reverse : 'a list → 'b list

- What does this mean?

  Since reversing a list does not change its type, there must be an error in the definition

  x is not used in "reverse(lst)"!

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The type inference algorithm

$f(x) = 2+x$ equiv $f = \lambda x.\ (2+x)$ equiv $f = \lambda x.\ ((\text{plus } 2)\ x)$

- Example

  - fun f(x) = 2+x;

    (val f = fn x => 2+x ;)

  > val f = fn : int $\rightarrow$ int

Detour: the λ-calculus

- "Entscheidungsproblem": David Hilbert (1928): Can any mathematical problem be solved (or decided) computationally?
- Subproblem: Formalize the notion of decidability or computability
- Two formal systems/models:
  - Alonzo Church (1936) - λ-calculus
  - Alan M. Turing (1936/37) – Turing machine
- λ-calculus → functional programming languages
- Turing-machines → imperative, sequential programming languages
- The models are equally strong (they define the same class of computable functions) (Turing 1936)

Detour: the λ-calculus

- Two ways to construct terms:
  - Application: F A (or F(A))
  - Abstraction: $\lambda x.e$

    If e is an expression on x, then $\lambda x.e$ is a function
    Ex:

    e = 3x+4 .

    $\lambda x.e = \lambda x.(3x+4)$ (fn x => (3x+4))

    compare with "school book" notation:

    if $f(x) = 3x+4$ then $f = \lambda x.(3x+4)$

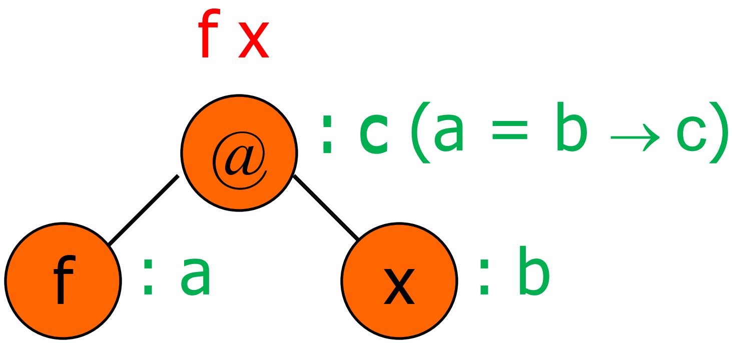
- Rules for computation

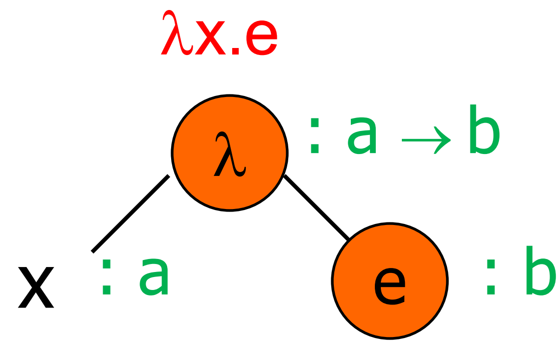
  $(\lambda x.(3x+4))\ 2 \rightarrow (3*2) + 4$

  $\lambda x.(3x+4) \rightarrow \lambda y.(3y+4)$ (α – conversion)

  $(\lambda x.(3x+4))\ 2 \rightarrow (3*2) + 4 \rightarrow 10$ (β – reduction)

Application and Abstraction





- **Application f x**
  - f must have function type domain→ range
  - domain of f must be type of argument x (b)
  - the range of f is the result type (c)
  - thus we know that $a = b \to c$
- **Abstraction λx.e** (fn x => e)
  - The type of λx.e is a function type domain→ range
  - the domain is the type of the variable x (a)
  - the range is the type of the function body e (b)

The type inference algorithm

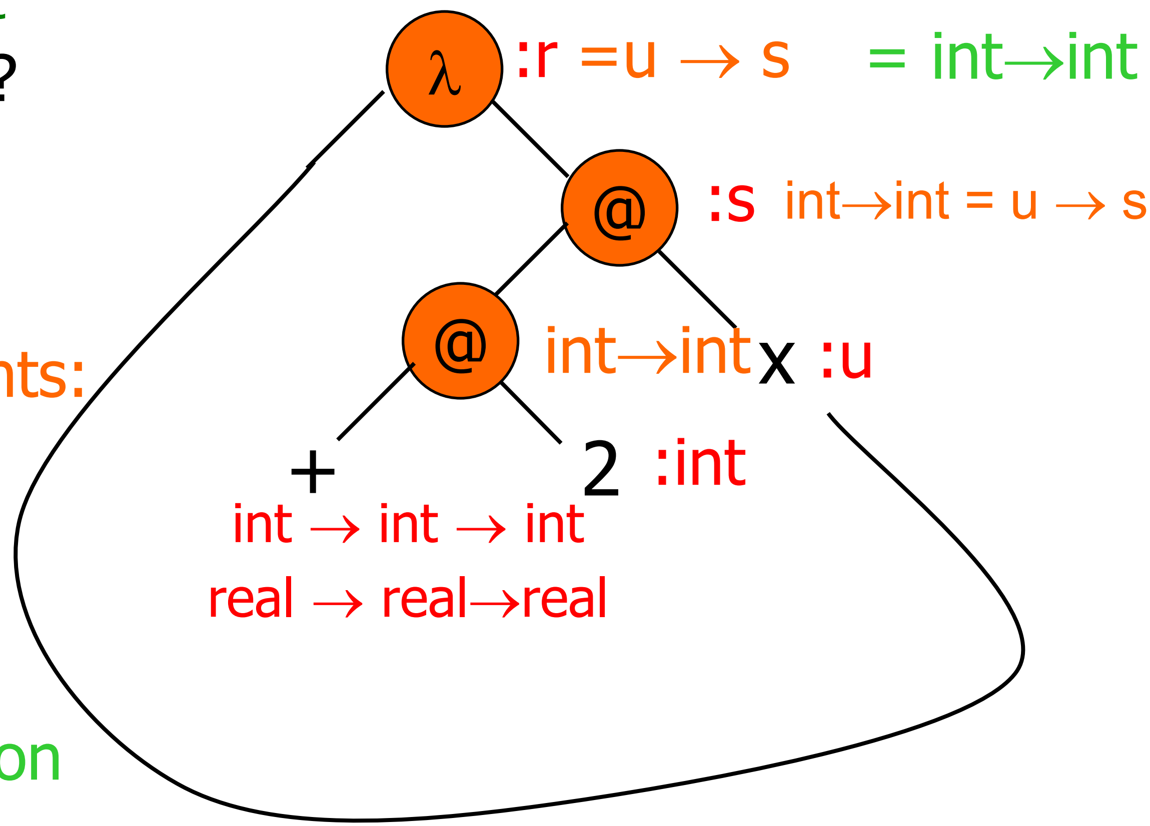
f(x) = 2+x equiv f = λx. (2+x) equiv f = λx. ((plus 2) x)

- Example
  - fun f(x) = 2+x;
  - (val f = fn x => 2+x ;)

  > val f = fn : int → int
- How does this work?

1. Assign types to expressions
2. Generate constraints:
   - int→int = u → s
   - r = u → s
3. Solve by unification/substitution

Graph for λx. ((+ 2) x)



Types with type variables

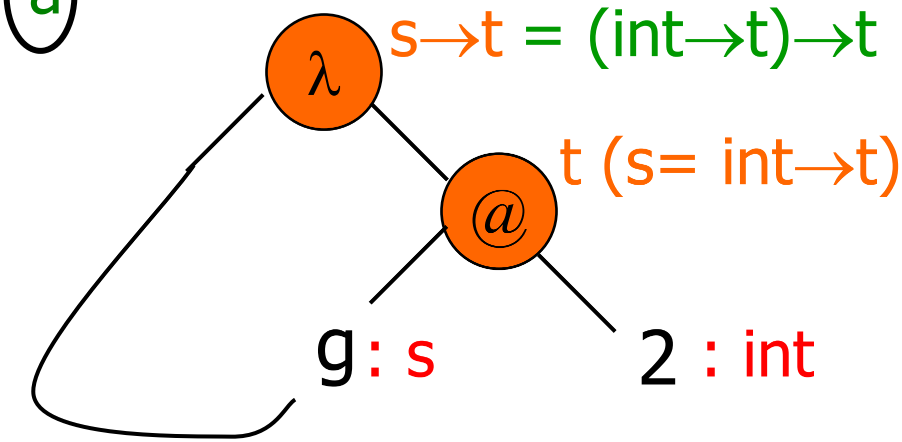
’a is syntax for “type variable” (t in the graph)

- Example
  - fun f(g) = g(2);

  > val f = fn : (int→’a)→’a
- How does this work?

1. Assign types to leaves
2. Propagate to internal nodes and generate constraints
3. Solve by substitution

Graph for λg. (g 2)



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Use of Polymorphic Function

- Function
  - fun f(g) = g(2);
  > val f = fn : (int→'a)→'a
- Possible applications

g may be the function:
- fun add(x) = 2+x;
> val add = fn : int → int
Then:
- f(add);
> val it = 4 : int

g may be the function:
- fun isEven(x) = ...;
> val it = fn : int → bool
Then:
- f(isEven);
> val it = true : bool

Recognizing type errors

- Function
  - fun f(g) = g(2);

  > val f = fn : (int→'a)→'a
- Incorrect use
  - fun not(x) = if x then false else true;

  > val not = fn : bool → bool

  - f(not);

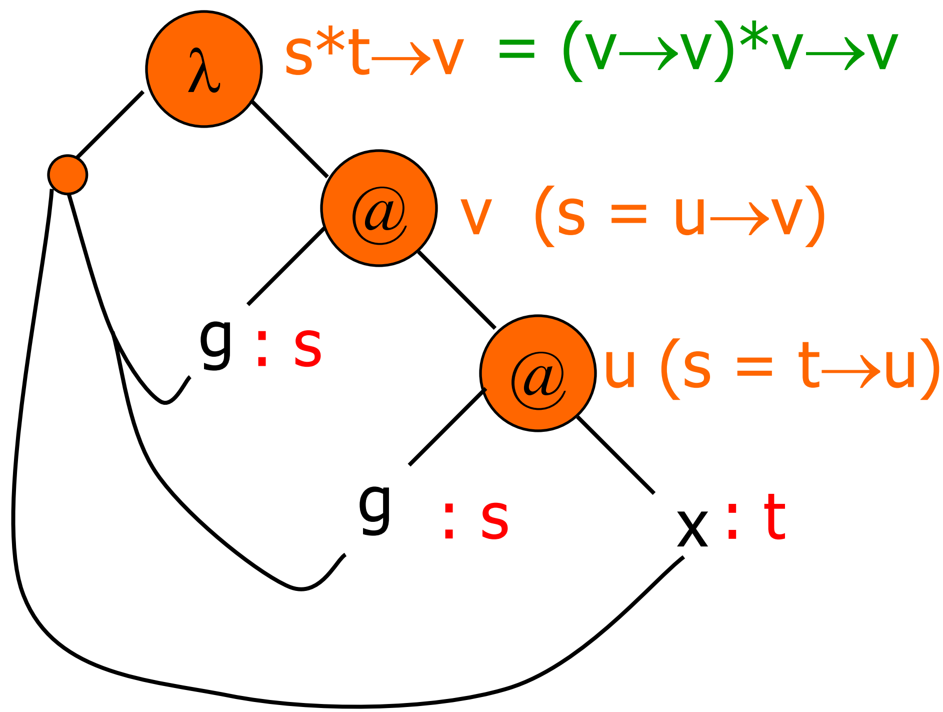
  Why?

  Type error: cannot make bool → bool = int → 'a

Another type inference example

- Function Definition
  - fun f(g,x) = g(g(x));

Graph for λ⟨g,x⟩. g(g x)



Assign types to leaves

Propagate to internal nodes and generate constraints:

s = t→u, s = u→v

t=u,u=v

t=v

Solve by substitution

Multiple clause function

- Datatype with type variable

  - datatype ’a list = nil | cons of ’a*(’a list);

  > nil : ’a list

  > cons : ’a*(’a list) → ’a list

- Polymorphic function

  - fun append(nil,l) = l

    | append (x::xs,l) = x:: append(xs,l);

  > val append= fn: ‘a list * ‘a list → ’a list

- Type inference
  - Infer separate type for each clause

    append: ‘a list * ‘b -> ‘b

    append: ‘a list * ‘b -> ‘a list
  - Combine by making the two types equal (if necessary) ‘b = ‘a list

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Main points about type inference

- Compute type of expression
  - Does not require type declarations for variables
  - Find *most general type* by solving constraints
  - Leads to polymorphism
- Static type checking without type specifications
- May lead to better error detection than ordinary type checking
  - Type may indicate a programming error even if there is no type error (example following slide).

Type inference and recursion

- Function definition
  - fun sum(x) = x + sum(x-1);

  > val sum= fn : ’int→’int

  sum = λx .((+ x) (sum((- x) 1)))

Outline

- Polymorphism*s*
- Type inference
- **Type declaration**

Type declaration

- Transparent: alternative name to a type that can be expressed without this name

- Opaque: new type introduced into the program, different to any other

ML has both forms of type declaration

Type declaration: Examples

◆ Transparent ("type" declaration)

```
- type Celsius = real;
- type Fahrenheit = real;
```

```
- fun toCelsius(x) = ((x-32.0)*0.5556);
> val toCelsius = fn : real → real
```

More information:

```
- fun toCelsius(x: Fahrenheit) = ((x-32.0)*0.5556): Celsius;
> val toCelsius = fn : Fahrenheit → Celsius
```

- Since Fahrenheit and Celsius are synonyms for real, the function may be applied to a real:

```
- toCelsius(60.4);
> val it = 15.77904 : Celsius
```

Type declaration: Examples

- Opaque ("datatype" declaration)

```
- datatype A = C of int;
- datatype B = C of int;
```

  - A and B are different types
  - Since B declaration follows A decl.: C has type int→B

  Hence:

```
- fun f(x:A) = x: B;
> Error: expression doesn't match constraint [tycon mismatch]
expression: A constraint: B
in expression:  x: B
```

Equality on Types

Two forms of type equality:

- Name type equality: Two type names are equal in type checking only if they are the same name
- Structural type equality: Two type names are equal if the types they name are the same

Example: Celsius and Fahrenheit are structurally equal although their names are different

Remarks – Further reading

- More on subtype polymorphism (Java): Mitchell's Section 13.3.5