

Logic - hrj

Arguing from assumptions

Language and calculus

Four important theorems

Cut elimination and interpolation

Language and calculus

- **Language**
 - **Signature** – literals, terms
 - **Logical superstructure** – connectives, quantifiers, equality
- **Calculus**
 - **Syntactic rules for sequents** – a tree of sequents
 - **Termination conditions** – branches contain axioms

Given a formula we get a tree of sequents above it.

Valid – satisfiable – falsifiable - contradictory

Important theorems

- **Completeness** (Skolem 1922)
 - The tree of sequents either contains a branch without axioms or all branches contain an axiom
- **Incompleteness** (Gödel 1930)
 - Theory of a datastructure – not done here
- **Cut elimination** (Gentzen 1936)
 - Using extra assumptions - this week
- **Interpolation** (Craig 1950)
 - Investigating threads in tree of sequents – next week

Cut elimination

- If G, F and $G, -F$ are derivable, then so is G
- Problems
 - Obstructs automating arguments
 - Proofs with cuts are short – same as using auxiliary lemmas. Without cut proofs may be practically impossible.

Interpolation

- If G, H is derivable, then there is I in the language common to both G and H such that both G, I and $\neg I, H$ are derivable
- This comes from an analysis of the threads in the tree given by the derivation of G, H