

INF4171 / INF3170

Exercises, week 35

Exercise 1

What is the meaning of the statement “have a nice day”? Can you find other syntactic representations of this meaning (or something close)?

Exercise 2

Proove, by structural induction, that propositional formulae contain 0 or more connectives.

Exercise 3

The operator NAND is defined as $A \text{ NAND } B \equiv \neg(A \wedge B)$. Write the other connectives (\neg , \wedge , \vee and \rightarrow) in terms of only NAND. Are the equivalences based on syntax or semantics?

Exercise 4

Explain the connection between satisfiable formulae and contradictions, and between falsifiable formulae and tautologies.

Exercise 5

For the following formulae, use a sequent calculus to either prove the formula to be valid or create a counter model.

- a. $(P \rightarrow Q) \rightarrow (\neg P \rightarrow \neg Q)$
- b. $(P \rightarrow Q) \rightarrow Q$
- c. $P \rightarrow (Q \rightarrow P)$
- d. $(P \rightarrow Q) \rightarrow P$

Exercise 6

Remove one of the rules in the sequent calculus. Explain why the resulting system is no longer complete.

Exercise 7

Give an example of a rule that, if added to the sequent calculus, would make the resulting system unsound.

Norwegian translations

Completeness Kompletthet¹

Contradiction Selvmotsigelse

Counter model Motmodell

Falsifiable Falsifiserbar

Propositional formula Utsagnslogisk formel

Propositional logic Utsagnslogikk

Satisfiable Oppfylldbar

Sequent Sekvent

Sequent calculus Sekventkalkyle

Soundness Sunnhet

Tautology Tautologi

Valid Gyldig

¹Here, completeness is translated to “kompletthet”. In this setting, completeness refers to a calculus’ ability to prove every valid formula. Where completeness refers to every sentence being either a tautology or a contradiction, completeness is translated “fullstendighet”.