Homework #7

For Friday, March 11

1. Finish reading Chapter 1 of van Dalen and start reading Sections 2.1–2.3.

⋆ 2. Do parts a, b, and e of problem 3 on page 39 of van Dalen.

3. Do problem 4 on page 39. Hints: For 4a, remember that if α and β are any formulas, from β you can conclude $\alpha \to \beta$. 4b is tricky, because it is not intuitionistically valid; you will need to use RAA. Note that from $\neg\alpha$ you can conclude $\alpha \to \beta$ using *ex falso* (show how).

4. Do problems 5, 7, and 8 on pages 39–40.

⋆ 5. Do problem 1 on page 47. If you claim the set is inconsistent, show that you can prove a contradiction from those assumptions. If you claim the set is consistent, demonstrate this by providing a valuation under which all the formulas are true. (Note that the completeness theorem implies that if a set of formulas is consistent, there will always be such a valuation.)

6. Do problems 2 and 3 on page 47.

7. A formula φ is said to be *independent* of a set of formulas Γ if $\Gamma \nvdash \varphi$ and $\Gamma \nvdash \neg\varphi$. Suppose Γ is a consistent set of formulas, φ is independent of Γ, and ψ is independent of $\Gamma \cup \{\varphi\}$. Show that there are at least three different maximally consistent sets containing Γ.

⋆ 8. Find a consistent set Γ that is not maximally consistent, but has the property that there is only one maximally consistent set containing it. In fact, find such a set Γ with the additional property that for some natural number k, every formula in Γ has length at most k.

∘ 9. Do problem 4 on page 47 of van Dalen.

∘ 10. Do problem 5 on page 48. In effect, you will be describing a computer program that prints out propositional formulas ad infinitum, in such a way that every propositional formula is printed sooner or later.

⋆ 11. Do problem 6 on page 48. Van Dalen's wording is awkward. What you need to prove is this: Suppose Γ is a consistent set of formulas with the property that for every formula φ, either $\varphi \in \Gamma$ or $\neg\varphi \in \Gamma$. Then Γ is maximally consistent.

⋆ 12. Show that if Γ is any consistent set, and φ is any formula, then either $\Gamma \cup \{\varphi\}$ or $\Gamma \cup \{\neg\varphi\}$ is consistent. (Hint: suppose they are both inconsistent…)

13. Say that a set of formulas Γ is finitely satisfiable if every finite subset of Γ is satisfiable. Note that the compactness theorem states

    > For every set of formulas Γ, if Γ is finitely satisfiable then Γ is satisfiable.

    Prove (directly) that if Γ is a finitely satisfiable set of formulas and φ is any formula, then either $\Gamma \cup \varphi$ or $\Gamma \cup \{\neg\varphi\}$ is finitely satisfiable.

14. Do problems 8 and 9 on page 48.

15. Do problem 11 on page 48.